High Frequency Tail Risk*

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Abstract

This paper proposes an alternative way to measure high-frequency Tail Risk directly extracted from stocks returns: A risk-neutral mean-adjusted expected shortfall. We rely on a non-parametric estimator for the state price density based on Hellinger's distance to risk-neutralize returns. Since the measure dispenses option prices, it can be potentially applied to a broader number of markets than corresponding option-based measures. Empirically, our tail risk factor extracted from S&P 500 returns has a 90% correlation with the VIX index. We document a persistent negative relation between tail risk and one-day ahead returns, for different assets. Consistent with the crash-insurance property of put options, tail risk predicts positive one-day ahead returns for portfolios long out-of-the-money, short in-the-money put options. An analysis of stock portfolios sorted on exposure to tail risk reveals a premium for bearing such a risk, even when controlling for known and established factors related to cross-section variability. The cross-sectional analysis is also robust to the inclusion of uncertainty indexes, macroeconomic and volatility measures.

Keywords: Tail Risk, Risk Neutral Measure, Expected Shortfall, Intra-day Market Returns.

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1 Introduction

Traditionally, since its introduction in 1993, the VIX index became a widespread indicator of investor sentiment, often viewed as a market fear gauge (Bollerslev et al., 2015). Since then, researchers have been trying to understand the relationship between volatility and returns (Glosten et al., 1993, Carr and Wu, 2008 and Bekaert and Hoerova, 2013, among others), its relationship to the cross section of stock returns (Ang et al., 2006), and the channels through which it affects the real economy (Bloom, 2009). Nonetheless, the VIX itself is designed to measure the expected volatility of the S&P 500 index over the next 30 days and abstracts from investors sentiment, market fear or tail risk realizations. Most importantly, the methodology on which it is based requires a minimum cross-section of options quotes such that reliable information can be extracted. Therefore, its applicability to markets that contain limited option data as well as to individual assets is compromised.

Following the 2007-2009 financial crisis, a substantial strand of the financial research community devoted efforts to better understand and disentangle the role of tail risk, the risk associated with low extreme events, volatility, and systemic risk. In particular, Bollerslev et al. (2015) rely on a nonparametric estimation of risk neutral tails to perform a decomposition of the variance risk premium into diffusive and jump components. This decomposition reveals that the latter is responsible for most of the return predictability. Given the intrinsic rare nature of extreme events on aggregate data, a second branch of the literature focused on the information embedded in the cross-section of returns rather than time series analysis (Almeida et al., 2016, Kelly and Jiang, 2014). With particular emphasis on the financial sector, motivated by Bear Stearns, Lehman Brothers and AIG failures (among several others), Allen et al. (2012) and Brownlees and Engle (2015) adopted VaR and Shortfall like measures to estimate systemic risks. Common to most of this literature is the fact that estimated measures are calculated on a monthly or weekly basis rather than daily.

In this paper, we combine three ingredients to come up with a new measure of tail risk: the use of high-frequency data, a risk neutralization algorithm, and a coherent measure of risk. We choose as a measure of tail risk, the expected shortfall, a well-known coherent measure of risk often adopted by practitioners and suggested by the Basel III agreement. In particular, we calculate a high-frequency version of this measure using intra-day data on market returns. Finally, we add to these two ingredients, motivated by Ait-Sahalia and Lo (1998) and Ait-Sahalia and Lo (2000), a non-parametrically risk neutralization distortion. By proceeding in this fashion, our tail risk measure is coherent, and is straightforwardly related to downside risk as opposed to the VIX index, which is based on expected volatility. In addition, it inherits all risk-adjustment properties of the risk neutralization, including putting more probability weight to extreme returns without neglecting pricing information coming from Euler equations. Moreover, by adopting a procedure that is not data intensive, our methodology can be easily applied to different markets and assets in which intra-day returns, at any frequency, are available.

The methodological aspect of the estimated risk measure builds on Almeida et al. (2016) who generalize the option-based risk neutral Value at Risk of Ait-Sahalia and Lo (1998) and Ait-Sahalia and Lo (2000). Considering a risk neutralization method based on a non-parametric estimation of the state price density using stock returns, Almeida et al. (2016) showed that is possible to recover risk neutral counterparts of traditional risk measures without the need of option data. In this paper, we re-interpret their technique introducing instead of daily returns, intra-day data. Increasing the frequency of data used in estimation allows us to by-pass the course of dimensionality that many tail risk procedures face, a trade-off between increasing the time series used for estimation and using less recent data to extract information. Besides, instead of obtaining a tail risk measure by investing in broadening the cross-section of assets as in Almeida et al. (2016) and Kelly and Jiang (2014), the high-frequency data allows us to focus, for each market, in the left tail dynamics of the unique one-dimensional series of market returns.

In order to offer a better view on the benefits of adopting an option-free measure, Table 1 provides information on the cross-section of index options for some of the leading stock markets, worldwide. Not surprisingly, the U.S. market has the richest cross-section, with a reasonably comprehensive set of liquid options. Surprisingly, however, is the fact that China, the second economy in the world, does not present an established index option market. This is also the case for three other economies that we analyze: Argentina, Saudi Arabia, and Russia. Moreover, even if we focus on other developed countries, the contrast to the U.S. market remains remarkable. In fact, Japan and Germany, the second and third largest markets, when size is measured by the availability of cross-sectional index options, have respectively, less than 30% and 20% of outstanding options relatively to the S&P 500 options market.

Concerning emerging economies, a first glimpse at the Brazilian IBOVESPA cross-section of options reveals that it appears to be a reasonably well-developed market. For instance, the collected cross-section is bigger than the ones from developed economies such as France, Italy, United Kingdom among others. Still, this figure can be misleading due to the high absolute value of the IBOVESPA index (around 50.000 points), which implies that across the moneyness range, options go out of the money quickly. This is emphasized by Astorino et al. (2015), who in an effort to estimate a Brazilian implied volatility index, highlight the problems related to the low-liquidity of its index options.

Nonetheless, when available, options contain substantial forward-looking information about future states of nature. In particular, option-based measures inherit these properties and allow researchers to disentangle the role of different sources of risk. Comparing our measure to Bollerslev et al. (2015)- an option-based measure designed to capture jump tail risk - we find that both co-move in a significant portion of the sample, with a correlation coefficient of approximately 60%. In a similar analysis for the VIX index, we find correlations as high as 90% for a smoothed version of our tail risk measure. In fact, figure 1 reveals that both series co-move in a large portion of the sample, providing some reassuring evidence on the "fear" nature of the volatility index. This heuristic evidence indicates that our measure might be capable of replicating at least some characteristics of option-based measures. Indeed, (Almeida et al., 2016) performed an extensive comparison between a lower-frequency (monthly) version of our Tail risk measure and a corresponding option-based measure built using Ait-Sahalia and Lo's (2000) methodology and found significant similarities concerning its empirical properties.

To assess the empirical relevance of our tail risk measure, we propose an extensive analysis of its relationship with several market features. First, we document a natural negative relationship between current tail risk and market returns. This negative relationship persists for one-day ahead predictive regressions, mean-reverting after two days. Additionally, we also find that the market implied tail risk is related to a "run to safety" effect: the contemporaneous relationship with Treasury ETF's returns is positive and mean-reverts after two days. In an effort to extract the maximum possible information from this analysis and to test for robustness of our results, we additionally select as crash sensitive measures, one macroeconomic (the Aruba, Diebold and Scotti index) and one representing uncertainty (the Economic Policy Uncertainty index of Baker et al., 2016) as control variables in the regressions framework. Overall our results are robust to the inclusion of these, and other alternative explanatory variables, which include the FEAR factor from Da et al. (2014), the VIX index, and a measure of realized volatility.

As in Bollerslev et al. (2015), we also analyze the relationship between our tail risk measure and returns from different portfolios. Two salient features arise from this analysis. First, all one-day ahead predicted betas for the portfolios analyzed were negative and statistically significant. Nonetheless, taking a closer look at the results for industry portfolios, a significant heterogeneity among sectors is noted. In particular, financial and mining sectors suffer the biggest losses after tail risk shocks. Also, while small and big firms react similarly to tail risk shocks, high minus low portfolios formed on Book to Market and Momentum reveal that one-day ahead returns for high book to market firms and looser firms have higher tail risk betas.

Given the fundamental relationship between jumps and tail risk, in particular in a high-frequency environment, we follow the methodology of Weller (2016) to construct a time series of realized intra-day jumps for market returns. Using this realized measure we show not only that tail risk is significantly related to contemporaneous jumps but also that it can predict realized jumps both one-day and one-week ahead. Again, this effect persists even when we control for several other explanatory variables. More interestingly, after controlling with our tail risk factor a regression of jumps on the VIX index, the VIX carries a negative relationship with realized jumps.

Despite deliberately avoiding the use of option data in the construction of our tail risk index, we acknowledge that these derivative contracts provide crash insurance against stock market meltdowns (Kelly and Jiang, 2014, Bollerslev and Todorov, 2011). Therefore, to investigate the relationship between our measure and option returns, we perform two separate complementary analysis. First, we construct five portfolios of put options on the S&P 500 index based on moneyness. Consistent with our conjecture, our tail risk measure predicts higher returns for deep out-of-the-money options, with monotonic decay across moneyness. In particular, the return of a portfolio long on deep out-of-the-money puts and short on deep in-the-money puts has a positive statistically significant beta. Additionally, we also calculate the alphas from standard Fama-MacBeth regressions controlling for the Fama-French-Cahart factors and document a negative correlation between portfolios' alphas and the tail risk exposures.

From an asset pricing perspective, theory suggests that assets whose returns co-move with bad states of nature, work as hedging instruments, making investors require lower returns to keep those assets in their portfolios (Almeida et al., 2016, Kelly and Jiang, 2014, Yuen, 2015). Our measure is positive and rises when aggregate market returns are low. Therefore, assets whose returns (payoffs) are high when our tail risk is high, provide insurance for severe stock market movements and therefore are expected to have a positive beta with respect to tail risk. Moreover, given the risk neutral characteristic of our measure, it naturally embeds investors' crash-aversion and therefore their desire for insurance. Thus, following the approach of Ang et al. (2006) and complementing the analysis performed in Almeida et al. (2016), we test this hypothesis by analyzing portfolio returns sorted on exposures to our tail risk measure. As in Almeida et al. (2016) and Yuen (2015), we find a positive, statistically significant, return associated with a long-short portfolio formed on tail risk exposure for all the holding periods analyzed. We also applied Ang et al. (2006)'s methodology, designed for testing the implications of VIX's innovations to the cross-section of stock returns and find similar results. Most importantly, the excess returns generated by the long-short portfolios are not captured by the traditional Fama-French-Cahart factors and are robust to several double-sort analysis.

Our paper is also intrinsically related to several branches of the financial literature. First, we contribute to the growing literature on the estimation of tail risk and systemic risk measures as in Kelly and Jiang (2014), Allen et al. (2012), Bollerslev et al. (2015), Siriwardane (2013), Brownlees and Engle (2015), Adrian and Brunnermeier (2014), Bali et al. (2009), among others. In contrast to these papers, we contribute to the literature by providing a methodology to compute tail risk on high-frequency environments that considers measures estimated on a daily frequency (or higher) and that is virtually applicable to any set of returns.

Our paper is close in spirit to Weller (2016) who developed a real-time tail risk measure based on intra-day bid and ask quotes. In contrast, however, while Weller focus on the natural relationship between tail risk and jumps, providing substantial evidence of jumps' prediction using intra-day data, we focus on the broader relationship between tail risk, market returns and the cross-sections of both stocks and options. Additionally, while our aim is to provide a measure that is not data intensive, Weller's measure on its baseline one-factor model, considers a panel of 2800 firms.

Aiming at capturing an "investor sentiment component" related to the VIX index, Da et al. (2014) rely on primary data from Google Trends to develop a daily sentiment index extracted from a pool of users. They document a negative relationship between their FEAR index and current market returns, and also a return-reversal feature for this index. Despite the fact that our measure is based on intra-day market data rather than investors' sentiment data, we show that the same qualitative results documented by Da et al. (2014) hold in our analysis.[1] With a similar methodology, Baker et al. (2016) extract information from newspapers to construct an Economic Policy Index (EPU). They find that their constructed index is positively associated with higher volatility, and expand results in Bloom (2009) in terms of macroeconomic forecasting ability. Our predictability results are robust to the inclusion of Baker et al. (2016)'s measure as a control.

Finally, our paper also expands the analysis in Bollerslev et al. (2016) and Yuen (2015). Bollerslev et al. (2016) considers an extension of the CAPM model that takes into account in the returns dynamics, not only a linear beta with market returns but also a beta coming from a jump component.[2] Using this decomposition Bollerslev, Li and Todorov show that while there is no premium associated with the market beta the converse is not true for the jump component. Related to this, Yuen (2015) considers the fear measure of Bollerslev et al. (2015) coupled with the cross-section analysis of Ang et al. (2006) to extract the tail risk premium embedded in the cross-section of asset returns. In contrast to these papers, we offer a new tail risk measure that is not dependent on options data and does not rely on any parametric dynamics for the market return. While this allows us to expand our analysis to settings where no option data is available, our proposal, being a reduced-form measure itself, also comes with some disadvantages. For instance, in contrast to Bollerslev et al. (2016) and Yuen (2015), we are not able to fully disentangle the effects of jumps and diffusion components on assets returns. Nonetheless, we are still able to isolate the effects of tail risk by controlling for additional risky measures that capture the diffusive component (as they do) and mitigate this issue. Moreover, overall we find that the main empirical results in Bollerslev et al. (2015) and Yuen (2015) hold when we consider our tail risk measure in replacement of theirs.

[1] Our predictive results are also robust to the inclusion of their FEAR measure when considering the sample where both tail risk measures overlap.

[2] In fact, their empirical analysis is even more general, since they also consider a beta coming from an overnight "jump".

The rest of the paper is organized as follows. Section 2 describes how we construct a risk-neutralized shortfall measure of tail risk based on the market returns. Section 3 starts with a brief outline of the empirical applications. Section 3.1 describe the high frequency data we use to estimate tail risk and the data considered in the empirical applications. Sections 3.2 and 3.3 relate our tail risk measure respectively to the VIX and to market (price) realized jumps. Section 3.4 and 3.5 analyzes the contemporaneous and prediction relationship between tail risk and several assets classes. Section 3.6 investigates the implications of tail risk for option portfolios. Section 3.7 presents results of a risk premium analysis using a cross-section of stock returns. Finally, section 4 concludes with a summary of the results.

2 A Nonparametric Tail Risk Measure

2.1 A First Look at the Tail Risk Measure:

In this paper, we aim at building a simple tail risk measure, based on aggregate stock returns, and being dependent only on the observed returns. Historically the Value-at-Risk (VaR) became the main benchmark for risk assessment between practitioners (e.g. Basel II agreement) and is widespread studied in the finance literature (Basak and Shapiro, 2001). Unfortunately, despite its intuitive interpretation, the simplicity of the VaR does not come without caveats. Among several criticisms highlighted by the literature (see also Basel III agreement for a practical discussion), two are relevant for the purpose of constructing an informative tail risk measure: (i) VaR is a point-wise measure and therefore ignores the behavior of the tail of the distribution beyond its threshold. (ii) due to the relative scarcity of tail events on historical time series, VaR modeling is challenging and can lead to overestimation of risks in calm periods (Berkowitz and OBrien, 2002) and underestimation during crisis (Jorion, Jorion).

Given the relative scarcity of extreme events in historical samples, one solution proposed in the literature is to look to past events in light of the risk neutral probabilities, intrinsically taking into account investors risk aversion. Ait-Sahalia and Lo (2000) and several other papers[3] lead this literature by proposing the estimation of risky measures

[3]See in particular Ait-Sahalia and Lo (1998), Breeden and Litzenberger (1978), Bates (1991), Rubinstein (1994), Longstaff (1995).

based on the risk neutral distribution extracted from option prices. By doing so, the implicit probabilities compensate the physical ones towards investor's attitude towards risk and, therefore, alleviate the lack of extreme, in sample, events.

Nonetheless, options arguably provide the best environment to estimate risk neutral densities and risky measures themselves. In fact, Andersen et al. (2016) estimates a left tail factor, for a panel of international data, that explicitly dependent on the risk neutral information embed in option prices. Andersen et al. (2016) also highlight that international derivative markets are expanding, although their analysis is restricted to seven well-developed economies. As shown in table1, aside from the U.S. economy, most of the markets around do not have a rich derivatives market, with an even more problematic scenario for emerging markets. To overcome this problem, in a tail risk estimation environment, recent papers focus on information about tail dynamics extracted from the cross section of returns, rather than the market returns [4].

Here we adopt the methodology explored in Almeida et al. (2016) to risk neutralize the returns and estimate a daily tail risk measure. Given our interest in the behavior of the entire tail of the distribution, and aiming at addressing point (i), we follow the main benchmark measure of Almeida et al. (2016) that is based on a risk neutral, mean adjusted, expected shortfall.[5] Differently from Almeida et. al however, we explore the richness of intraday returns, rather than the cross section of returns[6]. By increasing the frequency of the data used in the tail risk estimation, we can not only better characterize the tail behavior for the market returns but also to overcome the dimensionality problem faced by their methodology (which will be clear in the next section). We define the market excess expected shortfall as follows:

$$TR_t = E^{\mathcal{Q}(R)}[(VaR_\alpha(R_\tau) - R_\tau)^+] \tag{1}$$

where t is the day for which we are calculating the tail risk, τ denotes the possible states of nature, in this paper defined as the intra-day, realized returns for the main

[4]The most recent literature includes Kelly and Jiang (2014), Allen et al. (2012), and Adrian and Brunnermeier (2014) to cite a few.

[5]Relying on an equilibrium analysis,Basak and Shapiro (2001) show that the VaR generates counterintuitive results in terms of risk behavior. In contrast, expected shortfall measures overcome the VaR problematics. From a practitioner perspective, the Basel III agreement also directs to the use of shortfall measures for risk management.

[6]Although nothing prevents us to exactly replicate their methodology.

stock market index (S&P 500 for the U.S. economy), α is the VaR threshold and $\mathcal{Q}(R)$ indicates the risk neutral density. Note that in this design we model $\mathcal{Q}$ as a function of the observable returns. Throughout we calculate the tail risk at daily frequency using the intra-day realizations of the stock market returns to do so. Therefore there is no overlapping of data between days which allows the measure to quickly incorporate market information.

2.2 A Non-Parametric Risk-Neutral Density

Given the proposed measured, we are still missing one of its critical component: the state price density. As previously highlighted, we want our measure to be option independent, and thus we must search for alternatives estimation procedures. In a seminal paper, Hansen and Jagannathan (1991) proposed to estimate the stochastic discount factor via a minimization of a quadratic loss function given a set of basis assets. While insightful, this approach fails to explore higher order moments of the underlying returns - the estimated SDF is a linear combination of the basis assets. Expanding on the Hansen and Jagannathan idea, Almeida and Garcia (2016) generalized their methodology to estimators that inherit properties of higher order moments of the base assets returns.

Given a series of assets returns, in an incomplete market where there are more states of nature than assets, Almeida and Garcia (2016) find a family of SDFs that minimize convex functions defined in the space of admissible and strictly positive SDFs. These convex functions measure the distance between an admissible SDF and the constant SDF of a risk-neutral economy. Assuming a constant short-term rate and homogeneous physical probabilities, just as in a VaR historical simulation, we can obtain a direct correspondence between SDFs and RNDs.

Given the extensive analysis in Almeida et al. (2016) and Almeida and Garcia (2016) with respect to the properties of the nonparametric estimator, here we briefly describe the methodology adopted to calculate our benchmark measure. For the sake of simplicity, we specialize the problem of SDF estimation to the Hellinger estimator, a particular case of the general Cressie-Read family approach of Almeida and Garcia (2016). Let $(\Omega, \mathcal{F}, P)$ be a probability space, and R denote a K-dimensional random vector on this space representing the returns of K primitive basis assets. In this static setting, an admissible SDF is a random variable m for which $E(mR)$ is finite and satisfies the Euler equation:

$$E(mR) = 1_K, \tag{2}$$

where 1_K represents a K-dimensional vector of ones.

For a sequence of (m_τ, R_τ) that satisfy Equation (2) for all t, and observing a time series $\{R_\tau\}_{\tau=1,\ldots,T}$ of basis assets returns, we assume that the composite process (m_τ, R_τ) is sufficiently regular such that a time series version of the law of large numbers applies[7]. Therefore, sample moments formed by finite records of measurable functions of data R_τ will converge to population counterparts as the sample size T becomes large.

Given a sample of basis assets returns, the set of admissible SDFs will depend on the market structure. The usual case is to have an incomplete market, i.e., the number of states of nature (T) larger than the number of basis assets K. In such case, an infinity of admissible SDFs will exist, and if there is no in-sample arbitrage on the basis assets payoff space (see Gospodinov et al. (2016)), there will exist at least one strictly positive SDF (see Duffie, 2001). For each strictly positive SDF there will be a corresponding risk neutral density. The fundamental difference between this paper and Almeida et al. (2016) is that, instead of using daily realizations of R_τ as the states of nature, we rely on intra-day realizations of the market returns. By doing so, we can increase the number of states T by simply increase the frequency of the data used (say, from 1 hour to 15 minutes). Additionally, for tail risk estimation purposes, we are able to compute an aggregate market tail risk directly from the observed returns. From a factor model perspective, with the market return being the only source of risk, our non-parametric SDF approach resembles the CAPM. However, in contrast to the CAPM, our methodology generates a stochastic discount factor that is non-linear in the market returns, incorporating its higher order information.

Given the Hellinger discrepancy function $\phi(m) = -4(m^{1/2} - a^{1/2})$ the generalized the, in sample, minimum discrepancy problem proposed by Almeida and Garcia (2016) can be stated as:

[7]For instance, stationarity and ergodicity of the process (m_t, R_t) are sufficient (see Hansen and Richards, 1987). Also, we further assume that all moments of returns R are finite in order to deal with general entropic measures of distance between pairs of stochastic discount factors.

$$
\begin{aligned}
\hat{m}_{MD} = \underset{\{m_1,\ldots,m_T\}}{\arg\min} \quad & \frac{1}{T}\sum_{i=1}^{T}\phi(m_i) \\
\text{subject to} \quad & \frac{1}{T}\sum_{i=1}^{T} m_i(R_i - \frac{1}{a}1_K)] = 0_K \\
& \frac{1}{T}\sum_{i=1}^{T} m_i = a \\
& m_i \geq 0(\text{or } m_i > 0)\ \forall\ i
\end{aligned}
\tag{3}
$$

In this optimization problem, restrictions to the space of admissible SDFs come directly from the discrepancy function ϕ. The conditions $E(m\left(R - \frac{1}{a}1_K\right)) = 0_K$ and $E(m) = a$ must be obeyed by any admissible SDF m with mean a. In addition, whenever there is a strictly positive solution the implied minimum discrepancy SDF is compatible with absence of arbitrages in an extended economy that considers derivatives over the underlying basis assets[8]. The choice to impose a non-negativity or strict positivity constraint in the optimization problem is dictated by the choice of the discrepancy function $\phi(.)$ (see Almeida and Garcia (2016) for a detailed analysis).

Despite the straightforward interpretation of the problem in (3), its solution is not easy given that the number of unknowns is as large as the size of the sample. Therefore, Almeida and Garcia (2016) show that one can solve an analogous simpler dual problem:

$$
\hat{\lambda} = \underset{\alpha\in\mathbb{R},\ \lambda\in\Lambda}{\arg\sup}\ a * \alpha + \frac{1}{T}\sum_{i=1}^{T}\phi^{*,+}(\alpha + \lambda'(R_i - \frac{1}{a}1_K)) \tag{4}
$$

where $\Lambda \subseteq \mathbb{R}^K$ and $\phi^{*,+}$ denote the convex conjugate of ϕ restricted to the non-negative (or strictly positive) real line.

$$
\phi^{*,+} = \underset{w\in[0,\infty)\cap\text{domain } \phi}{sup} \quad zw - \phi(w) \tag{5}
$$

In this dual problem λ can be interpreted as a vector of K Lagrange multipliers that comes from the Euler equations for the primitive basis assets in (3). For the specific case of the Hellinger estimation, closed-form formulas are obtained for λ and $\hat{m}_{MD}$:

[8]It is important to note that the homogeneous probability assumption will not affect the key insights we derive from this methodology and, if desired, one could also consider a kernel density to model the physical probabilities without additional complications

$$\hat{\lambda}_H = \arg\sup_{\lambda\in\Lambda_{CR}} \frac{1}{T}\sum_{i=1}^{T}\left(2a^{1/2} - 2\left(a^{1/2} - \frac{1}{2}\lambda'\left(R_i - \frac{1}{a}1_K\right)\right)^{-1}\right) \tag{6}$$

and:

$$\hat{m}^i_{MD} = a\frac{\left(a^{1/2} - \frac{1}{2}\hat{\lambda}'_H\left(R_i - \frac{1}{a}1_K\right)\right)^{-2}}{\frac{1}{T}\sum_{i=1}^{T}\left(a^{-1/2} - \frac{1}{2}\hat{\lambda}'_H\left(R_i - \frac{1}{a}1_K\right)\right)^{-2}} \tag{7}$$

To obtain the risk neutral probabilities associated with each observation interpreted as a state of nature, we distort the usual $1/T$ measure by the computed SDF in (7) adjusted by the interest rate[9]

$$\pi_i^{RN} = \frac{m_i(1+r)}{T} \tag{8}$$

To understand the effect of the estimated risk neutral density on our tail risk measure, we derive a Taylor expansion of the expected value of $\phi(m) = -4(m^{1/2} - a^{1/2})$ around the SDF mean a. Noting that $\phi(a) = 0$, $\phi'(m) = -2m^{1/2}$, $\phi''(m) = m^{3/2}$, $\phi'''(m) = (-3/2)m^{5/2}$, $\phi''''(m) = (-3/2)(-5/2)m^{-7/2}$...,Taylor expanding ϕ and taking expectations on both sides we obtain:

$$E(\phi(m)) = \frac{a^{-3/2}}{2}E(m-a)^2 + \frac{-(3/2)a^{-5/2}}{3!}E(m-a)^3 + \frac{(15/4)a^{-7/2}}{4!}E(m-a)^4 + ... \tag{9}$$

Analyzing this Taylor expansion two important aspects regarding the weights attributed to skewness and kurtosis are notable. First, differently from the Hansen and Jagannathan (1991) approach, the Hellinger estimator is dependent on higher-order moments of the underlying returns. In particular, we see that the absolute weight given to kurtosis is smaller than the one given to skewness. Also, besides the differences in their magnitude, the weights given to skewness are negative, while the weights assigned to kurtosis are positive. In other terms, the state price density will be higher when kurtosis is higher or when skewness is lower, therefore revealing the skewness "preference" and kurtosis "aversion" characteristic of the implied estimator (which are in line with findings

[9]This framework is identical to the Entropic estimator for the SDF proposed by Stutzer (1996) with the only difference being the minimized function. In fact Almeida and Garcia (2016) showed that the minimum entropy estimator is a special case of the Cressie-Read function estimators.

on Kraus and Litzenberger (1976),Backus et al. (2011)).

3 Empirical Results

To evaluate our tail risk measure and its relationship to risky assets we perform an extensive analysis. Before getting into the details of the empirical results, we must point out the goals and the intuitions behind the empirical applications.

First, given the relative simplicity and the non-data intensive design of our measure we perform its estimation daily. This allows us to focus on a day-to-day analysis of risk returns and also extend the results for other time horizons. The empirical application starts by providing evidence on the relationship between tail risk and realized jumps on the market returns. Our methodology follows closely Weller (2016) approach using a daily framework instead of intra-day data.

The cornerstone of the empirical application is to verify the relationship between our measure and both contemporaneous and expected returns among different asset classes as in Bollerslev et al. (2015) and Da et al. (2014). To do so, we start by analyzing the contemporaneous and predictive properties of tail risk with respect to market returns. Then, we expand this framework to different asset classes and portfolios returns.

Given the natural insurance property of out of the money puts they provide the natural hedge derivative to tail risk (Kelly and Jiang, 2014). Therefore we expect that not only that put returns are intimately linked to tail risk but also that the latter provides additional information on future option returns.

Finally, complementing the cross section analysis of Almeida et al. (2016), Bollerslev et al. (2016) and Yuen (2015), we also provide evidence on the tail risk premium.

3.1 The Data and Series Construction

For the baseline measure estimated in this paper, we use 15 minutes intra-day data for the returns on the S&P 500 index, extracted from Bloomberg, from 01/02/2008 to 01/07/2015 similar to Andersen et al. (2016). We set the VaR threshold as $\alpha = 20\%$ so we can capture the daily dynamics of the tails of the returns distribution. Our sample starts 09:30 and ends at 16:00 every day. Therefore, we consider a total of 27 daily observations to estimate the tail risk measure. One could argue that the use of higher frequency data, say 1 minute, might translate to a more precise measure. We believe that, in line with Andersen et al. (2016), expanding the frequency of the data used might exacerbate the market microstructure related problems in the estimated measure.[10] For the sake of interpretation, all the results presented in the paper will be reported for standardized variables, including the controls we consider in the regressions framework.

We start our empirical exercises by comparing the Hellinger Tail Risk with realized jumps on the S&P 500 index. To do so we must then construct a realized measure of price jumps. We do this by modifying Weller (2016) approach. First, for each day, we calculate the number of price increments that exceeded 1-5 standard deviations relative to the previous 50 days mean/volatility. This allows us to control for the "magnitude" of the jumps across time, providing a simple alternative to parametric jump diffusion models. Our main jumps measure is, as in Weller (2016), a weighted average of the intra-day jump counts. Section 3.3 careful describe the computation of the measure.

To evaluate the relationship between tail risk and asset returns, we analyze a variety of asset classes and portfolio returns. First, given the clear relationship between our market tail risk measure and the market returns we evaluate it's contemporaneous and forecasting properties with the S&P 500 daily returns. However, different portfolio and assets classes might react differently to changes in risk and risk aversion. Naturally, the first candidate to expand this risk-return analysis is the returns on "safe" investments. Notably, in periods of stress, U.S. government bonds provide a safe run for investors. Given that constant maturity yields are estimated based on fitted data for true bonds, instead of traded data, we consider iShares U.S. Treasuries ETF, a daily traded fund invested in treasuries, to analyze the "fly-to-safety" aspect of high tail risk dates. Alternatively, we explore different portfolios exposure to tail risk by considering the following alternatives: portfolios formed on Size, Book to Market, Momentum and industry from Kenneth French library. We also consider portfolios formed on past market beta and volatility from CRSP.

For option data, we collect daily put options bids and asks from OptionMetrics U.S. Ivy database. As is standard in the literature we calculate option prices as the average between bid/ask closing values. With this data, we construct daily portfolios of put index option sorted on moneyness, defined as K/S. Portfolios are rebalanced daily, and the returns are calculated for a buy and hold strategy for the options. We calculate five

[10]For the sorting portfolio procedures we re-estimate our measure using one-minute data and provide some evidence that the overall results hold.

portfolios formed on moneyness as follows $0.9 < DOTM < 0.94$, $0.94 < OTM < 0.97$, $0.97 < ITM < 1.03$, $1.03 < ATM < 1.06$, $1.06 < DATM < 1.10$ for options with maturity between one and 45 days (results are robust to the maturity range we consider).

We also select individual stock returns from 1228 stocks that composed the S&P 500 and Russel 1000 index during the period of our sample for the intra-day returns[11]. All the data relative to these stocks returns are from CRSP, which also provide supplementary information needed to construct the control variables for the sorting portfolios procedure (see Appendix A for details on the control variables).

Additionally, for the regressions framework analyzed, we consider a handful amount of control variables. First, to control for volatility, we calculate a measure of realized volatility based on the sum of squared intraday returns following Andersen et al. (2016). For most of the regressions, we also use the CBOE VIX index as an explanatory variable given the forward-looking characteristic embed in the measure. To control for the macroeconomic business conditions we collect data for the Aruba, Diebold and Scotti index from the Philadelphia FRB. We also rely on data for economic uncertainty from the EPU index of Baker et al. (2016). Although we do not report the results, we also computed additional robustness test using the FEAR measure of Da et al. (2014) as a control variable for the restricted sample where our measure overlaps with theirs.

3.2 A first look at the tail risk measure over time

Figure 2 plots the daily estimated measure from 01/02/2008 to 01/07/2015 and the VIX index. These figures illustrate the ability of our tail risk to capture extreme stock market events as well as the correlation with economic conditions on a daily basis, corroborating the findings on Almeida et al. (2016). Notably, our measure is very volatile and features various peaks, often coinciding with periods of high expected volatility. More interestingly, the measure captures both the financial crisis and the European debt crisis, peaking in both of these events.

Perhaps the most striking feature of the estimated measure is its relationship with the VIX index. To put it into perspective we estimate our tail risk using only a risk neutralization procedure and intra-day returns on the S&P 500 index. In contrast, the VIX index is based on the expected volatility extracted from the hole cross section of

[11] Our data set is similar to Bollerslev et al. (2016) with additional stocks from the Russel 1000 index

option prices. Also, while the VIX measure the expected market volatility our measure is specifically designed to capture extreme movements in the stock market returns. Therefore, given both their methodological and theoretical differences, it is surprisingly how these two measures align.

Table 2 present the correlation between the estimated Hellinger Tail Risk, its physical counterpart and a moving average of the Hellinger Tail risk with several crashes sensitive measures. As it is evident from figures 2 and 1 the correlation with the VIX is high: 65% for the Hellinger Tail risk and 90% for the moving average. This high correlation is also noted for Bollerslev et al. (2015) fear measure, another option based tail measure. Interestingly, while also correlated to a realized variance measure, the Hellinger Moving average carries only 68% correlation with the realized volatility.

While it is expected for tail measures to co-vary together, we also compute the correlations of the Hellinger Tail Risk with uncertainty and macroeconomic based indexes. Table 2 reveal that our measure is positively correlated with uncertainty, measure as the Economic Policy Uncertainty (EPU) index from Baker et al. (2016) and negatively correlated to the Aruba, Diebold and Scotti (ADS) index, a pro-cyclical business condition index, from the Philadelphia Federal Reserve Bank. While similar in essence to the EPU index the correlation of our Hellinger Tail Risk with the FEAR index of Da et al. (2014) is almost neglectable. Not surprisingly, the correlation between our measure and an equity-based EPU is also positive and higher than the correlation with the EPU itself.

Finally, we note that the estimated tail risk measure is positively correlated with the realized jumps measure and several other crash sensitive market measures: Emerging Markets spread with the spot treasury curve from BofA, the Stock Market Crashes measure by the Cleveland FRB as the ratio of the current value for the S&P 500 index and its maximum over the last 365 days, the contribution of the banking sector to overall stock market volatility from the Cleveland FRB, the spread between Moody's Seasoned Baa Corporate Bond and 10-Year Treasury Constant Maturity and the noise measure of Hu et al. (2013).

3.3 Stock Market Jumps

Taken together, the heuristic results of the previous section provide some preliminary evidence that our daily tail risk measure capture fluctuations in market risks. In this section, we formally establish the relationship between tail risk and stock market jumps.

As stated in section 3.1 we measure aggregate stock market jumps as intra-day deviations from the historical, intra-day, mean for price increments. More specifically, let $\delta P(\tau - 1, \tau)$ denote the price variation between $\tau - 1$ and τ withing each intra-day price interval. We construct five preliminary downside jump measures as follows: $DJ_t(i) = \sum_{\tau=2}^{T} I[\mu_t - i\sigma_t < \delta P(\tau - 1, \tau) < \mu_t - (i+1)\sigma_t]$ for $i = 1, \ldots, 4$ and $DJ_t(5) = \sum_{\tau=2}^{T} I[\mu_t - 5\sigma_t < \delta P(\tau - 1, \tau)]$ where μ_t and σ_t are the respective mean and standard deviation of in price increments for the preceding 50 days. The aggregate jump measure is a weighted average of the five preliminary measures, as in Weller (2016), as follows:

$$\text{D. Jump}_t = \frac{\sum_{i=1}^{5} i DJ_t(i)}{\sum_{i=1}^{5} i} \tag{10}$$

By letting the both the mean and the standard deviation of the price increments vary over time, we allow our jump measure to capture jumps "conditional" on the volatility. This also avoids using future data on price increments when construction our realized jump measures or setting ad hoc thresholds for the jumps calculation.

Our key analysis in this section is a regression that takes the following form:

$$\text{D. Jump}_{t+h} = \alpha + \beta TR_t + \sum_{k=1}^{K} \gamma_k X_k + \epsilon_{t+h} \tag{11}$$

Where TR denotes the Hellinger Tail Risk measure and X indicates additional explanatory variables.

First, we set $h = 0$ and therefore look at the contemporaneous relationship between tail risk and jumps. Table 3 show the results of this analysis for the aggregate jump measure. Panel A present the estimated coefficients for univariate regressions. Except the ADS index, a business conditions macroeconomic based measure, all other explanatory variables have positive, statistically significant, betas. Among the selected set of explanatory variables, we note that the β with the highest magnitude is the one associated with tail risk, being almost 60% higher than the one for the realized variance, the second largest coefficient. More strikingly, while most of the explanatory variables have relatively small estimated R^2, the value associated with tail risk is as high as 16%, more than 100% higher than the one for the realized variance measure (the second largest).

Panel B of table 3 present the results for multivariate regressions where we add one explanatory variable at a time. Four features are notable in this table: first, for all four regressions, the coefficient on tail risk remains stable when compared to the univariate regression. Second, while the for the univariate regression the coefficient on the VIX index was positive and statistically significant, when we control for tail risk the coefficient becomes negative and statistically significant. We see this as evidence that, when controlled for negative tail realizations, the variance captures the positive "effect" on returns outcomes. Third, the magnitude of the estimated coefficient for realized variance (EPU index) is reduced by half (totally) when we control for tail risk. Finally, the estimated R^2 are only slightly higher when we add the additional explanatory variables when compared the tail risk univariate framework.

Table 4 further investigate this results for the multivariate regressions for the five components of the aggregate jump measure. Overall, we note that the main features persist for each component, with the tail risk being the main driver of the jump realizations. For each component, the univariate R^2 from tail risk regressions is also high when compared to the multivariate framework, representing on average more than 75% of the later. Also, while the tail risk beta is monotonic, being higher for smaller jumps, all the estimated betas are statistically significant. In fact, if one takes into account that the average for the $DJ_t(1)$ is approximate 2, meaning that for each day we observe on average two "jumps" between 1 and 2 standard deviations from the last 50 days mean, and the $DJ_t(5)$ average is approximate 0.07 the estimated coefficients for the tail risk convey additional meaning. That is, an increase of one standard deviation in current tail risk is associated with 0.26 more $DJ_t(1)$ on average, about 13% of the mean value. The same one standard deviation rise in tail risk is associated with an increase of 0.10 $DJ_t(5)$ on average, about a 100% increase relative to its average.

Table 5 present the results for one day ahead prediction regressions ($h = 1$). Similar to the contemporaneous setup, univariate regressions reveal that the most relevant variables in terms of prediction are the tail risk and realized variance. We must highlight however that the relationship between realized variance and future jumps are not surprising given the rolling window methodology to calculate the jump measure. By construction, increases in volatility imply more harsh jumps threshold on the future. Still, the coefficient on realized volatility is positive and of the same magnitude as for the tail risk in the prediction regressions. When comparing the obtained results with the contemporaneous regressions, we also find close patterns. In a multivariate setting, the tail risk coefficient is positive and statistically significant for all regressions, with the magnitude being almost unchanged when comparing to the univariate counterpart. As expected, while in contemporaneous regressions a one standard deviation increase in tail risk was associated with a 0.11 on average increase in the downside jump measure, the magnitude in the prediction regressions reduces to 0.02. Again, both the EPU and the ADS does not seem relevant in terms of jumps prediction.

When we move to the results in table 6 for the segregated jump prediction regression the picture is slightly different. With the one exception, for the $DJ_t(4)$ component, the tail risk beta is at least 25% higher than the one for the realized variance. Also, out of five components, three of the estimated betas for the realized variance are not significant, while all of them are for the tail risk (at a 10% significance level). The patterns for VIX, ADS and EPU are also stable, being for the most part irrelevant in statistical terms. Table 6 also reveal that for the jump measure components the R^2 of univariate regressions where the sole explanatory variable is the tail risk are relatively high, ranging from 20% to 50% when compared to the multivariate R^2.

Finally, in table 7 we present the results for weekly jump predictions. To avoid overlapping data, in the first day of each week, we compute all the measures used as explanatory variables. Then, we aggregate each jump component from the second day of each week to the first day of the next week. Results for these regressions revealed that, as before, the R^2 and the betas for the tail risk maintain their patterns. Interestingly, the realized volatility loose its explanatory power while the VIX estimates, a forward-looking measure, becomes statistically significant.

Overall, this section confirms the suggestive evidence in table 2 and figure 1. Our findings are also comparable to Weller (2016) although we only work with daily jumps while his analysis can be extended to both daily and intra-day settings. More importantly, we verify that our measure is intimately related to realized jumps and contribute to its understanding in ways not spanned by volatility.

3.4 Market Returns

As it is now evident from the tail risk literature, there is a long-term premium related bearing disaster risk ((Almeida et al., 2016), Kelly and Jiang (2014), Bollerslev et al. (2015)). However, while the long-term relationship between risk and returns is well understood, there is no clear evidence on the relationship between the short-term relationship between market returns and tail risk. Here, we explore the flexibility of our measure that allows for daily estimation and present some new evidence on this topic.

Our approach is similar to the one in the previous section. We want to establish the contemporaneous relationship between tail risk and returns and also analyze the predictability features of our tail risk measure. To do so we rely on the following regression, considering the same additional explanatory variables as before:

$$R_{t+h} = \alpha + \beta TR + \sum_{k=1}^{K} \gamma_k X_k + \epsilon_{t+h} \tag{12}$$

Table 8 present the results for this analysis when we take $h = 0, 1, 2$. Not surprisingly, both tail risk and the VIX index are associate with lower contemporaneous returns for the S&P 500 index. In contrast with the jump regressions, we note that both EPU and ADS coefficients are statistically significant for returns regressions. Among all the explanatory variables, the biggest coefficient in magnitude is the tail risk beta (note that all variables are standardized, these betas indicate standard deviations from the mean) stating that a one standard deviation increase in tail risk is associated with a -0.51% drop in the S&P 500 on average. For contemporaneous regressions, we also note that the R^2 for a univariate regression with the tail risk as a sole explanatory variable represents 66% of the total R^2.

In the predictions setting, taking $h = 1$ (a one day ahead prediction), while the coefficient for the VIX mean reverts, presenting almost the same magnitude as for $h = 0$, the coefficient for the tail risk remains negative and statistically significant. That is, increases in tail risk are not only associated with current negative returns but also to future, one day ahead, negative returns. While the realized variance and EPU have a positive statistically significant coefficient for contemporaneous predictions their effects are unclear in prediction setups.

Table 8 also reveal that, for $h = 2$, the effects of tail risk partially mean reverts. This result is also accompanied by a mean reversion in the coefficients for the ADS measure that starts one day after the shock and persists two days. Overall, the results presented in this section are consistent with previous evidence from Da et al. (2014) who estimate a FEAR measure based on internet searches. However, while Da et al. (2014) find a one-day mean reversion, we find that shocks in tail risk take longer to mean revert. Also, while Bollerslev et al. (2015) and Almeida et al. (2016) find a positive relationship between tail risk and future returns, a feature that is consistent with the risk-return characteristic of the market, in a higher frequency environment we note that there is a short-term persistence in responses to tail risk shocks.

3.5 Expanding the Analysis

Bollerslev et al. (2015) argue that different assets/portfolios might react differently to changes in risk, especially extreme event related risks. Also, Da et al. (2014) present some evidence on the various patterns of assets returns to raises in FEAR. To investigate the heterogeneity among assets concerning tail risk, we follow a similar approach and select a handful set of different assets/portfolios and re-estimate regression 12.

Complementing the analysis of Da et al. (2014), we start by focusing on the returns on iShares Treasury ETF. We select a traded ETF instead of returns on fixed maturity bonds to avoid using fitted data (fixed maturity bonds are constructed based on some methodology that interpolates observable bond prices/returns). Arguably, treasuries provide a safe haven in times of economic distress. Therefore, the ETF's allow us to analyze the market safety demand increase to shocks on tail risk. Table 9 present the results for the same set of regressions estimated for the market returns.

As in Da et al. (2014), we find a positive "fly-to-safety" effect associated with contemporaneous rises in tail risk. This is also true for all of the explanatory variables considered in the multivariate setting. In comparison to the market returns, the contemporaneous regressions betas for the explanatory variables are all of different signs and statistically significant. Again, the tail risk holds the one with the biggest magnitude, implying a 0.12% rise on Bond ETF's returns associated with a one standard deviation increase in tail risk. Differently from the market regressions are the mean reversion patterns. While the tail risk still mean reverts, the one day ahead coefficient is near zero. The case for the VIX is even more surprising, with no mean reversion detected. Additionally the realized variance, ADS and EPU does not seem to be related to future treasury returns.

To better comprehend the risk returns relationship between market tail risk and other assets we go forward and select portfolios formed on Size, Book to Market, Momentum, Market beta and variance and perform additional analysis. We first present the results for Market beta and variance sorted portfolios for which we can relate our results to Da et al. (2014) daily prediction regressions.

Tables 10 show the results for contemporaneous and prediction regressions for the Market beta. Given the central relationship between beta sorted portfolios and market returns, we expect that higher beta portfolios will be more exposed to the market tail risk. Indeed, column 1-10 of table 10, reveal that these portfolios have, both contemporaneous and prediction, tail risk betas that are negative and higher in magnitude than portfolios less exposed to the market returns. As an example, the portfolio formed with high market beta stocks have an average drop of -0.44% one day after a tail risk shock. In contrast, the same value for the portfolio with low market beta is around -0.11%. In fact, low beta stocks have a statically zero correlation with contemporaneous tail risk, revealing that not only these stocks are linearly orthogonal to market returns but also orthogonal to massive drops on the market's returns. This monotonic relationship between market beta portfolios and the estimated beta for the control variables are also verified for all the explanatory variables, although the coefficients are not statistically significant for the realized variance and VIX index. For the one day ahead predictions, similar to the market returns, we verify that the coefficients for the tail risk beta are negative, statistically significant, and usually the one with the biggest magnitude when compared to the control variables. This analysis also reveals that portfolios formed on low market betas minus high market beta have a statistically positive relationship with tail risk, and thus capable of hedging this risk.

Different from the results for the market beta portfolios, 11 reveals that there is no clear pattern in the estimated coefficients for the tail risk in the variance sorted portfolios. While the low variance minus the high variance portfolio still presents a positive, statistically significant, beta with tail risk contemporaneously, the statistical significance is only marginal for prediction regressions. We also do not find a monotonic relationship between the variance portfolios and tail risk exposure. Overall, contemporaneously the estimated beta ranges from -0.08 for the lower variance portfolio to -0.58 for intermediary portfolios.

As for the market returns, our results contrasts with the finds on Da et al. (2014) for their fear measure. While they find a significant mean reversion for both market beta sorted portfolios we find a significant persistence of this portfolio's returns to tail risk.

For the sake of brevity, we drop the contemporaneous returns regressions in the next analysis.

With respect to size-sorted portfolios, a clear distinction emerges. While we still find a negative relationship between future returns and current tail risk, there is no clear distinction between small and big firms. The estimated coefficients are of the same magnitude, around -0.30, for both portfolios, with an analogous magnitude in comparison to the estimated tail risk beta in S&P 500 index regressions. This is also the case for the VIX, and ADS index. We do find however, that big firms react more to rises in current realized variance, although individually the coefficients are not statistically significant (for the Small and Big portfolios). The results for high and low book to market firms are also similar, with no distinction between the estimated betas and the S&P 500 results. While tail risk is associated with more negative returns for both groups of portfolios, there is only a marginal difference across them.

Momentum portfolios, on the other hand, have a different dynamics. While the magnitude of the tail risk betas is still similar to the S&P 500 index regressions, ranging from -0.22 to -0.39 the portfolio formed with losers stocks is consistently more exposed to tail risk. In particular, the winners minus losers (WML) portfolio formed with small (big) firms have a 0.09 (0.17) beta with respect to a one-day lagged tail risk, both being statistically significant.

As a final analysis of portfolio returns exposure to tail risk, we select the 49 industry portfolios available at Kenneth French library and perform the same prediction regressions as before. While the previous analysis focuses on portfolios formed by a fixed firm characteristic, industry portfolios allow us to understand better how different sectors of the economy react to sharp declines in the market returns, measured by tail risk. Figure 3 plot the results for each of the 49 portfolios. In black, we present the estimated tail risk beta, in red the 10% confidence interval, in green the ADS beta and in purple the VIX beta. Although not presented in the figure we also control for the EPU index and the realized variance in these regressions. A careful analysis of the results reveal some interesting features. First, we find a significant heterogeneity on the one day ahead betas for different sectors. While the Beer sector has the smaller beta in magnitude, -0.11, the Coal industry have the biggest one, -0.53, followed by the Mining and Financial sector respectively. Not surprisingly the financial sector (Banks, Insurance and Real State and

Financial Firms) along with the mining sector (Gold, Mines, Coal and Oil) are the ones more exposed to tail risk shocks.

3.6 Crash Insurance

The previous sections documented that risky assets are typically negatively correlated to tail risk. These assets also appear to have a persistent component with respect to tail risk. On the other hand, we also verified that portfolios formed on low market betas minus high market betas provide some insurance against extreme market downturns. Moreover, the relationship between tail risk and Treasury invested ETFs implies a "fly to safety" reaction to rises in extreme, downside, market movements.

While we abstract from the use of option data in the construction of our tail risk measure, put index options naturally provide a hedge against negative market outcomes. Therefore, in this section, we focus on contracts that are explicitly designed to insure against "bad" scenarios. The following analysis relies on put option portfolios, sorted according to moneyness (here moneyness is defined as K/S where K is the strike price and S the spot price of the underlying asset). We form portfolios on a daily basis, with returns calculated as an equally average. A total of five portfolios are built with deep out of the money, out of the money, in the money, at the money and deep at the money options. Moneyness thresholds are defined as follows: $0.9 < DOTM < 0.94$, $0.94 < OTM < 0.97$, $0.97 < ITM < 1.03$, $1.03 < ATM < 1.06$, $1.06 < DATM < 1.10$ for options with maturity between one and 45 days. Although results are robust to the maturity range adopted, we focus on short-term options since they are more likely to react to short-term movements of the underlying index. As in the previous sections, although not reported, we control for the VIX index, realized variance, ADS index, EPU index and, additionally, to our aggregate downside jump measure. The addition of the jump measure in this analysis allows us to differentiate, to some extent, the effect of tail risk between that coming from a pure jump component.

Panel A of table 13 reports the contemporaneous results. In line with our intuition, given the implicit insurance provided by out of the money put, we find a positive monotonic relationship between tail risk and moneyness. In particular, while estimated the estimated beta for the deep out of the money portfolios is 0.09, the same value for the deep in the money options equals 0.04, a more than 50% difference. Also, the estimated beta for a portfolio long on deep out of the money options and short on deep in the money options is positive and statistically significant. In addition to the estimated coefficients, Panel A also present the values for the R^2 statistics for both the multivariate setting and a univariate setting where the sole explanatory variable is the Hellinger Tail Risk. Surprisingly, the tail risk measures explain, on average, approximately 9% of these portfolios variation. In a comparison to the multivariate setting, this number represents around 40% of the overall R^2, indicating a significant contemporaneous relationship between tail risk and put options returns.

Panel B of table 13 complements the preceding analysis by presenting the results for one day ahead prediction regressions. The first striking result is that the persistence of options returns to shocks in tail risk is much stronger in comparison to the market returns analysis. Indeed, the ratio between prediction and contemporaneous betas exceeds 75%, a higher number when compared to the 60% ratio for the market regressions. The case for the portfolios constructed with deep out the money puts options, the portfolio that provides the greatest insurance to market meltdowns, is, even more, stronger, with the estimated beta rising from 0.09 to 0.10 from contemporaneous to prediction regressions. Although smaller when compared to the contemporaneous analysis, the estimated R^2 for the univariate regressions remains high, ranging from 1% to 3%, in comparison to the multivariate R^2 between 3% to 6%.

The last piece of evidence we provide is an analysis of how tradition factor models alphas are related to the estimated portfolios tail risk exposure. Panel C report the estimated alphas for the Fama and French three-factor model and the Fama-French-Cahart four factor models. First, as previously documented in other papers (Kelly and Jiang, 2014), we find a negative, statistically significant, alpha for the DOTM portfolio. That is, selling out of the money puts produce, on average, positive returns that cannot be explained by traditional factor models. Second, the last column of this panel presents the correlation between the estimated prediction betas for the tail risk measure and the alphas for both factor models. Irrespective of the set of factors considered, we find a 72% correlations between the tail risk beta and portfolio alphas, revealing a strong relationship between the abnormal returns and tail risk exposure.

3.7 Portfolios Sorted on Tail Risk

Up until this point, our analysis focused on the contemporaneous and prediction relationship between tail risk and a set of investment returns. From an asset pricing perspective, the theory suggests that assets whose returns co-moves with bad states of nature provide a good hedge and therefore investors typically require low returns for these assets (Almeida et al. (2016), Kelly and Jiang (2014), Yuen (2015)). Our measure of tail risk is derived directly from market returns and is designed in such a way that negative shocks to the aggregate returns imply an increase in tail risk. Therefore, under this setting, the intuition is that assets whose returns (payoffs) are high when our tail risk is high provide insurance for severe stock market movements.

To investigate this risk-returns relationship, this section builds in a portfolio sort analysis using the cross section of returns from 1228 stocks that composed the S&P 500 and Russel 1000 index from 01/02/2008 to 01/07/2015. Altogether, these stocks account for a significant portion of the total market capitalization of the entire cross section available in the Center for Research in Securities Prices (CRSP) database. The sample considered here is comparable to Bollerslev et al. (2016) who selects the components of the S&P 500 index from 1993-2010.

The following analysis resembles the approach of Ang et al. (2006) , Yuen (2015) and complements Almeida et al. (2016). Specifically, we test the risk-returns hypothesis by analyzing returns on portfolios sorted based on tail risk beta. We divide our analysis into two steps. First, we present results for univariate sorts using both the Fama and French and Fama-French-Cahart linear factors models to explain the portfolios excess returns. Later, as in Bollerslev et al. (2016), Ang et al. (2006), Kelly and Jiang (2014), Yuen (2015) among others, we select several stock characteristics that have been previously documented in the literature (Appendix A), and perform double sorts on these characteristics and tail risk beta.

3.7.1 Uni-variate Sorts

To compute the uni-variate portfolio sorts we first need to measure the hedging capacity (or insurance value) of all stocks in the cross section. Resembling the approach of

Ang et al. (2006) and van Oordt and Zhou (2013) [12], we estimate the tail risk beta as follows:

$$\beta_{i,TR} = \frac{cov(R_{i,t}, TR_t)}{\sqrt{var(TR_t)}} \tag{13}$$

For all the empirical application that follows we rely on an overlapping, daily, rolling window regression using the previous 252 trading dates returns to compute the tail risk beta. Given the estimated tail risk betas, we then assign stocks into ten portfolios from the lowest hedging portfolio to the highest one. Given the portfolio's composition, we compute the post-formation returns associated with them over the next day, week (five days) and month (21 days).

Table 14 report the results for this procedure. There are three sets of results, each indicating a different holding period for the post formation portfolio returns. The last three lines present the High minus Low beta portfolio returns followed by the respective risk-adjusted returns for this portfolio respectively. We calculate risk-adjusted returns as the intercept of a time series regression that controls for either the Fama and French three-factor model or the Fama-Frech-Cahart four factor model.

First, is remarkable that for all three holding periods considered we find a strong monotonic relationship between the beta sorted portfolios and excess returns. In particular, the post-formation returns ranges from 3.17% to 3.35% per month for a one week and one month holding period for the portfolio formed by stocks with more negative tail risk beta respectively. The returns than decays monotonically reaching approximately 1% per month for all three holding periods for the portfolios less exposed to tail risk (the insurance portfolios). In fact, the High minus Low portfolio has a, statistically significant, average return of approximately -2.21% across the different holding periods. Moreover, while lower than the unadjusted returns, the estimated alphas for both FF3 and FF4 models varies from -0.72 to -1.30 with only the FF3 alpha for one month holding period not statistically significant.

This first analysis of portfolio sorts is based on raw exposure to tail risk. In a time series analysis we find that the AR(1) coefficient of our measure equals 0.48 and thus it

[12] In fact both Ang et al. (2006) and van Oordt and Zhou (2013) estimate conditional beta. While the first estimate betas conditional on market returns being below the historical average, the later uses a market VaR threshold. In our procedure, we do not need to condition on market returns given the downside characteristic of our measure.

is moderately persistent. Thus, to complement our initial approach and to present some robustness test, we follow Ang et al. (2006) and also re-calculate the portfolio returns for sorting on tail risk innovations. We measure innovations as: (i) the difference between current tail risk and a AR(7) model (the "best" time series model for tail risk). (ii) with parsimony in mind, we also consider a AR(1) version of the innovations. (iii) tail risk surprises measured as 1.65 standard deviations from its mean. As an additional robustness test, providing some evidence for the robustness of the data dimensionality used in the tail risk estimation, we re-estimate our tail risk measure with one-minute intra-day data and re-calculate the tail risk beta portfolio returns.

Table 15 present the results for the robustness uni-variate sorting procedure for a one-day holding period. Overall, irrespective of the measure considered, the main conclusions are the same as the ones reported for the baseline tail risk measure, reassuring our results. Concerning the previous literature that links downside risks and the cross-section of expected returns, we highlight that our results are broadly consistent with (Ang et al., 2006), Bollerslev et al. (2016), (Almeida et al., 2016) and Yuen (2015) with the high minus returns being with a similar magnitude.

The results in this section reveal that stocks with lower tail risk betas tend to have higher returns, in contrast, stocks with higher tail risk betas tend to have lower returns. Nonetheless, section 3.5 provide evidence that firms with different characteristics react differently with respect to tail risk. Therefore we must investigate if the variation in returns across tail risk beta sorted portfolios are related to previous firms characteristics that have been shown to help explain the cross-section of returns.

3.7.2 Bi-variate Sorts

To investigate the predictive power of our tail risk beta, when simultaneously controlling for additional firm-level characteristics, we select a handful of previously document characteristics that help to explain the cross-section of stock returns. In particular we deal with the following alternative characteristics: Size, following Fama and French (1993); Momentum, following Jegadeesh and Titman (1993); Monthly, weekly and daily return Reversal, following Jegadeesh (1990); Illiquidity, following Amihud (2002); Turnover, following Bali et al. (2016); Coskewness, following Harvey and Siddique (2000); Cokurtosis, following Ang et al. (2006); Downside Beta, following Ang et al. (2006); Downside Sigma, following Da et al. (2014); Idiosyncratic Volatility, following Ang et al. (2006) and Max Monthly Return, following Bali et al. (2011).

To implement the double sorts, we first sort stocks into five quantiles according to a given firm characteristics. Then, within each quantile, we sort the firms with respect to their tail risk beta into additional five quantiles. This procedure generates 25 portfolios sorted by tail risk beta and a given firm characteristic. To construct portfolios that are heterogeneous in the characteristic but homogeneous in tail risk beta we average the returns on the beta sorted quantiles across firms characteristics. By doing this, we are left with beta-sorted portfolios that have a small variation in the control variable. Again, we focus our analysis on one-day post formation holding period.

Table 16 present the results for the double sorts, where the last sort is in the tail risk beta. Each line indicates the control variable adopted while columns indicate the portfolios returns. The last column present the FF4 risk-adjusted alphas for the High minus Low portfolio formed on tail risk beta. Similar to the findings for the univariate sorts, we note a monotonic relationship in the portfolio returns. While this is true for all the control variables adopted a reasonable heterogeneity in the High minus Low returns appears. Conspicuously, the lowest value for the risk-adjusted returns is -0.84%, when we control for momentum, while the highest value is -1.74%, when the additional sorting variable is daily reversal. Still, all the returns for the High minus Low portfolios are statistically significant and translate into economically meaningful returns. In fact, at a 10% significance level, with two exceptions for momentum and turnover, all the risk-adjusted alphas are negative and statistically significant.

As in Bollerslev et al. (2016) we investigate is the tail risk beta is capable of "explaining" previously documented anomalies by relying on reverse double sorts. That is, given that for most of the previously documents characteristics, analyzed in table 16, the excess returns on tail risk beta remains significant we investigate the converse hypothesis: can the tail risk beta explain the anomalies previously documented? Table 17 present these results. Similar to Bollerslev et al. (2016) we find that, when we control for tail risk beta, several of the anomalies have a statistical zero risk-adjusted return. In fact, for momentum and turnover - the two characteristics for which the high minus low alphas were not significant in table 16 - both the high minus low returns and alphas are statistically zero. Similar to the results in Bollerslev et al. (2016) we also find a strong negative alpha related to the size effect and a high positive alpha related to the illiquidity effect. Additionally, all the reversals effects are statistically significant, with the monthly reversal having the same magnitude as in Bollerslev et al. (2016), and both the weekly and daily reversals presenting higher alphas.

Finally, an interesting result with respect to the downside beta is noted: while the high minus low alpha for the tail risk beta sorted portfolio remains negative and statistically significant in the double sorts controlling for downside beta the converse is not true. In other terms, after controlling for the tail risk beta we find that the downside anomaly dissipates. We interpret this result as evidence that investors price extreme market movements in contrast to the below average characteristic of the downside beta.

4 Conclusion

We propose a new measure of tail risk based on high-frequency risk-neutralized market returns. This measure presents useful characteristics that contribute to the tail risk literature: first, it can be estimated on a daily basis, without using overlapping data, based only on intraday returns. Second, it's non-parametric approach to risk neutralization offers quick reactions to changes in market conditions. Third, and perhaps most important, the fact that it does not depend on options data allows for possible extensions of our analysis to individual assets and to any market, which presents a series of high-frequency returns.

Our Tail risk series computed from 2008 to 2016, captures the 2008 financial crisis, the 2011 European debt crisis, and has a significant correlation with financial, uncertainty, and macroeconomic measures revealing a clear relationship between market prices and the real economy.

Our extensive analysis of the relationship between different assets' returns and tail risk reveals not only that several asset classes are exposed to market tail risk but also that some strategies provide a significant hedge against risks of extreme market return realizations. In particular, we document a strong fly-to-safety and mean reversion feature of Treasure ETF's, a strong negative relationship between various risky assets returns and tail risk, and the tail risk implications to expected returns on the cross section of assets. Additionally, we also show that tail risk is closely related to put options returns, with long-short strategies on deep out-of-the-money puts and deep in-the-money puts providing a significant hedge against tail risk.

Appendix A:

In this appendix, we give a brief description of the additional control variables we consider in the double sort procedures.

- SIZE: Following Fama and French (1993) firm size is measured at the end of each June by its market value, defined as the stock price multiplied by the number of shares outstanding. We update SIZE annually and use it to explain the following 12 months returns. If a stock is introduced in our dataset after the June cut-off we define its size as the stock price multiplied by the shares outstanding for the first day the stock appears in the data set and repeat this value until the next June breaking point. Following Bollerslev et al. (2016) the final value for SIZE is the natural logarithm of the firms' size.

- MOMENTUM: Following Jegadeesh and Titman (1993) we measure momentum as the gross return of the last 252 trading dates skipping the short term reversal, here defined as the last 21 trading dates. If, for a given date t, we do not have the last 252 days returns for a given stock than we calculate momentum using the maximum observable returns in our sample. If this is less than 50 trading days, we discard this stick for date t cross-section.

- REVERSAL: We calculate three reversal variables following Jegadeesh (1990) and Bali et al. (2016).

  - Monthly Reversal: Is defined as the aggregate return of the last 21 trading days.
  - Weekly Reversal: Is defined as the aggregate return of the last 5 trading days.
  - Daily Reversal: Is defined as the last trading day return.

- ILLIQUIDITY: Following Amihud (2002) we define illiquidity for each stock i at date t as follows:

$$ILLIQ_{i,t} = \frac{1}{T} \sum_{i=1}^{T} \frac{|r_{i,t}|}{volume_{i,t} price_{i,t}} \quad (14)$$

For each data t we use data for the preceding five trading days to calculate illiquidty. $volume_{i,t}$ is stock i date t trading volume, $price_{i,t}$ is stock i date t closing price.

- TURNOVER: Alternatively to illiquidity, following Bali et al. (2016), we define turnover as the average ratio between daily trades and total shares outstanding for the past five days.

- COSKEWNESS: Following Harvey and Siddique (2000), the daily firm co-skewness is defined as the β_{CS} estimate using daily data for the preceding 21 trading dates for asset i for the following regression:

$$R_{i,t} - Rf_t = \alpha + \beta_{MKT}(R_{m,t} - Rf_t) + \beta_{CS}((R_{m,t} - Rf_t))^2 + \epsilon_{i,t} \quad (15)$$

Where $R_{i,t}$, Rf_t and $R_{m,t}$ denote the asset i date t returns, the risk free rate and the market return respectively.

- COKURTOSIS: Similar to Ang et al. (2006), the daily firm co-kurtosis is defined as the β_{CK} estimate using daily data for the preceding 21 trading dates for asset i for the following regression:

$$R_{i,t} - Rf_t = \alpha + \beta_{MKT}(R_{m,t} - Rf_t) + \beta_{CS}((R_{m,t} - Rf_t))^2 + \beta_{CK}((R_{m,t} - Rf_t))^3 + \epsilon_{i,t} \quad (16)$$

Where $R_{i,t}$, Rf_t and $R_{m,t}$ denote the asset i date t returns, the risk free rate and the market return respectively.

- DOWNSIDE BETA: Following Ang et al. (2006), the daily downside beta is defined as follows:

$$\beta_i^- = \frac{cov(R_i, R_m | R_m < \mu_m)}{var(R_m | R_m < \mu_m)} \quad (17)$$

Where R_i, R_m, μ_m denote the asset i returns, the market returns and the market mean. We use the last 252 trading dates returns to calculate the downside beta. As for MOMENTUM, if, for a given date t, we do not have the last 252 days returns for a given stock than we calculate momentum using the maximum observable returns in our sample. If this is less than 50 trading days we discard this stick for date t cross-section.

- DOWNSIDE SIGMA: Following Da et al. (2014), the daily downside sigma is defined as follows:

$$\sigma_i^- = \sqrt{var(R_i | R_m < \mu_m)} \tag{18}$$

Where R_i, R_m, μ_m denote the asset i returns, the market returns and the market mean. We use the last 252 trading dates returns to calculate the downside beta. As for MOMENTUM, if, for a given date t, we do not have the last 252 days returns for a given stock than we calculate momentum using the maximum observable returns in our sample. If this is less than 50 trading days we discard this stick for date t cross-section.

- IVOL: Following Ang et al. (2006), the daily firm idiosyncratic volatility is compute as the standard deviation for the residuals from the following regression for the last 21 trading days:

$$R_{i,t} - Rf_t = \alpha + \beta_{MKT}(R_{m,t} - Rf_t) + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \epsilon_{i,t} \tag{19}$$

Where $R_{i,t}$, Rf_t, $R_{m,t}$, SMB_t and HML_t denote the asset i date t returns, the risk free rate, the market return, the SMB_t and HML_t portfolios of Fama and French (1993) respectively.

- MAX: Following Bali et al. (2011) MAX is defined as the maximum return of a given firm in the preceding 21 trading days.

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VIX vs. Hellinger Tail Risk Moving Average

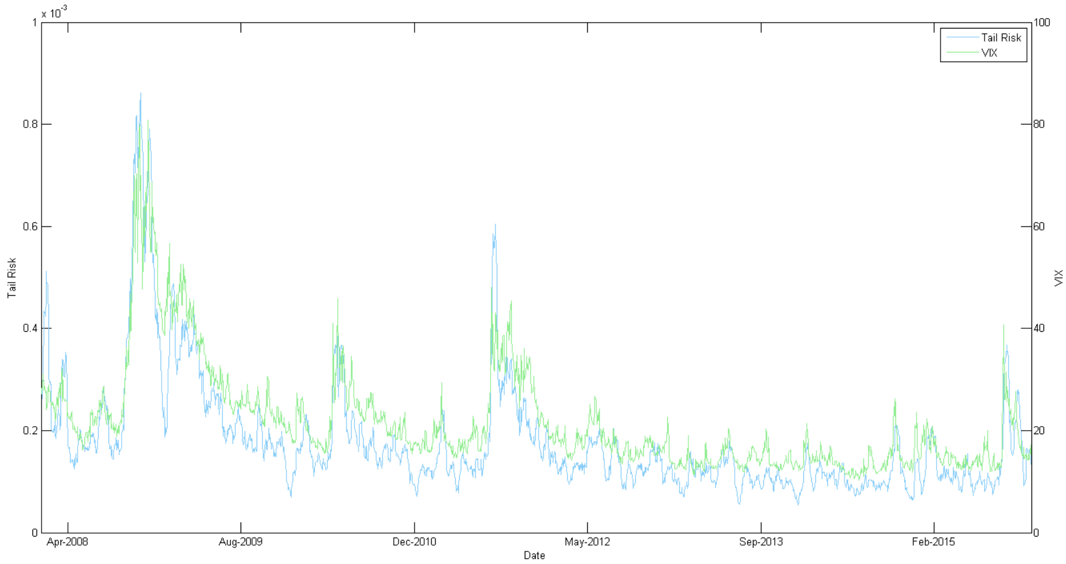

Figure 1: This figure plots a 10 days moving average for the estimated Hellinger Tail Risk and the daily VIX index.

VIX vs. Hellinger Tail Risk

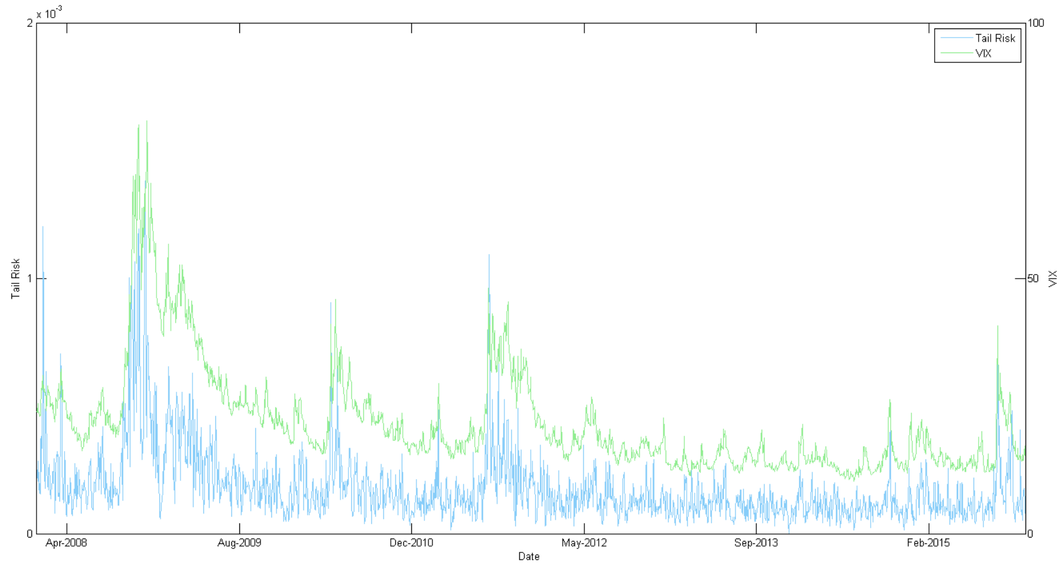

Figure 2: This figure plots the estimated, daily, Hellinger Tail Risk and the VIX index.

Industry Portfolios Betas

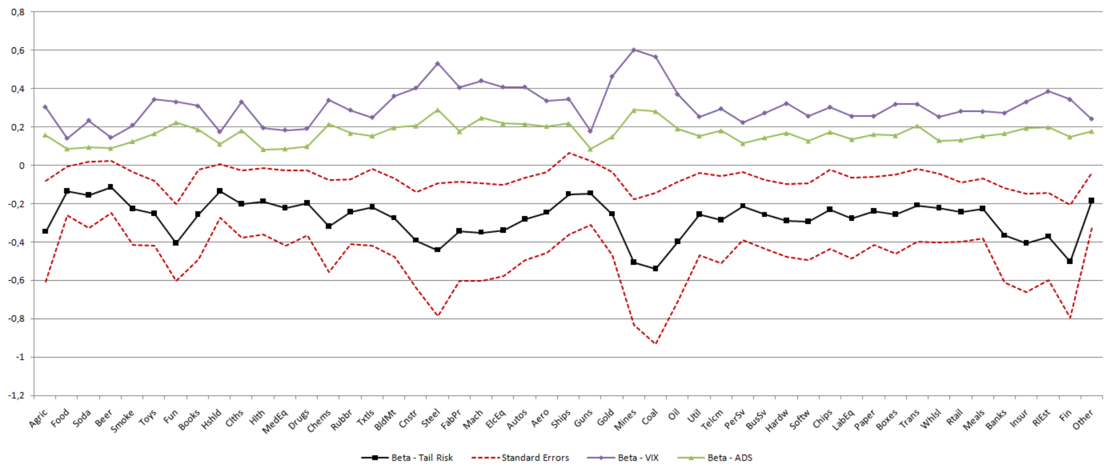

Figure 3: This figure plots the estimated betas for the Hellinger Tail Risk, CBOE VIX index and the Aruba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB for multivariate regressions where the endogenous variable are returns on the 49 Industry Portfolios from Kenneth French library. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance measured as the squared sum of intraday, 15 minutes interval, S&P 500 returns, the Aruba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). Red dashed lines indicate the 10% confidence bands for the Hellinger Tail Risk beta calculated using Newey and West variance matrix with five lags.

Table 1: **Option Cross Section Around the World**

Country	Strikes	Maturities	Outstanding	Volume	Date	Source
Argentina			As of 13/05/2016 inexistent.			
China			As of 13/05/2016 inexistent.			
Saudi Arabia			As of 13/05/2016 inexistent.			
Russia			As of 13/05/2016 inexistent.			
Brazil	59	10	1129	-	13/05/2016	TMX
Canada	93	6	810	-	20/04/2016	BM&F Bovespa
USA	277	30	9057	1164663	31/08/2015	WRDS
Japan	98	23	2443	-	13/05/2016	JPX
South Korea	45	10	250	-	13/05/2016	EUREX
Turkey	21	4	149	-	13/05/2016	BORSA ISTANBUL
Germany	129	16	1668	39078	30/12/2013	WRDS
France	50	13	626	19176	30/12/2013	WRDS
Italy	86	14	974	8750	30/12/2013	WRDS
United Kingdom	64	11	762	27068	30/12/2013	WRDS
Australia	68	10	611	-	13/05/2016	ASX

Table 2: **Correlations**

	Panel A:						
	S&P 500	VIX	RV	BTX	EPU	Eq. EPU	D. Jump
Hellinger Tail Risk	-0.24	0.65	0.65	0.59	0.34	0.41	0.39
Expected Shortfall	-0.37	0.61	0.76	0.51	0.29	0.40	0.37
Moving Average 10 Days	-0.04	0.90	0.68	0.68	0.43	0.47	0.17

	Panel A:						
	ADS	Noise	Spread	Crashes	Fin. Beta	Yield Spread	FEAR
Hellinger Tail Risk	-0.42	0.49	0.47	0.32	0.35	0.48	0.07
Expected Shortfall	-0.37	0.44	0.42	0.29	0.31	0.43	0.12
Moving Average 10 Days	-0.60	0.73	0.71	0.50	0.52	0.72	-0.05

This table present the correlation coefficients between the Hellinger Tail Risk, its physical counterpart, the 10 days moving average of the Hellinger Tail Risk and various other crash sensitive measures. VIX indicates the CBOE VIX index; RV stands for realized variance measure as the squared sum of intraday, 15 minutes interval, S&P 500 returns; BTX stands for the the left risk neutral jump tail variation of Bollerslev et al. (2015); EPU and Eq. EPU are the Economic Policy Uncertainty Index and Equity Uncertainty Index for the U.S. economy from Baker et al. (2016); D. Jump stands for the downside jump measure constructed as in 3.1; ADS stands for the the Aruba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB; Noise stands for the noise measure of Hu et al. (2013); Spread stands for the Emerging Markets spread with the spot treasury curve from BofA (Available at St. Louis FRED); Crashes stands for the Stock Market Crashes measure by the Cleveland FRB as the ration of the current value for the S&P 500 index and its maximum over the last 365 days; Fin. Beta stands for the measure of the contribution of the banking sector to overall stock market volatility from the Cleveland FRB; Yield Spread stands for the difference between Moody's Seasoned Baa Corporate Bond and 10-Year Treasury Constant Maturity from St. Louis FRED; FEAR stands for the Google Trends measure of investors sentiment from Da et al. (2014).

Table 3: **Contemporaneous Downside Jump Regressions**

	Tail Risk	VIX	RV	EPU	ADS
	Panel A: Univariate Regressions				
β	0.11	0.04	0.07	0.03	-0.01
$t-stat.$	(6.69)	(3.55)	(3.41)	(2.62)	(-0.78)
R^2	0.16	0.03	0.07	0.01	0.00
	Panel B: Multivariate Regressions				
Tail Risk		0.13	0.12	0.12	0.12
		(7.70)	(7.50)	(7.45)	(7.58)
VIX		-0.04	-0.05	-0.05	-0.03
		(-4.18)	(-4.38)	(-4.46)	(-2.12)
RV			0.03	0.03	0.02
			(1.49)	(1.49)	(1.42)
EPU				0.00	0.00
				(0.37)	(0.29)
ADS					0.00
					(0.29)
R^2		0.17	0.17	0.17	0.18

This table present the results for contemporaneous regressions where the endogenous variable is the Downside Jump measure calculated in 3.1. Panel A present the results for univariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance measured as the squared sum of intraday, 15 minutes interval, S&P 500 returns, the Aruba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). Panel B present multivariate regressions where additional to the tail risk measure we also control for the above mentioned crash sensitive indexes. t-statistics are calculated using Newey-West variance matrix with five lags.

Table 4: **Contemporaneous Downside Jump Regressions - Segregated**

	D. Jump 1	D. Jump 2	D. Jump 3	D. Jump 4	D. Jump 5	Aggregate
Tail Risk	0.26	0.24	0.11	0.07	0.10	0.12
	(4.39)	(4.99)	(5.04)	(4.30)	(5.75)	(7.58)
VIX	0.07	-0.08	-0.04	-0.03	-0.03	-0.03
	(0.72)	(-1.96)	(-2.20)	(-1.92)	(-2.46)	(-2.12)
RV	0.13	0.05	0.02	0.00	0.01	0.02
	(2.01)	(1.11)	(1.10)	(0.42)	(0.63)	(1.42)
EPU	0.04	-0.02	0.01	0.01	0.00	0.00
	(0.91)	(-0.78)	(0.79)	(0.70)	(-0.43)	(0.29)
ADS	0.11	0.04	0.04	0.01	0.03	0.03
	(1.23)	(1.12)	(2.33)	(0.51)	(2.17)	(1.92)
Constant	1.99	0.43	0.12	0.05	0.07	0.25
	(35.67)	(18.30)	(11.79)	(9.28)	(10.50)	(28.67)
R^2	0.05	0.08	0.08	0.06	0.09	0.18
R^{2*}	0.04	0.06	0.06	0.05	0.07	0.16

This table present the results for contemporaneous regressions where the endogenous variable is the Downside Jump measure calculated in 3.1 and its five components. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance measured as the squared sum of intraday, 15 minutes interval, S&P 500 returns, the Aruba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). In the last line we also present the R^2* for univariate regressios where the only explanatory variable is the Hellinger Tail Risk. t-statistics are calculated using Newey-West variance matrix with five lags.

Table 5: **Downside Jump Prediction Regressions**

	Tail Risk	VIX	RV	EPU	ADS
	Panel A: Bivariate Regressions				
β	0.02	0.01	0.03	0.01	-0.01
$t-stat.$	(3.03)	(1.43)	(4.25)	(0.88)	(-0.75)
R^2	0.22	0.22	0.23	0.22	0.22
	Panel B: Multivariate Regressions				
Tail Risk		0.03	0.02	0.02	0.02
		(3.11)	(2.15)	(2.14)	(2.14)
VIX		-0.01	-0.02	-0.02	-0.02
		(-1.50)	(-2.69)	(-2.77)	(-2.29)
RV			0.03	0.03	0.03
			(4.37)	(4.37)	(4.42)
EPU				0.00	0.00
				(0.19)	(0.19)
ADS					0.00
					(-0.08)
R^2		0.22	0.23	0.23	0.23

This table present the results for prediction regressions where the endogenous variable is the Downside Jump measure calculated in 3.1. Panel A present the results for bi-variate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance measured as the squared sum of intraday, 15 minutes interval, S&P 500 returns, the Aruba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016) in addition to the lag of the realized jump measure. Panel B present multivariate regressions where additional to the tail risk measure we also control for the above mentioned crash sensitive indexes. t-statistics are calculated using Newey-West variance matrix with five lags.

Table 6: **Downside Jump Prediction Regressions - Segregated**

	D. Jump 1	D. Jump 2	D. Jump 3	D. Jump 4	D. Jump 5	Aggregate
Tail Risk	0.26	0.07	0.05	0.02	0.04	0.02
	(4.49)	(3.12)	(2.73)	(1.69)	(2.86)	(2.14)
VIX	-0.11	-0.03	-0.02	-0.02	-0.03	-0.02
	(-1.56)	(-0.92)	(-1.09)	(-1.99)	(-2.87)	(-2.29)
RV	0.05	0.04	0.03	0.03	0.03	0.03
	(1.04)	(2.17)	(1.22)	(1.49)	(2.65)	(4.42)
EPU	0.05	-0.03	0.00	0.01	0.00	0.00
	(1.01)	(-1.53)	(-0.20)	(1.00)	(0.13)	(0.19)
ADS	-0.01	0.01	0.02	0.00	0.01	0.00
	(-0.14)	(0.21)	(1.55)	(-0.19)	(0.80)	(-0.08)
Lag Jump	0.27	0.25	0.14	0.00	0.07	0.42
	(11.07)	(7.00)	(2.26)	(0.09)	(1.61)	(9.94)
Constant	1.45	0.32	0.10	0.05	0.06	0.14
	(23.51)	(15.46)	(11.31)	(8.45)	(9.99)	(14.49)
R^2	0.11	0.09	0.05	0.02	0.04	0.23
R^2*	0.04	0.02	0.02	0.01	0.02	0.07

This table present the results for prediction regressions where the endogenous variable is the Downside Jump measure calculated in 3.1 and its five components. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance measured as the squared sum of intraday, 15 minutes interval, S&P 500 returns, the Aruba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). In the last line we also present the R^2* for univariate regressios where the only explanatory variable is the Hellinger Tail Risk. t-statistics are calculated using Newey-West variance matrix with five lags.

Table 7: **Weekly Downside Jump Prediction Regressions - Segregated**

	D. Jump 1	D. Jump 2	D. Jump 3	D. Jump 4	D. Jump 5	Aggregate
Tail Risk	1.13	0.51	0.24	0.23	0.12	0.20
	(2.92)	(2.19)	(2.00)	(3.52)	(2.08)	(2.90)
VIX	-0.68	-0.49	-0.14	-0.11	-0.15	-0.24
	(-1.57)	(-2.26)	(-1.58)	(-1.75)	(-2.55)	(-3.05)
RV	-0.30	-0.06	0.01	0.02	0.03	-0.03
	(-2.29)	(-0.65)	(0.14)	(0.38)	(1.18)	(-1.02)
EPU	-0.11	-0.01	0.03	-0.02	0.03	0.02
	(-0.29)	(-0.07)	(0.35)	(-0.59)	(0.48)	(0.29)
ADS	-0.31	-0.16	0.05	0.00	-0.02	-0.09
	(-0.70)	(-0.93)	(0.73)	(-0.09)	(-0.39)	(-1.35)
Lag Jump	0.36	0.32	0.19	0.14	0.25	0.51
	(6.21)	(5.90)	(3.27)	(1.91)	(3.65)	(9.61)
Constant	6.07	1.39	0.45	0.21	0.25	0.59
	(9.52)	(9.19)	(7.56)	(6.97)	(7.13)	(8.89)
R^2	0.17	0.14	0.08	0.13	0.10	0.31
R^2*	0.04	0.03	0.03	0.09	0.02	0.07

This table present the results for weekly prediction regressions where the endogenous variable is the Downside Jump measure calculated in 3.1 and its five components. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance measured as the squared sum of intraday, 15 minutes interval, S&P 500 returns, the Aruba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). In the last line we also present the R^2* for univariate regressios where the only explanatory variable is the Hellinger Tail Risk. t-statistics are calculated using Newey-West variance matrix with five lags.

Table 8: **S&P 500 Regressions**

	$S\&P500_t$	$S\&P500_{t+1}$	$S\&P500_{t+2}$
TR_t	-0.51	-0.30	0.15
	(-4.62)	(-2.42)	(1.69)
VIX_t	-0.17	0.28	-0.05
	(-1.60)	(3.09)	(-0.60)
$Variance_t$	0.29	0.11	0.08
	(2.18)	(0.86)	(1.12)
EPU_t	0.13	-0.04	0.01
	(2.79)	(-0.83)	(0.13)
ADS_t	-0.12	0.16	0.12
	(-2.43)	(3.05)	(2.32)
Cons	0.03	0.03	0.03
	(1.02)	(1.06)	(1.03)
$S\&P500_t$		-0.13	-0.05
		(-3.46)	(-0.87)
R^2	0.09	0.04	0.02
R^2*	0.06	0.00	0.01

This table present the results for contemporaneous and prediction regressions where the endogenous variable is the daily return on the S&P 500 index. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance measured as the squared sum of intraday, 15 minutes interval, S&P 500 returns, the Aruba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). In the last line we also present the R^2* for univariate regressios where the only explanatory variable is the Hellinger Tail Risk. t-statistics are calculated using Newey-West variance matrix with five lags.

Table 9: **Treasury Regressions**

	$Treasury_t$	$Treasury_{t+1}$	$Treasury_{t+2}$
TR_t	0.12	0.01	-0.04
	(5.45)	(0.42)	(-2.22)
VIX_t	0.05	0.00	0.06
	(1.73)	(-0.09)	(2.70)
$Variance_t$	-0.07	-0.01	-0.02
	(-2.12)	(-0.47)	(-1.38)
EPU_t	-0.04	0.01	0.00
	(-3.10)	(0.66)	(-0.12)
ADS_t	0.04	0.00	0.01
	(2.64)	(0.01)	(0.87)
Cons	0.01	0.01	0.01
	(1.09)	(1.05)	(1.04)
$Treasury_t$		-0.05	-0.03
		(-2.11)	(-1.27)
R^2	0.04	0.00	0.01
R^2*	0.02	0.00	0.00

This table present the results for contemporaneous and prediction regressions where the endogenous variable is the daily return on the iShares Treasury Bond ETF. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance measured as the squared sum of intraday, 15 minutes interval, S&P 500 returns, the Aruba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). In the last line we also present the R^2* for univariate regressios where the only explanatory variable is the Hellinger Tail Risk. t-statistics are calculated using Newey-West variance matrix with five lags.

Table 10: **Beta Sorted Portfolio Regressions**

Portfolio	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	High - Low
	Panel A: Contemporaneous Regressions										
SPX	-0.75	-0.65	-0.59	-0.52	-0.49	-0.44	-0.40	-0.34	-0.17	-0.04	0.71
	(-4.33)	(-4.41)	(-4.57)	(-4.53)	(-4.54)	(-4.61)	(-4.45)	(-4.64)	(-3.20)	(-1.06)	(4.31)
VIX	-0.31	-0.27	-0.24	-0.21	-0.18	-0.15	-0.13	-0.08	-0.04	0.02	0.33
	(-1.61)	(-1.65)	(-1.68)	(-1.62)	(-1.51)	(-1.41)	(-1.33)	(-1.02)	(-0.56)	(0.42)	(1.86)
RV	0.41	0.33	0.31	0.25	0.24	0.19	0.16	0.08	-0.03	-0.10	-0.51
	(2.30)	(2.18)	(2.24)	(2.16)	(2.19)	(1.94)	(1.83)	(1.28)	(-0.57)	(-1.58)	(-2.33)
EPU	0.24	0.21	0.18	0.15	0.13	0.13	0.10	0.08	0.05	0.04	-0.21
	(2.95)	(3.10)	(3.05)	(2.86)	(2.52)	(2.85)	(2.53)	(2.39)	(2.06)	(2.24)	(-2.64)
ADS	-0.29	-0.23	-0.21	-0.18	-0.16	-0.14	-0.13	-0.11	-0.08	-0.05	0.23
	(-2.59)	(-2.68)	(-2.76)	(-2.70)	(-2.63)	(-2.64)	(-2.57)	(-2.50)	(-2.19)	(-1.76)	(2.42)
R^2	0.06	0.07	0.08	0.07	0.07	0.08	0.08	0.08	0.06	0.04	0.07
	Panel B: Prediction Regressions										
SPX	-0.44	-0.38	-0.36	-0.33	-0.33	-0.29	-0.27	-0.23	-0.18	-0.11	0.34
	(-2.45)	(-2.38)	(-2.46)	(-2.34)	(-2.39)	(-2.36)	(-2.23)	(-1.96)	(-1.86)	(-2.22)	(2.27)
VIX	0.52	0.40	0.36	0.32	0.30	0.26	0.23	0.17	0.12	0.06	-0.45
	(3.39)	(3.11)	(3.12)	(2.92)	(3.00)	(2.93)	(2.77)	(2.24)	(2.14)	(1.53)	(-3.24)
RV	0.09	0.09	0.09	0.10	0.11	0.10	0.10	0.11	0.10	0.07	-0.05
	(0.51)	(0.56)	(0.60)	(0.72)	(0.79)	(0.76)	(0.82)	(0.82)	(0.84)	(1.19)	(-0.33)
EPU	-0.03	-0.02	-0.02	-0.02	-0.03	-0.02	-0.02	-0.02	-0.03	-0.01	0.03
	(-0.36)	(-0.33)	(-0.32)	(-0.37)	(-0.50)	(-0.44)	(-0.44)	(-0.53)	(-1.13)	(-0.57)	(0.33)
ADS	0.20	0.17	0.15	0.14	0.13	0.12	0.11	0.09	0.05	0.01	-0.20
	(1.85)	(2.02)	(1.97)	(2.12)	(2.17)	(2.23)	(2.27)	(2.12)	(1.66)	(0.23)	(-2.05)
Lag Ret.	0.02	-0.01	-0.04	-0.02	-0.04	-0.04	-0.02	0.00	0.09	0.23	0.01
	(0.51)	(-0.28)	(-1.03)	(-0.49)	(-1.03)	(-0.92)	(-0.51)	(0.06)	(1.59)	(6.52)	(0.14)
R^2	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.04	0.07	0.02

This table present the results for contemporaneous (Panel A) and prediction (Panel B) regressions for the NYSE/AMEX Market Beta sorted portfolios from CRSP. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance measured as the squared sum of intraday, 15 minutes interval, S&P 500 returns, the Aruba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016) and the lagged returns for prediction regressions. t-statistics are calculated using Newey-West variance matrix with five lags.

Table 11: **Variance Sorted Portfolio Regressions**

Portfolio	1	2	3	4	5	6	7	8	9	10	High - Low
					Panel A: Contemporaneous Regressions						
SPX	-0.51	-0.58	-0.56	-0.57	-0.53	-0.49	-0.44	-0.35	-0.26	-0.08	0.43
	(-4.36)	(-4.69)	(-4.53)	(-4.53)	(-4.41)	(-4.23)	(-4.42)	(-4.32)	(-4.00)	(-1.85)	(4.60)
VIX	-0.17	-0.21	-0.24	-0.23	-0.22	-0.21	-0.16	-0.10	-0.05	0.02	0.19
	(-1.29)	(-1.47)	(-1.68)	(-1.63)	(-1.67)	(-1.65)	(-1.48)	(-1.18)	(-0.73)	(0.45)	(1.75)
RV	0.13	0.25	0.30	0.31	0.28	0.25	0.20	0.11	0.04	-0.08	-0.21
	(1.70)	(2.11)	(2.24)	(2.24)	(2.17)	(2.12)	(2.02)	(1.46)	(0.61)	(-1.13)	(-2.51)
EPU	0.17	0.18	0.18	0.17	0.16	0.15	0.12	0.09	0.07	0.03	-0.14
	(2.84)	(3.06)	(3.09)	(3.12)	(3.01)	(2.75)	(2.74)	(2.37)	(2.12)	(1.28)	(-2.54)
ADS	-0.27	-0.23	-0.20	-0.19	-0.17	-0.17	-0.14	-0.11	-0.08	-0.02	0.25
	(-2.73)	(-2.60)	(-2.58)	(-2.67)	(-2.67)	(-2.83)	(-2.76)	(-2.63)	(-2.21)	(-0.60)	(2.89)
R^2	0.06	0.07	0.07	0.07	0.08	0.08	0.08	0.08	0.07	0.04	0.05
					Panel B: Prediction Regressions						
SPX	-0.28	-0.34	-0.35	-0.36	-0.32	-0.31	-0.28	-0.24	-0.19	-0.14	0.12
	(-2.22)	(-2.45)	(-2.60)	(-2.46)	(-2.45)	(-2.37)	(-2.21)	(-2.05)	(-1.74)	(-1.54)	(1.68)
VIX	0.32	0.38	0.38	0.35	0.31	0.28	0.24	0.20	0.14	0.08	-0.24
	(2.80)	(3.21)	(3.38)	(3.14)	(3.02)	(2.76)	(2.81)	(2.53)	(2.12)	(1.58)	(-2.64)
RV	0.06	0.06	0.06	0.09	0.08	0.11	0.11	0.12	0.13	0.13	0.06
	(0.46)	(0.44)	(0.48)	(0.64)	(0.64)	(0.78)	(0.81)	(0.87)	(0.94)	(1.08)	(1.05)
EPU	-0.03	-0.01	-0.03	-0.01	-0.02	-0.02	-0.02	-0.03	-0.03	-0.02	0.01
	(-0.44)	(-0.20)	(-0.43)	(-0.25)	(-0.28)	(-0.41)	(-0.50)	(-0.86)	(-0.86)	(-1.23)	(0.17)
ADS	0.07	0.13	0.15	0.16	0.15	0.13	0.12	0.11	0.09	0.05	-0.03
	(0.85)	(1.58)	(1.97)	(2.20)	(2.34)	(2.26)	(2.46)	(2.49)	(2.42)	(2.00)	(-0.39)
Lag Ret.	0.11	0.02	-0.04	-0.06	-0.05	-0.05	-0.04	-0.00	0.08	0.22	0.14
	(3.15)	(0.65)	(-0.99)	(-1.34)	(-1.08)	(-1.15)	(-0.95)	(-0.02)	(1.38)	(3.34)	(4.10)
R^2	0.03	0.02	0.02	0.03	0.02	0.03	0.03	0.03	0.04	0.09	0.03

This table present the results for contemporaneous (Panel A) and prediction (Panel B) regressions for the NYSE/AMEX Market Variance sorted portfolios from CRSP. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance measured as the squared sum of intraday, 15 minutes interval, S&P 500 returns, the Aruba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016) and the lagged returns for prediction regressions. t-statistics are calculated using Newey-West variance matrix with five lags.

Table 12: **Additional Portfolios Regressions**

	Small	Big	SMB	High	Lo	HML	Win S.	Looser S.	WML S.	Win B.	Looser B.	WML B.
Tail Risk	-0.31	-0.29	-0.03	-0.33	-0.27	-0.06	-0.30	-0.37	0.09	-0.22	-0.39	0.17
	(-2.65)	(-2.38)	(-0.81)	(-2.45)	(-2.32)	(-1.67)	(-2.50)	(-2.55)	(2.31)	(-2.00)	(-2.68)	(2.97)
VIX	0.30	0.27	0.02	0.28	0.26	0.02	0.30	0.37	-0.06	0.27	0.38	-0.11
	(2.95)	(3.11)	(0.62)	(2.39)	(3.28)	(0.43)	(3.01)	(3.11)	(-1.45)	(3.21)	(2.95)	(-1.50)
Variance	0.02	0.11	-0.08	0.09	0.10	-0.01	0.05	0.06	-0.04	0.04	0.17	-0.12
	(0.21)	(0.87)	(-2.06)	(0.61)	(0.84)	(-0.21)	(0.39)	(0.52)	(-1.26)	(0.39)	(1.10)	(-1.69)
EPU	-0.01	-0.04	0.03	-0.01	-0.04	0.03	-0.01	-0.03	0.02	-0.03	-0.03	0.00
	(-0.13)	(0.04)	(1.26)	(-0.20)	(-0.89)	(1.05)	(-0.17)	(-0.50)	(0.95)	(-0.72)	(0.06)	(0.07)
ADS	0.14	0.16	-0.02	0.16	0.15	0.01	0.17	0.12	0.03	0.18	0.19	-0.02
	(2.07)	(3.07)	(-0.82)	(2.09)	(3.04)	(0.28)	(2.62)	(1.45)	(0.86)	(3.44)	(2.13)	(-0.26)
Cons	0.05	0.04	0.01	0.04	0.05	-0.01	0.05	0.06	-0.01	0.04	0.03	0.00
	(1.25)	(1.38)	(0.44)	(0.99)	(1.73)	(-0.40)	(1.41)	(1.27)	(-0.44)	(1.43)	(0.76)	(0.15)
Lag Return	-0.11	-0.12	-0.08	-0.09	-0.12	-0.03	-0.09	-0.02	0.18	-0.08	-0.03	0.06
	(-2.96)	(-3.21)	(-2.79)	(-2.08)	(-3.07)	(-0.65)	(-2.47)	(-0.55)	(4.96)	(-2.35)	(-0.83)	(1.43)
R^2	0.03	0.04	0.02	0.02	0.04	0.00	0.03	0.02	0.04	0.03	0.02	0.01

This table present the results for one day ahead prediction regressions where the endogenous variables, indicated in each column, are the returns on (1) Small Firms Portfolio, (2) Big Firms Portfolio, (3) SMB Firms Portfolios, (4) High Book to Market Firms portfolios, (5) Low Book to Market Firms Portfolios, (6) HML Firms Portfolios, (7) Double sorted Winners and Small firms portfolios, (8) Double sorted Looser and Small firms portfolios, (9) WML Small firms portfolios, (10) Double sorted Winners and Big firms portfolios, (11) Double sorted Winners and Big firms portfolios, (12) WML Big firms portfolios. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance measured as the squared sum of intraday, 15 minutes interval, S&P 500 returns, the Aruba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). t-statistics are calculated using Newey-West variance matrix with five lags.

Table 13: **Options Portfolios Prediction Regressions**

	DOTM	OTM	ATM	ITM	DITM	DOTM-DITM	Corr.
			Panel A: Contemporaneous				
Tail Risk	0.09	0.08	0.07	0.05	0.04	0.05	
	(2.69)	(2.73)	(2.68)	(2.70)	(2.68)	(2.41)	
R^2	0.21	0.21	0.22	0.22	0.22	0.17	
R^2*	0.08	0.08	0.08	0.11	0.12	0.06	
			Panel B: Prediction				
Tail Risk -	0.10	0.08	0.06	0.04	0.03	0.06	
	(2.60)	(2.56)	(2.05)	(2.08)	(2.11)	(2.42)	
R^2	0.04	0.03	0.03	0.04	0.06	0.02	
R^2*	0.02	0.02	0.01	0.02	0.03	0.01	
			Panel C: Factor Model				
FF3 Alpha	-0.05	-0.03	0.03	0.04	0.02	-0.07	
	(-5.31)	(-3.99)	(3.87)	(11.33)	(10.72)	(-8.74)	-0.72
FF3+MOM Alpha	-0.05	-0.04	0.03	0.04	0.02	-0.07	
	(-5.40)	(-4.05)	(3.91)	(11.53)	(10.85)	(-8.85)	-0.72

This table present the results for contemporaneous and one day ahead prediction regressions where the endogenous variables are portfolios formed on index options according to the options moneyness for put options with expiration dates between 1 and 45 days from the portfolio formation date. The first three lines present the estimated beta, t-statistics and R^2 from multivariate contemporaneous regressions where the explanatory variables are are the Hellinger Tail Risk, the CBOE VIX index, the realized variance measured as the squared sum of intraday, 15 minutes interval, S&P 500 returns, the Aruba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016), the Downside Jump measure. The fourth line present the estimated R^2 for a regression where the Hellinger Tail Risk is the only explanatory variable. Lines 5-8 present the same statistics as lines 1-4 for prediction regressions where the lag of the portfolio return is a additional explanatory variable. Lines 9-12 present the alphas and t-statistics from Fama and French three factors model and Fama-French-Cahart four factor models respectively. The last column present the correlation between alphas and multivariate regressions Hellinger Tail Risk betas. t-statistics are calculated using Newey-West variance matrix with five lags.

Table 14: **Tail Risk Sorted Portfolios**

Portfolio	Ret.	t	Ret.	t	Ret.	t
1.00	3.28	(2.90)	3.17	(3.22)	3.35	(2.98)
2.00	2.31	(2.54)	2.20	(2.97)	2.32	(3.16)
3.00	2.01	(2.49)	1.97	(3.02)	2.01	(3.21)
4.00	1.83	(2.46)	1.78	(2.98)	1.85	(3.28)
5.00	1.68	(2.43)	1.70	(3.11)	1.73	(3.52)
6.00	1.56	(2.44)	1.42	(2.74)	1.48	(3.24)
7.00	1.41	(2.35)	1.42	(2.94)	1.45	(3.40)
8.00	1.38	(2.51)	1.27	(2.90)	1.30	(3.32)
9.00	1.08	(2.10)	1.13	(2.71)	1.24	(3.54)
10.00	0.97	(1.83)	1.03	(2.34)	1.17	(3.09)
High - Low	-2.31	(-3.03)	-2.14	(-3.08)	-2.18	(-2.50)
FF3 Alpha	-1.30	(-2.77)	-1.11	(-2.48)	-0.72	(-1.42)
FF4 Alpha	-1.24	(-2.73)	-1.06	(-2.53)	-0.76	(-1.99)

This table present the returns, Fama and French three factor model alpha and Fama-French-Cahart four factor model alpha for portfolios sorted according to their Hellinger Tail Risk Beta (we consider a total of 1228 stocks, the constituents of the S&P 500 and Russel 1000 index, ranging from 01/02/2008 to 11/12/2015). The Hellinger Tail Risk betas are estimating using returns over the 252 trading days prior to portfolio formation. After portfolio assignment we track returns one day, one week (5 days) and one month (21 days) post formation. To facilitate comparison we convert daily returns and alphas into monthly percentages by multiplying them by 21 and 5 for daily and weekly holding periods respectively.T-statistics are calculated using Newey and West variance matrix.

Table 15: **Tail Risk Sorted Portfolios - Robustness**

Model	(1)		(2)		(3)		(4)	
	Ret.	t	Ret.	t	Ret.	t	Ret.	t
1	2.95	(2.72)	2.94	(2.64)	3.06	(3.02)	2.97	(2.78)
2	2.00	(2.25)	2.14	(2.38)	2.14	(2.68)	1.97	(2.34)
3	1.92	(2.42)	2.01	(2.48)	1.77	(2.42)	1.63	(2.18)
4	1.65	(2.26)	1.73	(2.31)	1.72	(2.49)	1.65	(2.36)
5	1.65	(2.43)	1.74	(2.54)	1.70	(2.55)	1.67	(2.54)
6	1.49	(2.35)	1.56	(2.42)	1.48	(2.25)	1.50	(2.40)
7	1.44	(2.42)	1.53	(2.57)	1.43	(2.28)	1.64	(2.73)
8	1.38	(2.43)	1.46	(2.66)	1.36	(2.22)	1.39	(2.32)
9	1.12	(2.12)	1.41	(2.77)	1.38	(2.20)	1.46	(2.43)
10	1.49	(2.56)	1.32	(2.61)	1.47	(2.15)	1.61	(2.29)
High - Low	-1.46	(-2.31)	-1.61	(-2.24)	-1.60	(-3.02)	-1.36	(-2.12)
FF3 Alpha	-1.47	(-2.29)	-1.62	(-2.27)	-1.19	(-2.97)	-0.95	(-1.81)
FF4 Alpha	-1.49	(-2.32)	-1.62	(-2.28)	-1.11	(-2.94)	-0.72	(-2.06)

This table present the returns, Fama and French three factor model alpha and Fama-French-Cahart four factor model alpha for portfolios sorted according to their Hellinger Tail Risk Beta (we consider a total of 1228 stocks, the constituents of the S&P 500 and Russel 1000 index, ranging from 01/02/2008 to 11/12/2015). The Hellinger Tail Risk betas are estimating using returns over the 252 trading days prior to portfolio formation. In Model (1) stocks are sorted according to their exposures to $TR - E[TR]$ where $E[TR]$ is estimated by fitting a AR(7) model (the "best" fitting time series model for the Hellinger Tail Risk), in Model (2) we adopt a more parsimonious AR(1) setting to calculate $E[TR]$, in Model (3) stocks are sorted based on their exposures to surprises in the Hellinger Tail Risk, defined as the periods were the Tail Risk excedes 1.65 standard deviations from its mean, in Model (4) stocks are sorting according to their exposure to the Hellinger Tail Risk calculated using one minute intra day data. After portfolio assignment we track returns one day, one week (5 days) and one month (21 days) post formation. To facilitate comparison we convert daily returns and alphas into monthly percentages by multiplying them by 21 and 5 for daily and weekly holding periods respectively.T-statistics are calculated using Newey and West variance matrix.

Table 16: **Tail Risk Double Sorted Portfolios**

Portfolio	1.00	2.00	3.00	4.00	5.00	High - Low	FF4
SIZE	2.63	1.95	1.70	1.40	1.08	-1.55	-0.66
	(2.64)	(2.52)	(2.52)	(2.43)	(2.08)	(-2.65)	(-2.00)
MOM	2.36	1.71	1.65	1.51	1.52	-0.84	-0.19
	(2.58)	(2.28)	(2.47)	(2.48)	(2.67)	(-1.94)	(-0.76)
D. Rev.	2.77	2.01	1.66	1.27	1.03	-1.74	-0.91
	(2.82)	(2.64)	(2.52)	(2.17)	(1.94)	(-3.15)	(-2.96)
W. Rev.	2.66	1.98	1.58	1.35	1.18	-1.48	-0.71
	(2.73)	(2.58)	(2.39)	(2.30)	(2.19)	(-2.73)	(-2.25)
M. Rev.	2.67	1.98	1.62	1.35	1.13	-1.55	-0.76
	(2.76)	(2.60)	(2.44)	(2.31)	(2.06)	(-2.93)	(-2.45)
ILLIQ.	2.60	1.96	1.64	1.42	1.13	-1.47	-0.61
	(2.66)	(2.52)	(2.46)	(2.45)	(2.16)	(-2.62)	(-1.89)
Turn.	2.76	2.16	1.70	1.65	1.58	-1.18	-0.47
	(2.89)	(2.77)	(2.50)	(2.69)	(2.77)	(-2.27)	(-1.36)
COSKEW	2.71	2.01	1.59	1.34	1.11	-1.60	-0.72
	(2.72)	(2.62)	(2.38)	(2.31)	(2.13)	(-2.78)	(-2.30)
IVOL	2.42	2.02	1.59	1.53	1.20	-1.22	-0.48
	(2.65)	(2.62)	(2.32)	(2.49)	(2.24)	(-2.61)	(-1.77)
MAX	2.46	1.95	1.64	1.46	1.24	-1.23	-0.57
	(2.76)	(2.58)	(2.40)	(2.35)	(2.21)	(-2.85)	(-2.08)
COKURT	2.73	1.90	1.62	1.36	1.13	-1.59	-0.73
	(2.74)	(2.47)	(2.46)	(2.34)	(2.18)	(-2.77)	(-2.29)
D. Beta	2.47	1.80	1.59	1.47	1.42	-1.05	-0.67
	(2.95)	(2.50)	(2.37)	(2.30)	(2.20)	(-3.24)	(-2.59
D. Vol	2.29	1.93	1.65	1.59	1.29	-1.00	-0.44
	(2.69)	(2.57)	(2.39)	(2.48)	(2.25)	(-2.68)	(-1.81)

This table present the returns, Fama and French three factor model alpha and Fama-French-Cahart four factor model alpha for double sorted portfolios. Each day we first sort stocks into five groups based on firms characteristics. Within each characteristics group we then sort stocks according to tail risk beta (we consider a total of 1228 stocks, the constituents of the S&P 500 and Russel 1000 index, ranging from 01/02/2008 to 11/12/2015) estimated using returns over the 252 trading days prior to portfolio formation. To form portfolios that are heterogeneous on the characteristics but homogeneous on tail risk exposure we compute the means on the beta sorted portfolios across characteristics according to their Hellinger Tail Risk Beta. For details on the sorting characteristics please see appendix A. For all models portfolios are re-balanced daily, the holding period equal one day, and the betas are estimated using a univariate regression. To facilitate comparison we convert daily returns and alphas into monthly percentages by multiplying them by 21. T-statistics are calculated using Newey and West variance matrix.

Table 17: **Tail Risk Reverse Double Sorted Portfolios**

Portfolio	1.00	2.00	3.00	4.00	5.00	High - Low	FF4
SIZE	2.39	1.79	1.73	1.50	1.33	-1.06	-0.93
	(2.97)	(2.49)	(2.53)	(2.21)	(2.13)	(-3.36)	(-6.34)
MOM	2.05	1.63	1.59	1.54	1.94	-0.11	0.03
	(2.44)	(2.33)	(2.44)	(2.38)	(2.72)	(-0.26)	(0.17)
D. Rev.	2.36	2.10	1.76	1.35	1.17	-1.19	-1.05
	(3.00)	(3.06)	(2.67)	(2.03)	(1.59)	(-3.43)	(-3.10)
W. Rev.	2.44	1.90	1.65	1.40	1.37	-1.07	-0.88
	(3.07)	(2.78)	(2.49)	(2.11)	(1.88)	(-3.31)	(-2.77)
M. Rev.	2.27	1.83	1.69	1.54	1.41	-0.86	-0.68
	(2.87)	(2.66)	(2.54)	(2.31)	(1.96)	(-2.45)	(-1.98)
ILLIQ.	1.38	1.50	1.64	1.84	2.39	1.02	0.84
	(2.26)	(2.27)	(2.33)	(2.50)	(2.98)	(3.13)	(5.32)
Turn.	2.06	1.96	1.94	1.98	1.89	-0.17	-0.43
	(2.94)	(2.86)	(2.76)	(2.78)	(2.40)	(-0.48)	(-1.29)
COSKEW	1.94	1.51	1.60	1.70	2.00	0.06	0.01
	(2.62)	(2.28)	(2.43)	(2.51)	(2.60)	(0.24)	(0.04)
IVOL	1.43	1.52	1.65	1.87	2.28	0.86	0.40
	(2.50)	(2.42)	(2.41)	(2.48)	(2.60)	(2.07)	(1.61)
MAX	1.53	1.57	1.64	1.66	2.35	0.83	0.29
	(2.81)	(2.50)	(2.39)	(2.16)	(2.64)	(1.84)	(1.16)
COKURT	1.97	1.68	1.57	1.69	1.85	-0.12	-0.12
	(2.61)	(2.51)	(2.39)	(2.51)	(2.44)	(-0.54)	(-0.57)
D. Beta	1.57	1.62	1.58	1.80	2.17	0.60	-0.09
	(2.97)	(2.65)	(2.30)	(2.37)	(2.36)	(1.26)	(-0.35)
D. Vol	1.34	1.48	1.65	1.77	2.51	1.18	0.54
	(2.53)	(2.43)	(2.38)	(2.31)	(2.71)	(2.34)	(1.93)

This table present the returns, Fama and French three factor model alpha and Fama-French-Cahart four factor model alpha for double sorted portfolios. Each day we first sort stocks into five groups based on the Hellinger Tail Risk beta, estimated using returns over the 252 trading days prior to portfolio formation. Within each beta group we then sort stocks according to firms characteristics (we consider a total of 1228 stocks, the constituents of the S&P 500 and Russel 1000 index, ranging from 01/02/2008 to 11/12/2015). To form portfolios that are heterogeneous on the tail risk beta but homogeneous on the characteristics we compute the means on the characteristics portfolios across beta groups. For details on the sorting characteristics please see appendix A. For all models portfolios are re-balanced daily, the holding period equal one day, and the betas are estimated using a univariate regression. To facilitate comparison we convert daily returns and alphas into monthly percentages by multiplying them by 21. T-statistics are calculated using Newey and West variance matrix.