

Pacioli in the computer age

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The authors would like to thank Mico Loretan for his thorough revision of the text and his many valuable suggestions.

Summary

This paper is inspired by the brilliant insights of Luca Pacioli who, we will argue, more than 500 years ago proposed a solution for the long-standing data and IT problems that financial institutions are facing today. We start out by taking a fresh look at Pacioli's *Summa*, with focus on his recognition of the importance of properly recording known future events. We also look at how computers entered the financial world at an early stage via their use in relatively straightforward accounting tasks, and we discuss the unintended adverse consequences of this early development. We imagine how – based on the *Summa* – Pacioli could have solved these issues had he had access to modern computers. The central idea is the paradigm of a standardized representation of financial contracts, which includes both the data and the mathematical/algorithmic representation of the embedded contractual future obligations. We discuss the benefits of such a system and how to overcome obstacles to adopting it.

Introduction

Luca Pacioli, ca. 1445 to 1517, a Renaissance genius¹ and contemporary and friend of Leonardo da Vinci is generally considered to be the father of modern accounting. In formalizing the “Venetian” accounting method in his magnum opus, the *Summa*,^{2 3} he proposed the components required to record all information essential to managing a business. The objective of his treatise was to describe the entire system of record keeping, of which accounting is merely a subset. The three components of Pacioli's system that are still

¹ Pacioli is clearly nowhere near as famous as Da Vinci and other Renaissance geniuses such as Michelangelo and Brunelleschi. However, considering his many “firsts”, he has clearly earned a recognized place among that august company. Among these “firsts” is creating the first printed book on mathematics, bringing Arabic numerals into the mainstream in Europe, and formalizing accounting. He likely taught Da Vinci mathematics, especially proportions and perspective. He is also the first mathematician to have been depicted in an oil painting, possibly together with Albrecht Dürer the Younger. See (Gleeson-White, 2013) pages 49-89.

² The full title of Pacioli's treatise is *Summa de Arithmetica, Geometria, Proportioni et Proportionalità*; the literal translation is “Summary of arithmetic, geometry, proportion and proportionality”. The book was first published in 1494 in Venice.

³ Pacioli (1494). Pacioli formalized accounting in Section Nine, Treaties XI. We use the English translation from Geijsbeck (J.B. Geijsbeck, 1914)

in use today are the Journal⁴, General Ledger, Balance Sheet, and P&L Statement. The historically inclined may also remember the Memorandum. A careful reading, however, reveals that Pacioli was deeply concerned, not only with the accounting records, but with the processes that must be in place *before* accounting itself can begin. Furthermore, he emphasizes the importance of organizing these business processes effectively.

Much of his treatise is concerned with the activities of merchants, who must be vigilant, “must never rest”,⁵ and must take note of and organize everything in multiple records, each of which is responsible for specific parts of the system. One of the components of Pacioli’s system, which is rarely mentioned today, but is of particular importance for our paper, is the Ricordance, which records all future obligations, including all future payments and receipts, on a day-by-day basis.

The accounting books are created using the information contained in the various business records. Creating the accounts also demands much diligent work. The famous piece of advice, “not to go to bed as long as debit and credit does not tally,” is widely attributed to him and has been known to every accountant since his day⁶.

In this article we will focus on applications of Pacioli’s system to firms in the banking sector. Banks and other financial institutions are unique among businesses in that their balance sheets are largely made up of financial contracts such as loans, deposits, mortgages, swaps, and futures.⁷ While the approach we use in this paper could be applied to other businesses, say, manufacturing and services, such extensions are the topic for follow-on articles.

We explore how Pacioli’s formalization of business record keeping might look like, had he had access to today’s information technology. This is obviously a thought experiment. Nevertheless, we argue that one can establish a remarkably detailed roadmap on the basis of the clear structure of Pacioli’s system and the books that make up his system. With this starting point we can “stand on Pacioli’s shoulders” to build a model of how Pacioli might have further evolved his system had he had access to IT technology that is now available to financial institutions. Furthermore we argue that this model has important implications for today’s banks and the banking systems.

⁴ For non-accountants: Journal and General Ledger are important concepts in accounting. Accountants use the Journal to keep track of each relevant financial event. The Journal entries are then transferred into the General Ledger, which is organized at the highest level into assets, liabilities, equity, revenues, expenses, and net profit.

⁵ (Pacioli, 1494) chapter 4.

⁶ Interestingly one does not find this often quoted saying in Pacioli’s work itself. To be sure, Pacioli talked again and again of the importance of diligent and long work, from which the saying can be derived. That said, the saying has been on the mind of every accountant since long before the arrival of computers. A small deviation between the sum of the debits and credits would cause sleepless nights for every accountant, and such occurrences were not rare events.

⁷ We restrict ourselves to applications in the financial sector because we believe that this sector is organized chaotically with regard to information technology -- and therefore could benefit disproportionately from better adhering to Pacioli’s system.

Banks, computers and accounting

Jumping ahead about 450 years from Pacioli's time, banks were among the earliest non-governmental adopters of computers. When the Bank of America developed ERMA (which stands for "Electronic Recording Machine, Accounting") in cooperation with Stanford University in the early nineteen-fifties, Pacioli's debit/credit concerns were well present. The first problem computers helped solve for banks was the challenge of eliminating errors in the accounts that led to mismatches between debits and credits that kept accountants up at night. Balancing debits and credits were a major pain point for banks, tailor-made for computers and relatively easy to solve.

The management of banking until the advent of computers was largely based on accounting, starting with the Journal. All business figures rolled into the Journal and from there to the General Ledger, the balance sheet, and P&L. Bank management focused on the P&L and balance sheet, which are directly related to debits and credits. Errors in debits and credits create errors in the P&L and balance sheet. Fixing this problem was obviously important for banks.

The first computer programs deployed by banks were all linked to accounting in connection with current accounts. Martins Bank of Liverpool proudly announced in the early sixties: "Our bank is the first bank in this country to use an electronic computer for the complete current account bookkeeping including the statements for customers"⁸.

In retrospect, it is clear that while deploying computers in this manner helped solve certain basic problems, an opportunity was missed to make even more profound improvements to the entire system of record keeping. Computers were used to simply replace existing paper-based processes without taking into account their inherent capabilities to do far more. We believe that Pacioli – had he had computers rather than only paper, pen and ink, brain power, and hardworking people – would have brought about a far more robust solution.

Why is solving the problem of accounting a sub-optimal starting point? Accounting is the end point of a complex system of record keeping and computations starting with the facts of the business and ends with the balance sheet and the P&L statement. Starting with accounting means trying to solve a problem backward from the end. This approach has much in common with the efforts of alchemists up to the 19th century. Obsessed with the desire to create gold they mixed everything with anything achieving next to nothing⁹. Chemistry only became a true science with the discovery of atomic structures and its formalization within the model of Niels Bohr. True science – and sound business, as we learn from Pacioli – must begin with the foundational elements from where it progresses to the end points.

⁸ See, for example, this article from BCS The Chartered Institute: <https://www.bcs.org/articles-opinion-and-research/when-computers-changed-banking/> retrieved on October 23, 2023.

⁹ It is said, that they found gun powder by chance. Hower even this seems to be a tale.

Refocusing on the Memorandum and records before that

Pacioli is generally remembered for having defined the Memorandum¹⁰ (or daybook), the Journal and the General Ledger. Of these three books, the Memorandum has largely dropped from our memories, leaving us today with the Journal and Ledger.

What has also been forgotten are the significant parts of Pacioli's treatise that focus on the many details which need to be recorded even *before* information is entered into the Memorandum. Pacioli talks again and again of the many notes that must be taken. In section 35 he lists all the letters and agreements that have to be carefully collected, ordered, marked and be put into "a little desk until the end of the month". Then they are put into pouches, clipped together etc. in good chronological order. All letters have to be listed again in another book (Memoria) in order not to miss one single important agreement or promise. There are at least three other books,¹¹ which are a receipt book, the book of important correspondence, and the Ricordance, or "recording book". This last book is especially important since its purpose is to record all promised future events on a timeline. We will focus especially on the Ricordance which has, unfortunately, been almost lost from memory.

Pacioli first mentions the Ricordance in section 35 of the *Summa*. However, in section 38 he dedicates an entire chapter (the last one) to this special recording book¹². The Ricordance is a book of records that should contain all the facts that "if they are lost, may cause great danger". In chapter 38 he lists ten specific items which need to be recorded. Most of them are related to agreements which unfold over time, such as payments related to future events or payments that are part of financial obligations such as the repayment of a loan given or taken¹³. The information contained in the Ricordance is entered to the Memorandum when the recorded item is actually executed on a given day.

While it is true that Pacioli describes in the *Summa* the double entry system of debits and credits and how it relates to the Memorandum, Journal, and General Ledger, it is obvious that he equally cares about all the processes which must be well organized and present even before entries are made into the Memorandum. His many invocations to God, the Cross and Jesus are clearly meant to spur merchants to organize and execute their business in the most diligent and optimal way.

Pacioli's thinking clearly starts with the facts of the business, as registered in the Ricordance and other sources, from which the Memorandum is derived day by day and from which, say on a weekly cycle, the Journal is derived, and on even longer cycles, the General Ledger. Finally, at the very end of an established reporting period come the balance sheet and P&L statement.

¹⁰ In the Memorandum the accountants registered each day all the necessary actions derived from other sources. For example if an interest payment was due at a specific day. It was a daily to-do list.

¹¹ See (Alan Sangster, 2012)page 4.

¹² (Alan Sangster, 2012) The entire article is dedicated to the Ricordanze.

¹³ Even after mentioning so many books and the steps involved, he warns the businessman not to get tired since "it is impossible to give rules for every little thing....". In section 4 he says: "More bridges are necessary for a good merchant than a lawyer can make."

When we talk about the Ricordance in the following sections, we will mean the entire process which goes before the Memorandum. We will also mean that these processes should be organized optimally and registered and executed with the most diligent care.

The consequences of starting “at the end”

Bankers to begin with focus on the balance sheet and P&L statement then and now. By solving the debit/credit problem first, this focus of the bankers was carried over to the IT world. Only later were systems added in an ad hoc manner that used computers to manage business activities, such as the registration and life cycle management of loans, deposits, derivatives etc., information that would be part of Pacioli’s Ricordance. These systems provided the necessary information for the accounting system, but they were not part of an overall integrated architecture.

The result of this ad hoc evolution of the use of computers in banking is seen today in the chaotic IT infrastructure of banks¹⁴. Chaos appears to have reigned in banks already in Pacioli’s time: In section 17 he says: “Woe to you if you have anything to do with these people (the bankers). Therefore be very careful when dealing with them” because “...in these offices they often change their clerks, and as each one of these clerks likes to keep the books in his own way, he is always blaming the previous clerks, saying that they did not keep their books in good order, and they always try to make you believe that their way is better than all others, so that at times they mix up the accounts in the books of these offices in such a way that they do not correspond with anything”.

Pacioli singles out banks with this caustic remark; no other industry gets a similar treatment. However, in the defense of the bankers he says: “Maybe they mean well, nevertheless they show ignorance.” If the chaos was remarkable then, Pacioli would be even more shocked today.

How would Pacioli have addressed the problem had he had access to modern computers? What solution would he have had for the banking industry in the computers age?

An approach based on Pacioli’s system

We believe that Pacioli would have taken the same approach as proposed in his Summa. He would first have taken a close look at the business of banking. He would have realized that banking was far more complex than in his time when banking was mainly keeping track of current accounts¹⁵. He would find long-term loans and mortgages, leasing contracts, fixed and variable rate loans possibly with caps if floating, swaps, futures, options and many securitized loans such as ABS, MBS, CDO’s. He would understand that the business of banking was about managing all aspects of financial contracts. This would require an organization optimized to manage the origination, life-cycle management, trading, and

¹⁴ A fate which other industries managed better to avoid.

¹⁵ Pacioli is in his treatise mainly concerned with merchants selling and buying goods. He does talk about banking in different sections such as the end of section 3, 17, 19 and 24. In all these chapters he describes the case of current accounts.

analysis of such contracts in the broadest possible sense including keeping track of the accounting books. He would also see that all these contracts have a clear relationship with time and that everything associated with these financial contracts needs to be ordered on the timeline.

Studying all these financial instruments he would have understood that financial contracts were simply about the exchange of cash-flows i.e. promises to pay how much, to whom, and when. Secondly, he would realize that cash-flows were mere numbers¹⁶. And third, as a first-rate mathematician, he would quickly see and understand the algorithmic nature of the cash-flow obligations of the financial contracts.

The descriptive term “algorithms” is used because each action and especially each payment would follow specific mathematical rules. In some cases, the rules would depend on external factors such as the setting of a floating interest rate. However, once the state of these factors is clear, the algorithms can be applied. He would understand that computing the payment obligations of such algorithmic-based contracts is something computers are made for.

Pacioli would see that many of these financial contracts follow the same or similar patterns independent of the product names given by some lawyer or marketing specialist. For example, a typical amortizing mortgage, a leasing contract and even a life annuity follow the same pattern. He would realize that the big bulk of all financial contracts follow a half a dozen patterns and that less than three dozen patterns cover the relevant universe of finance¹⁷.

The payment obligations generated by the algorithms would be the modern iteration of the Ricordance and a perfect use case for the computer. Computers can understand numbers which follow time ordered patterns in which each pattern has an initial state (the parameters of a financial Contract such as initial exchange date, maturity date, notional, interest rate etc.), and state transition functions (or a set of algorithms that lead to a new state) which is a computer executable program.

Since the number of exchange patterns is limited, it is possible not only to define and standardize the data, but also to standardize and code its associated algorithms.

We believe Pacioli would have realized that computers would be able to perform in a standardized fashion and error free all tedious tasks of manually performing computations and record keeping in the world of finance.

From the Ricordance to a deeper understanding of finance

Following Pacioli's system we can use the algorithmic representation of the computerized Ricordance not only for accounting, but for any financial analysis. Instead of only backward-looking accounting, it is now possible to perform any forward-looking financial analysis since any such forward-looking analysis is a function of the state contingent future cash-flows. In

¹⁶ In Pacioli's time money was gold in many currencies such as ducats, florins, guilders etc. He might be shocked by this observation.

¹⁷ See (ACTUS, 2024) for the patterns.

other words, we have a system that covers all needs of risk management and finance: From simple accounting to the most advanced accounting techniques, from simple risk to rocket science risk management.

Valuation and accounting

Most likely Pacioli would have first understood the power of the new Ricordance for valuation. Based on the discounted future cash-flows derived from the Ricordance it would be possible to derive fair value. The information needed for discount rates would be derived from the observed state of the world as expressed in yield curves, FX-rates and so on¹⁸.

In addition it would be possible to derive amortized cost, nominal value, or any other cash-flow based valuation method in a fully consistent way because the underlying state contingent cash-flow would be the same in all cases. This can be done at any point in time going forward.

This calculation could be performed for every loan, deposit, mortgage, swap etc. in short for every financial agreement.

This capability would lead to automated financial accounting on the single contract level. Not only is it possible to calculate value at any point in time, but it is also possible to calculate for each financial instrument, multiple values based on different accounting systems or criteria such as nominal value, fair value, IFRS value, local GAAP value, etc. These values would be 100% consistent because different valuation methods are simply different interpretations of the expected future cash flow. The expected cash-flows can be derived uniquely from the computerized and well organized and standardized “Ricordance” and they are independent of the applied valuation technique. Since all valuation methods can now be based on the same expected cash-flows, the different accounting methods are consistent by design.

Since all valuations would happen on the single contract level, a balance sheet is then a simple aggregation executed with commands such “group by” and “sum”.

Since value can be calculated at any point in time a P&L statement is reduced to a qualified delta calculation between two points in time¹⁹.

Such a system based on single contract calculations supports profitability analysis since profitability can be derived from delta calculation on value. Transfer rates can be added, which leads to Funds Transfer Pricing and profitability analysis. Since all values are available on the single contracts, profitability can be aggregated along any dimension such as customer, profit-center, product line and so on.

¹⁸ We speak of “state contingent cash-flows”, the “state” meaning the state of the financial world such as interest rates, FX rates etc. upon which a financial contract might be dependent.

¹⁹ We say «qualified delta calculation» because we need to distinguish between payments which are not P&L relevant (such as principal payments) and payments that are P&L relevant (such as interest payments). In addition we have to figure out value changes which are P&L relevant. All this information can be derived from the Memorandum.

Analysis beyond the balance sheet and P&L statement

Beyond this, Pacioli would understand the need for more highly sophisticated analysis. Modern finance has become far more complex than in Pacioli's time when financial analysis was limited to the accounting view. Pacioli would closely examine the breath of financial analysis and discerned that all types of analysis are interpretations of the expected future cash-flow. Since this is true even for valuation, he would understand that he has created with his computerized Ricordance not only a basis for the life cycle management of financial contracts, but at the same time the perfect basis for all financial analyses. Any cash-flow based analysis becomes possible²⁰ and accounting is only a specific case.

The treasurer would see liquidity in the form of the expected cash-flows on any granularity level. The Asset and Liability manager would focus on the interest rate gap and sensitivity measures like duration and convexity. The risk manager would see value at risk numbers, expected shortfall, stress tests, and so on.

Budgeting and strategic business planning would include an understanding of the contracts rolling off the balance sheet and the potential new business, its time-related characteristics and pricing.

Even regulation could be improved. Having all contractual facts in a standardized form with respect to data and cash-flow generating algorithms would enable regulators to create digital twins of the regulated banks. Regulators would then have the same analytic capabilities as the risk manager of any well managed bank. Instead of asking ever new reports in response to emerging risks, regulators could independently perform analytic tasks such as stress testing. This would enable regulators to understand in real time how the condition of regulated financial institutions would change as the state of the world evolved. This is the analytical foundation that would metaphorically enable regulators to carry out their responsibilities by looking through the windshield rather than the rear-view mirror²¹.

The new world of finance

Pacioli could in our time transform how computers improve bank operations and analysis. Rather than first solving accounting problems, the industry – under his guidance – could start at the very beginning of the record keeping process and create a consistent system from beginning to end for both operational and analytical purposes.

Based on this system he would first create standardized machine-readable financial contracts. The algorithms of the standardized contracts would be first used in the life cycle

²⁰ While it is true, that the algorithms which describe the life cycle of financial contracts are the same as the ones needed for financial analysis, there is however a difference with respect to the external factors such as interest rates that affect the contracts. Life cycle management depends on observed external factors. Financial analysis depends on forecasted external factors.

²¹ Fulfilling the expectation of Senator Chris Dodd in 2010 when he introduced the Dodd-Frank Act by saying "...in addition to looking in the rear-view mirror, we must look through the windshield. There will be shocks to our system in the future for certain. And we need an early warning system so that the next time it occurs our system is prepared to deal with it."

management of these contracts. Then he would automate valuation on the single contract level. From this he would derive the General Ledger, the balance sheet and P&L's. Further he would use the same data and logic to derive any financial analysis from the simplest to the most complex.

The new system would be better, more cost effective and transparent in a good way. Unlike the current system it would be fully consistent from beginning to end.

Can this become a reality?

People will quickly agree that a new bank should be built on such thinking and especially such a Standard as proposed above. Opinions could differ when it comes to legacy banks.

Practitioners might say "all well and good, but Pacioli is long dead and the current mess in the financial sector is a fact. Changing this would be like changing engines in full flight, an impossible task²².

It is not. The best way to think about it is to imagine a bank as having two stories. On the lower floor we have the transaction processing systems (TPS) and on the upper floor the analytic systems (AS):

- TPS: TPS's compute on a daily basis both the bank's payment obligations and expected contractual receipts for that day. It is responsible for maintaining the records of what is added to and runs off of the balance sheet on a day-by-day basis, and any required payments during the life of the asset, liability or off-balance-sheet position. These two functions are generally referred to as deal acquisition and deal processing. For example: A mortgage is made by the bank and kept on its balance sheet. After it is created it is entered into the TPS that handles all the payments of the mortgage through its maturity date unless it is sold or securitized.
- AS: The analytic level corresponds to the bank management level. Bank management can be roughly grouped into three views: risk management, finance, and regulatory reporting. However, all analyses have in common, that they are an interpretations of the future state contingent cash-flow.

Both levels are at their cores concerned with cash-flows. On the transaction processing level, the principal focus is on the next event and its calculation may be based on observed external factors, such as the setting of an interest rate in a swap. On the analytic level all future events and especially future cash flows are important. The same swap is calculated using the same algorithm as in the TPS, but the yield curve has to be forecasted to the maturity date of the swap in order to calculate all future cash-flows. Based on this, any value can be calculated and any other analysis performed.

Financial analysis means analysis of the contracts maintained in the TPS. Therefore, it should start with the same cash-flow obligations maintained and managed by the TPS. This

²² A study (Blattmann U. and Buschor F., 2021) shows, that basically all banks want to get out of the chaotic architecture. At the same time, nobody believes, that it is possible to do so.

perspective gives direct insight into the value of algorithmically defined contracts for more efficient and higher quality bank operations. Because both the TPS's and the AS's start fulfilling their tasks using the same cash-flow generated by the individual financial contracts, the same algorithmically defined can be used on both levels.

The reality, however, is far different from this ideal solution. Banks have many systems on both the TPS and the AS level. The transaction processing level might have "m" systems and on the analytic level might have "n". "m" might be a number close to 100 and "n" is often 20 or 30 in a large banks. Each of these systems has its own data dictionary and cash-flow generating mechanism to represent the contractual obligations. Different systems that process the exact same contractual obligation differ. This is due to the IT architecture that started with the debit/credit problem explained above.

Alternative attempts have been deployed to solve the problem. The first iteration was the data warehouse, which mutated to data marts, and then the data lakes. Will the next iteration be data swamps? The mutation of the names reflects the reality that just focusing on contract terms cannot solve the problem. Solutions along this line will always require interfaces between all TPS' and AS' systems which make mxn interfaces. In a large bank mxn can easily be a few thousand. Each interface is extremely expensive because what needs to be translated are not simply data fields, but data fields with their associated logic.

Maintaining such an architecture is also very expensive. Changing anything within the system becomes almost impossible due to potential of many unintended consequences. We end up with an expensive architecture and a chaotic and sclerotic system.

It is obvious that a Financial Contract Standard that not only tries to standardize data fields as it happens in data warehouses, but at the same time standardizing the associated cash-flow generating algorithms is a genuine and highly efficient solution.

How can such a solution be implemented? Today's efforts contemplate on first solving the problem on the transaction processing level and only then addressing the issues on the analysis level. This approach is not likely to succeed because: 1) TPS's are critical to running the real business²³ and any disruptions cannot be countenanced, 2) the mxn interfaces will all be affected leaving the bank without reliable information needed to manage the bank for a very long time.

We see the only promising way forward is by adopting the Financial Contract Standard first. Given the Standard the bank has a roadmap on how to proceed. Then the next task is to solve the problem at the AS level. This can be achieved by mapping all data on the many TP's into the Financial Contract Standard. This is noninvasive since it is a "read only" activity. All cash-flow dependent analyses can be handled now with the same cash-flow generating mechanism for all reports. This reduces the number of interfaces from mxn to mx1, say from 3000 to 100, which in itself would be tremendous progress. Given this transformation, the transformation of the TPS's can now be undertaken without excessive pressure. This solution can succeed, because it will not disrupt the real operations of the bank. The TPS systems as

²³ This is the big bang problem. The failed projects are legion.

such work well enough for the specific task they are performing. Systemic problems and the high cost are created by the unstandardized treatment of the different systems. Once the mapping is completed, a large portion of the inefficiency is solved. Over time however, as TPS systems are renewed anyway for other reasons, they can be replaced with a new system that is built on the Financial Contract Standard. At the very end mxn is reduced to 1x1.

Coming back to Pacioli and his system described in the Summa, we will have a well-organized system that collapse the Recordance, all proceeding books including the Memorandum into one golden source. On the one hand this system tells us every day what needs to be done, and at the same time all future events that are needed for valuation, accounting and all other analyses²⁴. The balance sheet will be created by simple database queries with commands such as group and sum. The P&L will be a delta calculation between any two points in time.

References

ACTUS, Foundation. (2024, 01 13). *ACTUS Taxonomy*. Retrieved from ACTUS:
<https://www.actusfrf.org/taxonomy>

Alan Sangster, e. (2012). "Pacioli's forgotten book: The Merchants's Ricordanze". *Accounting Historians Journal*, Vol. 39,, 4.

Blattmann U. and Buschor F. (2021). *Kernbankensysteme – Zukünftige Anforderungen und mögliche Entwicklungen, HLU 2021*. Luzern: Hochschule Luzern.

Gleeson-White, J. (2013). *Double Entry*. Allen & Unwin.

J.B. Geijsbeck, M. (1914). *Ancient Double Entry Bookkeeping*. Denver.

Pacioli, L. (1494). "*Summa de arithmetica, geometria. Proportioni et proportionalita*". Venice: Paganino Paganini.

²⁴ Of course, these events are generated under a given state of the risk factors.