A robust methodology for modeling and forecasting the CBOE Volatility Index*

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Abstract

The paper proposes the M-estimator technique, that relies on the Huber-Bisquare objective function with a scale function based on the Median Absolute Deviation, centered around the Median (MADMED), for modeling and forecasting the CBOE Volatility Index. Under the so called Heterogeneous Market Hypothesis, which recognizes the presence of heterogeneity across traders, the theoretical framework is derived. The robust methodology is compared with several benchmarks, ARMA(3,3)-TARCH(1,1), ARMA(3,3)-EGARCH(1,1), ARMA(3,3)-PARCH(1,1), revealing the out-performance of the methodology based on several metrics of accuracy.

JEL Classification: C32, C53, G13

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1. Introduction

Forecasts of future price variability are needed to measure the risk of a portfolio and to value financial instruments. A vast empirical and theoretical literature is focused on this topic, proposing new methods for estimating volatility or comparing the effectiveness of techniques already in-use, with the aim to evaluate the in sample and out-of-sample performance.

This paper belongs to that stream of literature which explores the pros and cons of a robust methodology, based on the Huber M-estimator, for modeling and forecasting the Chicago Board Options Exchange (CBOE) Volatility Index (VIX), comparing it with several alternative methodologies (benchmarks), in order to derive some conclusions useful to market participants.

The theoretical framework of the paper is constructed under the so called Heterogeneous Market Hypothesis, which recognizes the presence of heterogeneity across traders, where a financial market is composed of participants having a large spectrum of trading frequency that impacts on the prices of the securities. The theory argues that the financial markets are composed of non-homogeneous market participants with different trading appetite and opportunities.

Since 1993, the VIX is computed by the CBOE, with the aim to measure the market expectations of the near-term volatility implied by stock index option prices. The VIX considers a model-free estimator of the implied volatility and is able to offer a forward looking estimate of thirty days volatility, providing a measure of market risk as well as investor's sentiment (Hentschel 2003). Low VIX levels would mirror complacency among market participants, setting up the market for disappointment and raising the likelihood of a market correction (Fernandes et al. 2014), whereas, high VIX levels typically reflect pessimism, that causes equity prices to overshoot on the downside and thus leading to subsequent rallies.

Academic studies (Canina and Figlewski (1993), Jorion (1995), Fleming (1998), Martens and Zein (2004), Bandi and Perron (2006)) argue that options-implied volatility is typically more informative than time-series volatility models, based on stock market index returns for forecasting purposes, though the latter may sometimes carry further information.

The rest of this paper is organized as follows: Section 2 describes the theoretical framework; Section 3 discusses the data and provide some summary statistics. Section 4 proposes the statistical methodology. In Section 5 are reported the empirical results; whereas, Section 6 concludes the paper.

2. The theoretical framework

The theoretical framework for determining the price assumes that the observed price of an asset ($\hat{p}$), at a certain time t, consists of two components $\tilde{p}$, that is the unobserved price of an asset also caused by the arrival of new information at a certain time t and p that represents the transaction costs incurred in making an exchange of a certain asset, at time t. Therefore,

$$\hat{p}_t = \tilde{p}_t + p_t. \tag{1}$$

The unobserved price ($\tilde{p}$) and the transaction costs (p) incurred in making an exchange of the asset at time t, can be written in the following way:

$$\tilde{p} = \tilde{p}_{t-1} + Q_t \cdot Z_t + e_t \tag{2}$$

and,

$$p_t = f\,(Q_t\,,\,C_t) \tag{3}$$

where, the quantity Q_t represents the unobserved indicator for the bid/ask classification and takes a value equals to $+1$, if the transactions were initiated by a buyer and a value equals to -1, if the transactions were initiated by a seller. The quantity Z_t represent the adverse selection components that also depend on the order sizes arrived, since well informed traders maximize the returns to their perishing information, impacting on the level of the asymmetric information.

The quantity $Q_t \cdot Z_t$ represent the products between the unobserved indicator for the bid/ask classification and the adverse selection components, conditional on the arrival of new orders. Assuming a positive quantity of Z_t, a buy/sell order respectively creates a potential increase/decrease of the unobserved price $(\tilde{p})$, with sizes that are in absolute values respectively equal to Z_t. The quantity $\tilde{p}_{t-1}$ represents the unobserved price of an asset at time $t-1$; whereas, the quantity e_t^o represents the innovations for the unobserved price of an asset, that depend on the arrival of public information, from time $t-1$ to t and has a distribution equals to G, with observations that are independent and identically distributed (i.i.d), provided that the mean is respectively equal to μ and with variance equals to v^2, at time t.

The component p_t is a function $f(\cdot)$ of the unobserved indicator for the bid/ask classification (Q_t) and the unobserved transitory component (C_t), that also depend on the order sizes. As such, the equality (1) can be rewritten in the following way:

$$\hat{p}_t = \tilde{p}_{t-1} + Q_t \cdot Z_t + f\left(Q_t\, ,\, C_t\right) + e_t. \tag{4}$$

Under the so called Heterogeneous Market Hypothesis, which recognizes the presence of heterogeneity across traders, a financial market is composed of participants having a large spectrum of trading frequency. The main idea is that agents with different time horizons perceive, react to, and cause different types of price components. Therefore, there are short-term traders with daily trading frequency, the medium-term investors who typically rebalance their positions weekly, and the long-term agents with a characteristic time of one, three and six months.

The observed price $(\hat{p})$ at time t is a combination of the price one day before $(\hat{p}_{t-1})$, the price one week before $(\hat{p}_{t-1w})$ and the prices one month $(\hat{p}_{t-1m})$, three months $(\hat{p}_{t-3m})$ and six months ($\hat{p}_{t-6m}$) before. According to this specification, the observed price at time t can be written in the following way:

$$\hat{p}_t = \hat{c} + \hat{\beta}^{(d)} \cdot \hat{p}_{t-1} + \hat{\beta}^{(w)} \cdot \hat{p}_{t-1w} + \hat{\beta}^{(m)} \cdot \hat{p}_{t-1m} + \hat{\beta}^{(3m)} \cdot \hat{p}_{t-3m} + \hat{\beta}^{(6m)} \cdot \hat{p}_{t-6m}, \qquad (5)$$

where, $\hat{\beta}^{(d)}, \hat{\beta}^{(w)}, \hat{\beta}^{(m)}, \hat{\beta}^{(3m)}\, and\, \hat{\beta}^{(6m)}$ respectively represent the sensitivities of the observed prices at one day, one week, one month, three months and six months before, with respect to the observed price at time t.

3. Data and descriptive statistics

The empirical analysis considers the Chicago Board Options Exchange (CBOE) Volatility Index (VIX), from January 1st, 1992 to October 9th, 2020. The index is computed on a real-time basis through each trading day and was introduced by Whaley (1993). It represents the market's expectations of thirty days forward looking volatility and provides a measure of market risk as well as investor's sentiment. The original index was based on the prices of eight at-the-money index calls and puts for the S&P 100 Index, that accounted for 75% of the total index option volume in 1992. Indeed, the average trading volume for the calls was equal to 120,475 and for the puts was equal to 125,302. Over the years, the option market on the S&P 500 Index became more active and for this reason the VIX was computed on the calls and puts of this index that respectively reached a level of 525,460 and 909,748 call and put option contracts in the first ten months of 2008.

The shift in market dominance from options on the S&100 to S&P500 is based on the remark that the index portfolios have a high correlation and seem perfect substitutes, with the means and the standard deviations that are nearly identical. As of October 2008, all S&P100 stocks were contained within the S&P 500 index and the highest market cap stocks were the same.

[Please insert Table 1 around here]

The recent financial crisis shows several spikes of the CBOE VIX that reacts in response to unexpected market and world events. The average values and the standard deviations are respectively above 19.496 and 8.087 during the entire period of observation. The bursting of the United States housing bubble, culminating with the bankruptcy of Lehman Brothers on September 15, 2008, as well as the lack of investor confidence in bank solvency and declines in credit availability rapidly spread into a global economic shock, reporting several bank and business failures, reflecting these conditions with the spikes of the CBOE VIX. The household wealth felt around $ 14 trillion USD, resulting in a decline of the consumption and a decline of the business investment.

In the fourth quarter of 2008, the quarter-over-quarter decline in real GDP in the U.S. was 8.4%, with a progressive level of unemployment increasing along the time and a decrease of the average number of hours per work week. In the aftermath of each spike the CBOE VIX returns to more normal levels. Although the weekly closing levels of the VIX appears to spike in opposite directions of the S&P 500 index, there are also times when a run-up in stock prices is accompanied by a run-up in volatility (Whaley 2009).

Another example of spikes for the CBOE VIX is followed by the European sovereign debt crisis, which began with a deficit of the Greek economy in late 2009, and the 2008–2011 Icelandic financial crisis, which involved the bank failure of the major banks in Iceland. During this period, the financial assistance of the European Central Bank (ECB) or the International Monetary Fund (IMF) were extremely important for several eurozone member states. The Greek government disclosed that its budget deficits were far higher than previously thought and several European nations implemented a series of measures in 2010, such as the European Financial Stability Facility (EFSF) and the European Stability Mechanism (ESM).

These news created the premises for the spikes of the CBOE VIX. The economies were unable to reimburse or rollover their government debts and bail out over-indebted banks under their national supervisions. The ECB also lowered the interest rates with the aim to provide cheap loans of more than one trillion euro, in order to maintain money flows between European banks.

During the period from the third quarter of 2009 to the last quarter of 2013, the mean and the standard deviation of the CBOE VIX were respectively above 19.9 and 6.3, implying a level of the CBOE VIX volatility lower than the standard deviation computed over the entire period of observation. It is important to remark the EU-IMF bailout for Ireland and Portugal in November 2010 and May 2011 as well as the second Greek bailout in march 2012, with rescue packages for Spain and Cyprus in June 2012.

The circumstances that allow the spikes of the CBOE VIX are also relevant during the period referred to as The Coronavirus crash, that began on February 20th, 2020 and ended on April 7th, 2020. Table 1 respectively shows the mean value and the standard deviation for the CBOE VIX that are above 50.25 and 17.3. These values are the highest over the entire period and the analyzed subperiods. The crash is the most disastrous since the Wall Street Crash of 1929 and characterized the beginning of the COVID-19 recession. The Coronavirus crash follows a decade of economic prosperity and sustained growth from the global financial crisis that began in July 2007. The selling activity was intensified during the first half of March to mid-March, with the largest drops on March 9th, 2020, on March 12th, 2020 and March 16th, 2020. To deal with the panic, banks and reserves across the world cut their interest rates as well as offered unprecedented support to investors and markets.

4. The statistical methodology

This section proposes the statistical methodology for modeling and forecasting the CBOE Volatility Index. The methodology relies on the Huber M-estimator, based on the Huber-Bisquare objective function for the residuals, with a scale function based on the Median

Absolute Deviation, centered around the Median (MADMED), where, the estimation of the co-variance matrix relies on the Type I technique for the Huber function.

The **M**-estimation technique can be implemented considering the left hand side (LHS) of the regression as y_i and β the coefficient of the covariates at the right hand side (RHS) of the regression. Then, the residuals that are possible to redefine with r_i can be written as a function of the quantity β in the following way:

$$r_i\left(\beta\right) = y_i - X_i^{'}\beta. \tag{6}$$

Therefore, the Huber M-estimator computes the coefficient values that minimize the summed values of a function of the residuals:

$$\hat{\beta}_M = argmin_{\beta} \sum_{i=1}^{N} \rho_c \left(\frac{r_i\left(\beta\right)}{\sigma \omega_i} \right), \tag{7}$$

where, σ is a measure of the scale for the residuals, c is an arbitrary positive tuning constant associated with the Huber-Bisquare function and ω_i are the individual weights that are generally set to 1, but may be set to the quantity below:

$$\omega_i = \sqrt{1 - X_i\left(X'X\right)^{-1} X_i^{'}}. \tag{8}$$

If the scale σ is known, then the $k - vector$ of coefficient estimates $\hat{\beta}_M$ may be found using standard iterative techniques for solving the k nonlinear first-order equations:

$$\sum_{i=1}^{N} \psi_c \left(\frac{r_i\left(\beta\right)}{\sigma \omega_i} \right) \frac{x_{ij}}{\omega_i} = 0 \quad j = 1, ..., k \tag{9}$$

for β,where $\psi_c\left(\cdot\right) = \rho_c^{'}\left(\cdot\right)$ is the first derivative of the function $\rho_c\left(\cdot\right)$ and x_{ij} is the value of the $j - th$ covariate for the observation i.

If the scale function σ is not known, then a sequential procedure is necessary with the aim to solve the equation n. 9. In particular, computing updates estimates of the scale $\hat{\sigma}_{(j+1)}$given the coefficient estimates $\hat{\beta}_{(j)}$and using iterative methods to find the $\hat{\beta}_{(j)}$. The initial $\hat{\beta}_{M,(0)}$is obtained from ordinary least squares. The initial coefficients are used to compute a scale estimate, $\hat{\sigma}_{(1)}$, and from that are formed new coefficient estimates $\hat{\beta}_{M,(1)}$, followed by a new scale estimate $\hat{\sigma}_{(2)}$, and so on until convergence is reached. Given an estimate $\hat{\beta}_{M,(s-1)}$, the updated scale $\hat{\sigma}_{(s)}$ is estimated using the following quantity:

$$\hat{\sigma}_{(s)} = median\left[\frac{|\left(r_{i,(s-1)} - median\left(r_{i,(s-1)}\right)\right)|}{0.6745}\right]. \tag{10}$$

5. Empirical results

This section discusses the estimates and the empirical results of the statistical methodology reported in Section 4. In particular, Table 2 reports the values of the sensitivities for the observed prices at one day, one week, one month, three months and six months before. These values decrease in magnitude from 0.891 to 0.004, revealing how the more recent prices tend to weight more than the old prices. The Rw-squared is equal to 99.59% and the coefficients of the statistical methodology are all positive and statistically significant.

[Please Insert Table 2 around here]

The results also emphasize more the Monday effect that refers to the theory that Monday market returns follow those of the previous Friday. The Monday effect is related with the tendency of companies to release bad news on a Friday, after markets close, which then depresses stock prices on Monday. Some scholars argue that the Monday effect might be attributed to short selling, which would affect stocks with high short interest positions. Alternatively, the effect could simply be a result of traders' fading optimism between Friday and Monday.

The empirical results of the robust methodology are compared with the estimates of alternative models (see Table 3) that allow to derive some conclusions regarding the out-of-sample performance. The analysis proposes the ARMA(3,3)-TARCH(1,1), the ARMA(3,3)-EGARCH(1,1), the ARMA(3,3)-PARCH(1,1), with t-student distributions computed on the logarithmic variations, as benchmarks for comparing the prediction of the CBOE VIX. The coefficients that depict the volatility components are all statistically significant, across the specifications.

[Please Insert Table 3 and Table 4 around here]

Table 4 respectively reports the metrics of accuracy related to the one-step ahead forecasts for 50% of the time frame and for the sub-periods related to the financial crisis, the European sovereign debt crisis and the Coronavirus crash. Three out of five metrics of accuracy report an out-performance of the robust methodology respect to the benchmark models. In particular, the robust methodology reports the lowest Mean Absolute Error (MAE), the lowest Mean Absolute Percentage Error (MAPE) and the lowest Symmetric Mean Absolute Percentage Error (SMAPE); whereas, the Root Mean Squared Error (RMSE) and the Theil Inequality Coefficient are slightly higher. If the one step ahead forecast considers the Coronavirus crash period, the robust methodology reports the lowest RMSE and the lowest Theil Inequality Coefficient.

6. Conclusions

The main goal of this paper is to investigate the role of a robust methodology for modeling and forecasting the CBOE VIX. The methodology relies on the Huber M-estimator, based on the Huber-Bisquare objective function for the residuals, with a scale function based on the Median Absolute Deviation, centered around the Median (MADMED).

The robust methodology, constructed under the so called Heterogeneous Market Hypothesis, which recognizes the presence of heterogeneity across traders, out-performs the alternative models (ARMA(3,3)-TARCH(1,1), ARMA(3,3)-EGARCH(1,1), ARMA(3,3)-PARCH(1,1)) used as benchmarks.

The study can be used by market participants for constructing forecasts based on an alternative statistical framework and for measuring the risk of a portfolio.

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Table 1.
Descriptive Statistics

The table reports the descriptive statistics of the CBOE VIX during the following sub-periods: (i) Entire sample (01/01/1990 – 04/30/2021); (ii) The financial crisis (Q3 2007 – Q1 2009); (iii) The European Sovereign Debt crisis (Q3-2009 until Q4-2013); (iv) the 2020 stock market crash or Coronavirus crash (February 20th, 2020 – April 7th, 2020).

Summary Statistics	Entire Period	Financial Crisis	European Sovereign Debt Crisis	Coronavirus Crash
Mean	19.496	30.940	19.901	50.273
Median	17.540	24.520	18.095	52.225
Max.	82.690	80.860	48.000	82.690
Min.	9.140	14.720	11.300	15.560
Std. Dev.	8.087	14.662	6.319	17.376

Table 2.
Empirical Results

The table reports the robust regressions for predicting the level of the CBOE VIX. The regressions are based on the **M-estimation** technique that relies on the Huber-Bisquare objective function and the type I covariance matrix, with median centered scale estimates.The standard errors are reported in the brackets. The significance levels at 1%, 5% and 10% are respectively represented in the following way: ***, **, *.

	(1)	(2)	(3)	(4)	(5)
Price(-1)	0.974*** (0.002)	0.905*** (0.005)	0.897*** (0.005)	0.892*** (0.005)	0.891*** (0.005)
Price(-5)		0.069*** (0.005)	0.057*** (0.005)	0.056*** (0.005)	0.055*** (0.005)
Price(-22)			0.021*** (0.003)	0.013*** (0.003)	0.013*** (0.003)
Price(-66)				0.016*** (0.002)	0.014*** (0.003)
Price(-132)					0.004*** (0.002)
Monday	0.722*** (0.044)	0.685*** (0.044)	0.664*** (0.045)	0.602*** (0.048)	0.584*** (0.050)
Tuesday	0.350*** (0.043)	0.331*** (0.043)	0.315*** (0.044)	0.256*** (0.047)	0.238*** (0.049)
Wednesday	0.333*** (0.043)	0.322*** (0.043)	0.302*** (0.044)	0.244*** (0.047)	0.228*** (0.049)
Thursday	0.327*** (0.043)	0.312*** (0.043)	0.293*** (0.045)	0.235*** (0.047)	0.218*** (0.050)
Friday	0.179*** (0.043)	0.166*** (0.043)	0.140*** (0.045)	0.079*** (0.047)	0.060*** (0.049)
Rw-squared	99.59%	99.61%	99.59%	99.59%	99.59%

Table 3.
Alternative models

The table reports the estimates of alternative models with the aim to predict the level of the CBOE VIX. The standard errors are reported in the brackets. The significance levels at 1%, 5% and 10% are respectively represented in the following way: ***, **, *.

	ARMA(3,3)-TARCH(1,1)	**ARMA(3,3)-EGARCH(1,1)**	**ARMA(3,3)-PARCH(1,1)**
β_0	-0.002* (0.001)	-0.002*** (0.000)	-0.002*** (0.000)
β_1	0.063 (0.157)	0.005 (0.070)	0.013 (0.045)
β_2	-0.295*** (0.119)	-0.338*** (0.047)	-0.332*** (0.020)
β_3	0.833*** (0.150)	0.778*** (0.060)	0.786*** (0.035)
β_4	-0.146 (0.116)	-0.099* (0.059)	-0.105*** (0.041)
β_5	0.229*** (0.084)	0.262*** (0.039)	0.258*** (0.017)
β_6	-0.909*** (0.108)	-0.866*** (0.048)	-0.871*** (0.029)
θ_0 (x10)	0.004*** (0.000)		
θ_1	0.171*** (0.040)		
θ_2	-0.183*** (0.062)		
θ_3	0.802*** (0.053)		
ϕ_0		-0.489*** (0.115)	
ϕ_1		0.140*** (0.021)	
ϕ_2		0.121*** (0.014)	
ϕ_3		0.933*** (0.018)	
γ_0			0.004*** (0.001)
γ_1			0.078*** (0.012)
γ_2			-0.910*** (0.161)
γ_3			0.857*** (0.023)
γ_4			1.108*** (0.111)
t	5.188*** (0.340)	5.245*** (0.332)	5.262*** (0.332)

Table 4
One-step ahead forecasts

The table reports the metrics of performance (RMSE, MAE; MAPE: SMAPE; Theil Inequality Coefficient) related to the one-step ahead forecasts for the models discussed. The forecasting period consider 50% of the observation (Panel 4.1), as well as the financial crisis period (July 2007 – March 2009), the European sovereign debt crisis (October 2009 – December 2013) and the Coronavirus crash (February 20th, 2020 – April 7th, 2020) reported in Panel 4.2. The models are numbered as (1), (2), (3), (4). The model n. (1) is the proposed robust methodology; (2) is ARMA(3,3)-TARCH(1,1); (3) is ARMA(3,3)-EGARCH(1,1); (4) is ARMA(3,3)-PARCH(1,1).

Panel 4.1: One step ahead forecasts for 50& of the observations

	50% of the observations			
	(1)	(2)	(3)	(4)
RMSE	1.98051	1.99639	1.97494	1.97495
MAE	1.10354	1.11810	1.11805	1.11814
MAPE	5.13477	5.19444	5.20509	5.20553
SMAPE	5.21294	5.27163	5.27522	5.27536
Theil Inequality Coeff.	0.04596	0.04628	0.04567	0.04567

Panel 4.2: One step ahead forecasts for sub-periods

	Financial crisis (July 2007 - March 2009)				European sovereign debt crisis (October 2009 - December 2013)				Coronavirus crash (February 20th, 2020 – April 7th, 2020)			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
RMSE	3.17275	3.18653	3.11895	3.11887	1.74543	1.74687	1.73196	1.73212	8.41247	8.68370	8.45656	8.45669
MAE	1.94610	1.97806	1.96209	1.96189	1.04776	1.06491	1.06475	1.06482	5.82037	5.81200	5.86123	5.86147
MAPE	5.68366	5.79919	5.79692	5.79669	4.83389	4.92683	4.93294	4.93330	11.77831	11.61261	11.84920	11.84746
SMAPE	5.81227	5.90148	5.86363	5.86320	4.89706	4.97989	4.98591	4.98596	12.78471	12.76284	12.83624	12.83339
Theil Inequality Coeff.	0.04694	0.04702	0.04580	0.04580	0.04203	0.04199	0.04159	0.04160	0.08142	0.08489	0.08151	0.08151