A simple framework for predicting the overnight returns and the change of illiquidity*

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Abstract

The paper proposes the augmented Diagonal BEKK(1,1) with a *multivariate t-student* distribution of the innovations for predicting the gaps that usually occur when there are adverse news announcements, which can cause a substantial variation from the previous day's closing price. The analysis, applied to the CBOE Volatility Index (VIX), also discusses: (i) a theoretical framework for deriving the conditional illiquidity at the open and close of a trading day as well as the change of conditional illiquidity during the overnight and the daytime; (ii) the quantile and the robust regressions for measuring the errors between the realized overnight variations and the predicted ones.

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1. Introduction

The opening and closing of trading on the Chicago Board Options Exchange (CBOE) provide an excellent laboratory for investigating the effect of a particular market structure on the overnight returns and the level of illiquidity. The 2020 stock market crash, also referred to as the Coronavirus crash of 2020, has focused increasing attention on the effect of market structure on the behavior of securities prices around the overnight period of time.

The call to academics for predicting the overnight returns was received, delivering an econometric methodology based on a bivariate augmented Diagonal BEKK(1,1), with a multivariate *t-student* distribution of the innovations. The error computed between the realized overnight variations and the predicted overnight variations is estimated via quantile and robust regressions, that respectively rely on the Markov Chain Marginal Bootstrap and the MM-estimation technique, with the aim to control the accuracy of the econometric methodology and being able to manage the gap risk that usually occurs when there are adverse news announcements, which can cause a substantial variation from the previous day's closing price.

The academic literature documents that return volatility is much larger when the market is open than when it is closed and at other times of the day (French and Roll 1986, Wood et al. 1985). Stoll and Whaley (1990) point out the role of private information revealed in trading and temporary price deviations induced by specialists and other traders affecting more the return volatility at the open. Hong and Wang's (2000) framework predicts lower returns during overnight periods than during daytime periods. Conversely, Longstaff's (1995) model predicts higher returns during overnight periods than during daytime periods to compensate liquidity providers for considering additional risk. George and Hwang (2001) examine the rate of information flow and find that the daytime information rate is about seven times higher than the overnight rate, and the variances of pricing errors at the open are not different from those at the close of trading. Barclay and Hendershott (2003) document that price changes are larger, because information asymmetry declines over the day, reflecting more private information, and are less noisy before the open than after the close.

With respect to return patterns over daytime and overnight periods, the empirical evidence relies on GARCH methodologies (Bollerslev et al. 1992, Glosten et al. 1993) and it is not consistent across empirical studies, pointing out some anomalies. Susmel and Engle (1994) estimate a GARCH model for overnight and daytime returns using the FT 30 Stock Index for a two-year period beginning in January 1987; Masulis and Ng (1995) propose a GARCH methodology for modeling overnight and daytime stock return dynamics on the London Stock Exchange, discussing the results around the 1986 market restructuring (Big Bang) and the 1987 stock market crash.

French (1980) identifies the weekend effect using U.S. daily stock returns from 1953 to 1977, finding that Monday's mean return is significantly negative, while the other day-of-the-week returns are significantly positive. Rogalski (1984) studies the weekend effect for the U.S. stock market, decomposing the close-to-close returns into an overnight and daytime return. Cliff et al. (2008) document the U.S. equity premium is entirely due to overnight returns, with returns during the day are close to zero and sometimes negative. Tompkins and Wiener (2008) find significant differences between daytime and overnight period returns. For the U.S. market, the mean return is higher for the daytime period than for the overnight period, with the overnight period reports a lower variance. For the four non-U.S. stock markets, the overnight period return is significantly higher than that of the daytime period. The anomaly appears to be due to differences in regulatory risk management requirements for equity derivative market-makers, amplified by Basel I capital requirements.

The timing of earning announcements, asset liquidity, and investor trading heterogeneity have been proposed as arguments for explaining the overnight effect. Although the empirical evidence is not consistent with such arguments (Patell and Wolfson, 1982; Doyle and Magilke, 2009), the timing-of-earnings-announcement hypothesis suggests that managers tend to disclose good news during the overnight period, particularly before the opening of the markets. The asset liquidity hypothesis (Amihud 2002) suggests that the expected market illiquidity positively affects the ex ante stock excess return. The investor trading heterogeneity during daytime and overnight periods has also been suggested as a contributor to the effect. Berkman et al. (2012) and Lou et al. (2018) link investor heterogeneity to the persistence of the overnight and intraday components of returns. The authors document that the trading activity of individual investors plays an important role in explaining higher overnight returns due to their behaviour, which pushes opening prices up.

Another part of the literature derive propositions about the short term behavior of prices and the implications on the bid-ask spread (Roll 1984, McInish and Wood 1989), decomposed into two components, one due to asymmetric information and one due to inventory costs, specialist monopoly power, and clearing costs (Glosten and Harris 1988). Stoll (1989) shows the relation between the square of the quoted bid-ask spread and the serial covariance of transaction returns as well as the serial covariance of quoted returns. The serial covariance as estimator overestimates the serial covariance of the underlying stock values (Harris 1990). As pointed out by Niederhoffer and Osborne (1966), the negative serial dependence in observed price changes should be anticipated when a market maker is involved in transactions. Therefore, Richardson and Smith (1991, 1994) discuss a general approach to testing serial dependence restrictions implied from financial models, further extended with the aim to test serial correlations in stock returns.

Finally, the academic literature also documents the relations between return sensitivity and market liquidity (Pastor and Stambaugh 2003), prices and market liquidity (Amihud and Mendelson 1986) and that liquidity comoves with returns and predicts future returns (Amihud 2002, Chordia et al. 2001a).

2. The Theoretical Framework

The opening of the Chicago Board Options Exchange resembles to a call auction market, where there is a specialist, the "auctioneer", that accumulates orders and determines the prices with the assistance of the opening automated reporting system. The specialist might participate at the open, with the aim to offset order imbalances, although the transactions are usually among investors.

The theoretical framework for determining the open and the close prices relies on Roll (1984), Glosten and Harris (1988), Corvasce (2016) and assumes that the observed price of an asset ($\hat{p}$), at a certain time t, consists of two components $\tilde{p}$, that is the unobserved price of an asset also caused by the arrival of new information at a certain time t and p that represents the transaction costs incurred in making an exchange of a certain asset, at time t. These quantities are observed at the open (o) and the close (c) of a trading day. Therefore,

$$\hat{p}_t^o = \tilde{p}_t^o + p_t^o \tag{1}$$

$$\hat{p}_t^c = \tilde{p}_t^c + p_t^c \tag{2}$$

where,

$$\tilde{p}_t^o = \tilde{p}_{t-1}^o + Q_t^o \cdot Z_t^o + e_t^o \tag{3}$$

$$\tilde{p}_t^c = \tilde{p}_{t-1}^c + Q_t^c \cdot Z_t^c + e_t^c \tag{4}$$

whereas,

$$p_t^o = f\left(Q_t^o\,,\, C_t^o\right) \tag{5}$$

$$p_t^c = f\left(Q_t^c\,,\, C_t^c\right) \tag{6}$$

In particular, the quantities Q_t^o and Q_t^c respectively represent the unobserved indicators for the bid/ask classifications, at the open and close of a trading day and take a value equals to $+1$, if the transactions at the open and close were initiated by a buyer and a value equals to -1, if the transactions were initiated by a seller. The quantities Z_t^o and Z_t^c respectively represent the adverse selection components that also depend on the order sizes arrived at the open and close of a trading day, since well informed traders maximize the returns to their perishing information, impacting on the level of the asymmetric information, available at the open and close of a trading day.

Therefore, the quantities $Q_t^o \cdot Z_t^o$ and $Q_t^c \cdot Z_t^c$ respectively represent the products between the unobserved indicators for the bid/ask classification and the adverse selection components at the open and close of a trading day, conditional on the arrival of new orders. Assuming a positive quantity of $Z_t^o$and Z_t^c, a buy/sell order respectively creates a potential increase/decrease of the unobserved price $(\tilde{p})$, at the open and close of a trading day, with sizes that are in absolute values respectively equal to $Z_t^o$and Z_t^c. The quantities $\tilde{p}_{t-1}^o$ and $\tilde{p}_{t-1}^c$ represent the unobserved prices of an asset, at the open and close for the time $t-1$; whereas, the quantities e_t^o and e_t^c represent the innovations for the unobserved prices of an asset, that depend on the arrival of public information, from time $t-1$ to t and have distributions equal to $G^o$and G^c, with observations that are independent and identically distributed (i.i.d), provided that the means are respectively equal to μ^o and μ^c with variances equal to $v^{2,o}$ and $v^{2,c}$, at time t.

The components p_t^o and p_t^c are functions $f(\cdot)$ of the unobserved indicators for the bid/ask classifications $(Q_t^o and\, Q_t^c)$ and the unobserved transitory components $(C_t^o and\, C_t^c)$, that also depend on the order sizes, at the open and close of a trading day. As such, the equalities (1) and (2) can be rewritten in the following way:

$$\hat{p}_t^o = \tilde{p}_{t-1}^o + Q_t^o \cdot Z_t^o + f\left(Q_t^o\, ,\, C_t^o\right) + e_t^o \tag{7}$$

$$\hat{p}_t^c = \tilde{p}_{t-1}^c + Q_t^c \cdot Z_t^c + f\left(Q_t^c\, ,\, C_t^c\right) + e_t^c. \tag{8}$$

Following Roll (1984), Kyle (1985), Easley and O'Hara (1987), Glosten (1987b), Glosten and Harris (1988) and Corvasce (2016), the framework assumes that the observed price changes at the open and close of a trading day are respectively equal to the following quantities:

$$\Delta \hat{p}_t^o = \Delta \tilde{p}_{t-1}^o + \Delta \left(Q_t^o \cdot Z_t^o \right) + \Delta f \left(Q_t^o \, , \, C_t^o \right) + \Delta e_t^o \tag{9}$$

$$\Delta \hat{p}_t^c = \Delta \tilde{p}_{t-1}^c + \Delta \left(Q_t^c \cdot Z_t^c \right) + \Delta f \left(Q_t^c \, , \, C_t^c \right) + \Delta e_t^c . \tag{10}$$

For the purpose of the analysis, the variations of the observed prices at the open and close can be rewritten in the following way:

$$\Delta \hat{p}_t^o = \delta^o + \Delta \hat{p}_{t-1}^o + \Delta \hat{p}_{t-1}^c \tag{11}$$

$$\Delta \hat{p}_t^c = \delta^c + \Delta \hat{p}_{t-1}^c + \Delta \hat{p}_{t-1}^o , \tag{12}$$

where, $\delta^o$and δ^c are respectively the constant terms; whereas, $\Delta \hat{p}_{t-1}^o$ and $\Delta \hat{p}_{t-1}^c$ are the changes of the observed prices at the open and close for the previous trading day. Therefore, the changes of the adverse selection costs components also depend on the changes of the observed prices at the open and close for the previous trading day, on the changes of the unobserved prices at the open and close for the previous trading day, on the change of the innovations for the unobserved prices of an asset. The equalities (11) and (12) can also be simply written as follows:

$$\Delta \hat{p}_t^o = \Delta \tilde{p}_t^o + \Delta p_t^o \tag{13}$$

$$\Delta \hat{p}_t^c = \Delta \tilde{p}_t^c + \Delta p_t^c , \tag{14}$$

where, $\Delta \tilde{p}_t^o$ and $\Delta \tilde{p}_t^c$ are respectively the unobserved open and close price changes at time t; Δp_t^o and Δp_t^c are the changes of the transaction costs components for the open and the close prices at time t. Using the return relations, the derivation of the conditional illiquidity at the open and close of a trading day is based on the j-th order serial conditional covariances. It is not necessarily inconsistent with efficiency since expected returns may be conditional on factors that are serially dependent. As such,

$$Cov\left(\Delta\hat{p}_t^o\,,\,\Delta\hat{p}_{t-j}^o\,|\,F_{t-j-1}\right) = Cov\left(\Delta\tilde{p}_t^o + \Delta p_t^o\,,\,\Delta\tilde{p}_{t-j}^o + \Delta p_{t-j}^o\,|\,F_{t-j-1}\right) \tag{15}$$

$$Cov\left(\Delta\hat{p}_t^c\,,\,\Delta\hat{p}_{t-j}^c\,|\,F_{t-j-1}\right) = Cov\left(\Delta\tilde{p}_t^c + \Delta p_t^c\,,\,\Delta\tilde{p}_{t-j}^c + \Delta p_{t-j}^c\,|\,F_{t-j-1}\right). \tag{16}$$

Therefore, the conditional levels of illiquidity at the open and close of a trading day are computed in the following way:

$$LAGIlliquidity_t^o = -Cov\left(\Delta\hat{p}_t^o\,,\,\Delta\hat{p}_{t-j}^o\,|\,F_{t-j-1}\right) \tag{17}$$

$$LAGIlliquidity_t^c = -Cov\left(\Delta\hat{p}_t^c\,,\,\Delta\hat{p}_{t-j}^c\,|\,F_{t-j-1}\right). \tag{18}$$

The quantities respectively depict the conditional time varying surprises that are possible to discover during the evolution of the observed open and close prices and the time varying co-movements between the unobserved price variations and the changes in transaction costs at the open and close of a trading day. The quantities (17) and (18) allow to compute the changes of the conditional illiquidity during the overnight and the daytime periods of time. As such,

$$Overnight\_Illiquidity_t = LAGIlliquidity_t^o - LAGIlliquidity_{t-1}^c \tag{19}$$

$$Daytime\_Illiquidity_t = LAGIlliquidity_t^c - LAGIlliquidity_t^o. \tag{20}$$

A level of *Overnight_Illiquidity* greater than zero identifies a higher level of conditional illiquidity at the open of a trading day than the level of conditional illiquidity at the close of the previous trading day. Therefore, the potential loss incurred by an investor for selling or exchanging a certain asset at the open of a trading day is greater than the potential loss incurred for selling or exchanging a certain asset at the close of the previous trading day.

This means that the asset can be sold more quickly at the end of the previous trading day than the open of a trading day. Therefore, a level of $Overnight\_Illiquidity$ greater than zero defines a situation in which a seller offers an asset at the open of a trading day for a knockdown price in order to drum up interest that is in relative terms higher than the price that might potential receive at the close of the previous trading day selling or exchanging the same asset. These conditions also have implications on the quantity $Daytime\_Illiquidity$, computed as a difference between the conditional illiquidity at the close of a trading day and the conditional illiquidity at the open. Indeed, a level smaller than zero defines a situation in which the potential loss incurred by an investor for selling or exchanging a certain asset at the close of a trading day is smaller than the potential loss incurred for selling or exchanging a certain asset at the open of a trading day.

3. Data and descriptive statistics

The empirical analysis considers the Chicago Board Options Exchange (CBOE) Volatility Index (VIX), from January 1st, 1992 to October 9th, 2020. The index is computed on a real-time basis through each trading day and was introduced by Whaley (1993). It represents the market's expectations of thirty days forward looking volatility and provides a measure of market risk as well as investor's sentiment. The original index was based on the prices of eight at-the-money index calls and puts for the S&P 100 Index, that accounted for 75% of the total index option volume in 1992. Indeed, the average trading volume for the calls was equal to 120,475 and for the puts was equal to 125,302. Over the years, the option market on the S&P 500 Index became more active and for this reason the VIX was computed on the calls and puts of this index that respectively reached a level of 525,460 and 909,748 call and put option contracts in the first ten months of 2008.

The shift in market dominance from options on the S&100 to S&P500 is based on the remark that the index portfolios have a high correlation and seem perfect substitutes, with the means and the standard deviations that are nearly identical. As of October 2008, all S&P100 stocks were contained within the S&P 500 index and the highest market cap stocks were the same.

[Please insert Table 1 around here]

The recent financial crisis shows several spikes of the CBOE VIX that reacts in response to unexpected market and world events both at the open and close of a trading day. The average values and the standard deviations are respectively above 19.3 and 8.2 during the entire period of observation and the overnight variation is equal to 0.074 with a standard deviation equals to 0.939. During the global financial crisis, both the averages and the standard deviations for the close and open values of the CBOE VIX are respectively above 30 and 14.6.

The bursting of the United States housing bubble, culminating with the bankruptcy of Lehman Brothers on September 15, 2008, as well as the lack of investor confidence in bank solvency and declines in credit availability rapidly spread into a global economic shock, reporting several bank and business failures, reflecting these conditions with the spikes of the CBOE VIX. The household wealth felt around \$ 14 trillion USD, resulting in a decline of the consumption and a decline of the business investment. In the fourth quarter of 2008, the quarter-over-quarter decline in real GDP in the U.S. was 8.4%, with a progressive level of unemployment increasing along the time and a decrease of the average number of hours per work week. In the aftermath of each spike the CBOE VIX returns to more normal levels.

Another example of spikes for the CBOE VIX both at the open and the close of the trading days is followed by the European sovereign debt crisis, which began with a deficit of the Greek economy in late 2009, and the 2008–2011 Icelandic financial crisis, which involved the bank failure of the major banks in Iceland. During this period, the financial assistance of the European Central Bank (ECB) or the International Monetary Fund (IMF) were extremely important for several eurozone member states. The Greek government disclosed that its budget deficits were far higher than previously thought and several European nations implemented a series of measures in 2010, such as the European Financial Stability Facility (EFSF) and the European Stability Mechanism (ESM).

These news created the premises for the spikes of the CBOE VIX. The economies were unable to reimburse or rollover their government debts and bail out over-indebted banks under their national supervisions. The ECB also lowered the interest rates with the aim to provide cheap loans of more than one trillion euro, in order to maintain money flows between European banks.

During the period from the third quarter of 2009 to the last quarter of 2013, the means and the standard deviations of the CBOE VIX for the open and close values were respectively above 19.9 and 6.3, implying a level of the CBOE VIX volatility lower than the standard deviations computed over the entire period of observation. The maximum values of the CBOE VIX at the open and close are below the levels of 48; whereas, the overnight variations report an average value that is equal to 0.105 and a standard deviation of 1.112. During the European sovereign debt crisis, it is important to remark the EU-IMF bailout for Ireland and Portugal in November 2010 and May 2011 as well as the second Greek bailout in march 2012, with rescue packages for Spain and Cyprus in June 2012.

The circumstances that allow the spikes of the CBOE VIX are also relevant during the period referred to as The Coronavirus crash, that began on February 20th, 2020 and ended on April 7th, 2020. Table 1 respectively shows the mean values and the standard deviations for the CBOE VIX that are above 49.6 and 17.3. These values are the highest over the entire period and the analyzed subperiods. The crash is the most disastrous since the Wall Street Crash of 1929 and characterized the beginning of the COVID-19 recession. The Coronavirus crash follows a decade of economic prosperity and sustained growth from the global financial crisis that began in July 2007. The selling activity was intensified during the first half of March to mid-March, with the largest drops on March 9th, 2020, on March 12th, 2020 and

March 16th, 2020. To deal with the panic, banks and reserves across the world cut their interest rates as well as offered unprecedented support to investors and markets.

3.1. Interpolating the Data

The interpolation procedure for a series fills in missing values, within a series by interpolating from values that are non-missing. This sub-section proposes the *cubic spline interpolation* technique as a solution widely used in finance for filling potential missing values with the analyzed time series. A cubic spline is a function $f : \mathbb{R} \rightarrow \mathbb{R}$ that relies on the cubic polynomials $p_k(x)$ on different intervals $\left[x^{[k]}, x^{[k+1]}\right]$. It has the following form:

$$f(x) = \begin{cases} p_1(x) & x^{[1]} \leq x \leq x^{[2]} \\ p_2(x) & x^{[2]} \leq x \leq x^{[3]} \\ . & . \\ . & . \\ p_{m-1}(x) & x^{[m-1]} \leq x \leq x^{[m]} \end{cases} \tag{21}$$

A cubic spline is constructed by interpolating a cubic polynomial p_k between each pairs of consecutive points $\left(x^{[k]}, y^{[k]}\right)$ and $\left(x^{[k+1]}, y^{[k+1]}\right)$, considering the points $\left(x^{[1]}, y^{[1]}\right), ..., \left(x^{[m]}, y^{[m]}\right)$, *with* $x^{[1]} < x^{[2]} < ... < x^{[m]}$ and according to the constraints that can be summarized in four steps:

1. *Each polynomial passes through its respective end points* :

$$p_k\left(x^{[k]}\right) = y^{[k]} \text{ and } p_k\left(x^{[k+1]}\right) = y^{[k+1]}$$

2. *First derivatives match at interior points* :

$$\frac{d}{dx} p_k\left(x^{[k+1]}\right) = \frac{d}{dx} p_{k+1}\left(x^{[k+1]}\right)$$

3. *Second derivatives match at interior points* :

$$\frac{d^2}{dx^2} p_k \left(x^{[k+1]} \right) = \frac{d^2}{dx^2} p_{k+1} \left(x^{[k+1]} \right)$$

4. *Second derivatives vanish at the end points* :

$$\frac{d^2}{dx^2} p_1 \left(x^{[1]} \right) = 0 \ \ and \ \ \frac{d^2}{dx^2} p_{m-1} \left(x^{[m]} \right) = 0.$$

This technique delivers a polynomial that is smoother and has a smaller error, avoiding the problem of oscillation at the edges of an interval that occurs when using polynomial interpolation with polynomials of high degree over a set of equispaced interpolation points (*Runge's phenomenon*).

4. The Econometric methodology

This section proposes the econometric methodology for predicting the overnight returns that occur when there is a substantial drop in demand for an asset from the close of a trading day to the open of the next trading day. The framework is applied to the study and prediction of the overnight returns for the CBOE VIX, computed as the difference between the open price and the close price of an asset at the previous trading day. The econometric methodology relies on a bivariate augmented Diagonal BEKK model (Baba et al. 1985) for depicting the dynamics of the conditional variances and covariance between the close-to-close returns and the open-to-open returns for the CBOE VIX. It also considers the CBOE Skew Index at five days before the trading day, as an exogenous factor of the conditional variance/covariance matrix. This indicator typically ranges from 100 to 150 and estimates the skewness of the S&P 500 returns at the end of a 30 days horizon and considers the implied volatility of out-of-the-money strikes. The higher is the rating, the higher is the perceived tail risk and a change for a *black swan* event. The indicator is considered at 5 days before the trading day, for depicting the so called Friday effect observable for stock returns that tend to be higher on Fridays than on Mondays.

Therefore, the arithmetic and the natural logarithmic variations for the daily observed

CBOE VIX at the open and close can be computed in the following way:

$$\Delta\hat{p}_t^c = \frac{\hat{p}_t^c - \hat{p}_{t-1}^c}{\hat{p}_{t-1}^c} \simeq log\left(\frac{\hat{p}_t^c}{\hat{p}_{t-1}^c}\right) = \alpha_0 + \alpha_1 \cdot \Delta\hat{p}_{t-1}^c + \alpha_2 \cdot \Delta\hat{p}_{t-1}^o + \epsilon_t^c \tag{22}$$

$$\Delta\hat{p}_t^o = \frac{\hat{p}_t^o - \hat{p}_{t-1}^o}{\hat{p}_{t-1}^o} \simeq log\left(\frac{\hat{p}_t^o}{\hat{p}_{t-1}^o}\right) = \alpha_3 + \alpha_4 \cdot \Delta\hat{p}_{t-1}^o + \alpha_5 \cdot \Delta\hat{p}_{t-1}^c + \epsilon_t^o \tag{23}$$

where, the quantities α_0 and α_3 are respectively the constants of the mean equations; the quantities α_1 and α_4 are respectively the coefficients of the mean equations related to the close and the open for the previous trading day; the coefficients α_2 and α_5 are the coefficients related to the open and close of the previous trading day that have influence on the observed close-to-close and open-to-open variations. The quantities ϵ_t^c and ϵ_t^o are the residuals of the mean equations. The innovations of the residuals follow an augmented Diagonal BEKK(1,1) (Baba et al. 1985), with a multivariate *t-student* distribution and an unknown parameter t, able to quantify the degrees of freedom. This assumption, regarding the distribution of the disturbances, allows to depict the stylized facts (i.e. asymmetry and fat tails), concerned about the distributions of the variations for the CBOE VIX at the close and the open.

Therefore, the conditional variances and covariance for the observed variations of the close and the open, provided the information set F at time $t-1$, are computed in the following way:

$$E\left[\epsilon_t^{2,c} \,|\, F_{t-1}\right] = \sigma_t^{2,c} = M(1,1) + A(1,1)^2 \cdot \epsilon_{t-1}^{2,c} + B(1,1)^2 \cdot \sigma_{t-1}^{2,c} + E(1,1) \cdot SKEW_{t-5} \tag{24}$$

$$E\left[\epsilon_t^{2,o} \,|\, F_{t-1}\right] = \sigma_t^{2,o} = M(2,2) + A(2,2)^2 \cdot \epsilon_{t-1}^{2,o} + B(2,2)^2 \cdot \sigma_{t-1}^{2,o} + E(2,2) \cdot SKEW_{t-5} \tag{25}$$

$$E\left[\epsilon_t^c \cdot \epsilon_t^o \,|\, F_{t-1}\right] = cov_t^{co} = A(1,1) \cdot A(2,2) \cdot \epsilon_{t-1}^c \cdot \epsilon_{t-1}^o + B(1,1) \cdot B(2,2) \cdot cov_{t-1}^{co} + E(1,2) \cdot SKEW_{t-5}. \tag{26}$$

The quantities $M(1,1)$ and $M(2,2)$ are the diagonal coefficients that depict the long term components of the conditional variances and the conditional covariance; $A(1,1)$ and $A(2,2)$ are the diagonal coefficients that depict the influence of the squared residuals at time $t-1$;whereas, $B(1,1)$ and $B(2,2)$ are the diagonal coefficients that depict the persistence of the conditional variance. The coefficients $E(1,1)$, $E(2,2)$ and $E(1,2)$ depict the influence of the CBOE Skew Index, at five days before the trading day on the conditional variance/covariance matrix.

5. Empirical Results

This section discusses the estimates and the empirical results of the econometric methodology proposed in Section 4 for predicting the overnight differentials, based on the prediction of the open and the close values for the CBOE VIX. The estimation results rely on the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm that is an iterative method for solving unconstrained nonlinear optimization problems. It belongs to quasi-Newton methods and seeks a stationary point of a function, reachable when the gradient is zero. The optimization algorithm begins at an initial estimate for the optimal values and proceeds iteratively to get a better estimates at each stage, till when there is a convergence for finding the solutions. For simplicity, the maximum number of iterations is fixed to n. 5,000 and the convergence rate to 1e-06.

The step method is based on the Levenberg-Marquardt algorithm that is more robust than the Gauss-Newton algorithm, since it allows to derive solutions even if the algorithm starts very far off from the final minimum. In cases with multiple minima, the algorithm converges to the global minimum only if the initial guess is already somewhat close to the final solution. The estimation procedure also accommodates the Huber-White estimator that allows to derive the variance/covariance matrix considering the heteroscedasticity of the residuals.

[Please Insert Table 2 around here]

The results of the estimates are reported in Table 2. It allows to figure out the statistical implications related to the assumption of arithmetic variations rather than logarithmic variations of the CBOE VIX. Although the arithmetic variations aggregate well across portfolios, the logarithmic variations are time additive, they are not impacted by the compounding frequency and with small variations, the logarithmic variations are approximately equal to arithmetic ones and can be considered as normally distributed.

The constants of the mean equations for the close-to-close variations are negative and highly statistically significant; whereas, the constant for the open-to-open logarithmic variations is negative and significant and becomes statistically irrelevant for arithmetic variations. The coefficients that depict the influence of the previous close-to-close variations on the actual close-to-close variations are negative and not statistically significant; whereas, the coefficients for the previous open-to-open variations on the actual close-to-close variations are negative and statistically significant. Further, the coefficients that depict the influence of the previous open-to-open variations and the previous close-to-close variations on the actual open-to-open variations are negative.

The coefficients that depict the persistence of the conditional variances are positive and statistically significant. Therefore, an increase of the conditional variances for the previous trading day allows to increment the actual conditional variances. The same implications are related to the indicator of tail risk. A higher level of the CBOE Skew Index, at five days before the trading one also increases the actual levels of conditional variances and covariance. Therefore, a higher level of the rating and so a higher chance of a black swan event increases the actual levels of conditional variances and covariance.

[Please Insert Table 3 around here]

The estimation results allow to derive the predictions of the CBOE VIX at the open and close of a trading day compared to the realized values. The results are crucial for computing the prediction of the overnight variations between the open values for a particular trading day and the close values of the previous trading day. Therefore, Table 3 summarizes the descriptive statistics for the predicted close and open values of the CBOE VIX over different sub-periods. During the financial crisis, the error computed as a difference between the average realized close values and the average predicted close values is respectively equal to 0.008 and 0.054, considering arithmetic and logarithmic variations with a difference of standard deviations respectively equal to -0.013 and 0.118. The error components respectively change to 0.069 and -0.01 for the prediction of the open values of the CBOE VIX, with a difference of standard deviations respectively equal to 0.033 and 0.05.

The inaccuracy of the econometric methodology increases during the so called Coronavirus crash, when the CBOE VIX reports several days of spikes within a short period of time. Indeed, the error components for the close values are respectively equal to 0.953 and -0.262, considering arithmetic and logarithmic variations with a difference of standard deviations respectively equal to -1.082 and 0.993. The error components respectively change to 0.591 and 0.057 for the prediction of the open values of the CBOE VIX, with a difference of standard deviations respectively equal to -0.234 and 0.218.

5.1 Measuring the Conditional illiquidity for the CBOE VIX

This subsection discusses the conditional illiquidity for the CBOE VIX, based on the framework proposed by Corvasce (2016) that relies on Roll (1984), Glosten (1987b), Glosten and Harris (1988). The first order serial conditional covariance that allows the computation of the conditional illiquidity is computed for the close-to-close and the open-to-open arithmetic and logarithmic variations.

[Please Insert Table 4 around here]

The estimation results are reported in Table 4. The diagonal coefficients that depict the influence of the squared residuals at time $t-1$ as well as the coefficients that depict the persistence of the conditional variances are positive and highly statistically significant. A higher level of the coefficients produce a high level of the actual conditional variances and covariance, generating an increase of the conditional illiquidity, also during days of spikes for the CBOE VIX. The first decile of the days characterized by a high level at the close of the CBOE VIX reports a value of the CBOE Skew Index equal to 118, with an average overnight return of 0.378; whereas, the first decile of the days characterized by a high level at the open of the CBOE VIX reports an average overnight return of 0.392.

[Please Insert Figure 1 and Figure 2 around here]

The conditional level of illiquidity for the close and the open values of the CBOE VIX, based on arithmetic and logarithmic computations, is respectively reported in Figure 1 and Figure 2. Following the arithmetic computations, the days characterized by a high level of conditional illiquidity at the close of a trading day show an average overnight return of 0.20 for the first decile of the days analyzed; whereas the days characterized by a high level of conditional illiquidity at the open of a trading day report an average overnight return of -0.154. The logarithmic computations allow to derive similar results. A high level of conditional illiquidity at the close shows an average overnight return of 0.208; whereas a high level of conditional illiquidity at the open of a trading day reports an average overnight return of -0.136.

[Please Insert Figure 3 and Figure 4 around here]

The changes of the conditional illiquidity during the overnight and the daytime periods of time are respectively reported in Figure 3 and Figure 4. Both arithmetic and logarithmic computations show how the first decile of the days characterized by a high change of the conditional illiquidity during the overnight period of time reports an average overnight return of -0.14; whereas, the first decile of the days characterized by a high daytime conditional illiquidity report an average overnight return of 0.14.

5.2 Measuring the GAP Risk

This subsection provides the tools for measuring the GAP Risk incurred from one day to the next, relying on quantile and robust regressions, with the aim to study the error component between the realized overnight variations and the predicted overnight variations. The gap usually occurs when there are adverse news announcements, which can cause a substantial variation from the previous day's closing price.

[Please Insert Table 5 around here]

The quantile regressions are computed at the 1st percentile and rely on the Markov Chain Marginal Bootstrap (MCMB) method developed by Kocherginsky et al. (2005). The MCMB-A method distinguishes itself from the usual bootstrap since it involves solving only one-dimensional equations for parameters of any dimension and produces a Markov chain rather than a (conditionally) independent sequence. Therefore, the method alleviates computational burdens often associated with bootstrap in high-dimensional problems and can be applied for solving the quantile regressions. The sparsity estimation rely on the Chamberlain bandwidth with a level of the bandwidth equals to 0.0050196 and the number of bootstrap replications is equal to 10,000.

The differences between the realized overnight variations and the predicted overnight variations are explained as a function of several covariates. Following the arithmetic and the logarithmic computations, the coefficients related to the levels of conditional volatilities for the previous day's closing price and for the actual open price, the overnight and daytime variations for the previous trading day and the level of conditional illiquidity at the open are negative and statistically significant. Further, the differences between the realized and the predicted overnight variations are higher on Mondays and lower on Fridays. Indeed, the variations of the CBOE VIX on Mondays are often significantly lower than those of the immediately preceding Fridays, with a general tendency to release bad news on a Friday after the markets close, which then depresses the level of the CBOE VIX on Monday.

[Please Insert Table 6 around here]

The analysis is also corroborated by robust regressions that rely on the *MM-estimation* technique. This statistical procedure attempts to retain the robustness and resistance of S-estimation (Rousseeuw and Yohai 1984), whilst gaining the efficiency of M-estimation. The technique proceeds by finding a highly robust and resistant S-estimate that minimizes an M-estimate of the scale of the residuals. The estimated scale is then held constant whilst a close by M-estimate of the parameters is originated (the second M). Both the computations based on arithmetic and logarithmic variations report high levels of statistical significance for the coefficients related to the conditional volatilities for the previous day's closing price and for the actual open price, the overnight and daytime variations for the previous trading day and the level of conditional illiquidity.

6. Conclusions

The manuscript investigates the structure of the market at the open and close on the Chicago Board Options Exchange (CBOE), delivering an econometric methodology, based on a bi-variate augmented Diagonal BEKK(1,1), able to describe the dynamics of the prices, with the aim to predict the gaps that usually occur when there are adverse news announcements, which can cause a substantial variation from the previous day's closing price.

The error computed between the realized overnight variations and the predicted overnight variations is estimated via quantile and robust regressions, with the aim to manage the gap risk. The paper documents that the coefficients related to the levels of conditional volatilities for the previous day's closing price and for the actual open price, the overnight and daytime variations for the previous trading day and the level of conditional illiquidity at the open are negative and statistically significant. Further, the differences between the realized and the predicted overnight variations are higher on Mondays and lower on Fridays, due to a general tendency to release bad news on a Friday after the markets close, which then depress the level of the CBOE VIX on Monday.

The analytical framework allows to derive the dynamic of the conditional illiquidity at the open and close, considering the time period between January 1st, 1992 and October 9th, 2020. A higher level of the coefficients produce a high level of the actual conditional variances and covariance, generating an increase of the conditional illiquidity, also during days of spikes for the CBOE VIX.

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Table 1.
Descriptive Statistics

The table reports the descriptive statistics for the OPEN (Panel 1.1) and CLOSE (Panel 1.2) values of the CBOE VIX as well as for the Overnight variation (Panel 1.3) during the following sub-periods: (i) Entire sample (01/01/1992 – 10/09/2020); (ii) The financial crisis (Q3 2007 – Q1 2009); (iii) The European Sovereign Debt crisis (Q3-2009 until Q4-2013); (iv) the 2020 stock market crash or Coronavirus crash (February 20th, 2020 – April 7th, 2020).

Panel 1.1: The OPEN Values of the CBOE VIX

Summary Statistics	**Entire Period**	**Financial Crisis**	**European Sovereign Debt Crisis**	**Coronavirus Crash**
Mean	19.412	31.078	20.017	49.611
Median	17.130	24.800	18.210	51.675
Max.	82.690	80.740	47.660	82.690
Min.	9.010	14.960	11.520	14.540
Std. Dev.	8.370	14.728	6.345	18.354

Panel 1.2: The CLOSE Values of the CBOE VIX

Summary Statistics	**Entire Period**	**Financial Crisis**	**European Sovereign Debt Crisis**	**Coronavirus Crash**
Mean	19.339	30.939	19.901	50.273
Median	17.080	24.520	18.080	50.225
Max.	82.690	80.860	48.000	82.690
Min.	9.140	14.720	11.300	15.560
Std. Dev.	8.294	14.664	6.319	17.376

Panel 1.3: The Overnight rate of the CBOE VIX

Summary Statistics	**Entire Period**	**Financial Crisis**	**European Sovereign Debt Crisis**	**Coronavirus Crash**
Mean	0.074	0.200	0.105	0.289
Median	0.030	0.000	0.035	0.080
Max.	14.570	10.010	6.470	8.040
Min.	-12.300	-6.760	-12.300	-6.540
Std. Dev.	0.939	1.255	1.112	3.506

Table 2.
Empirical Results

The table reports the estimation results of the econometric methodology. The augmented Diagonal BEKK (1,1) considers the evolution of the CBOE Skewness, at five days before the trading day, for modelling the dynamics of the conditional variance/covariance matrix between the CLOSE values and the OPEN values of the CBOE VIX. Panel 2.1 reports the values of the model that considers the ARITHMETIC changes of the OPEN and CLOSE values for the CBOE VIX; whereas, Panel 2.2 reports the values of the model that considers the LOGARITHMIC values. In particular, Panel 2.1 reports the following results: α_0 is the constant of the mean equation related to the arithmetic CLOSE changes of the CBOE VIX; α_1 is the coefficient of the mean equation that depicts the influence of the arithmetic CLOSE changes of the CBOE VIX, at the previous trading day; α_2 is the coefficient of the mean equation that depicts the influence of the arithmetic OPEN changes of the CBOE VIX, at the previous trading day; α_3 is the constant of the mean equation related to the arithmetic OPEN changes of the CBOE VIX; α_4 is the coefficient of the mean equation that depicts the influence of the arithmetic OPEN changes of the CBOE VIX, at the previous trading day; α_5 is the coefficient of the mean equation that depicts the influence of the arithmetic CLOSE changes of the CBOE VIX, at the previous trading day; $M(1,1)$ is the constant values of the conditional variance related to the CLOSE arithmetic values of the CBOE VIX; $A(1,1)$ is the coefficient that depicts the squared residuals of the conditional variance related to the CLOSE arithmetic values of the CBOE VIX; $B(1,1)$ is the coefficient that depicts the persistence of the conditional variance related to the CLOSE arithmetic values of the CBOE VIX; $E(1,1)$ is the coefficient that depicts the influence of the CBOE Skewness, at five days before the trading day, to the conditional variance related to the CLOSE arithmetic values of the CBOE VIX; $M(2,2)$ is the constant values of the conditional variance related to the OPEN arithmetic values of the CBOE VIX; $A(2,2)$ is the coefficient that depicts the squared residuals of the conditional variance related to the OPEN arithmetic values of the CBOE VIX; $B(2,2)$ is the coefficient that depicts the persistence of the conditional variance related to the OPEN arithmetic values of the CBOE VIX; $E(2,2)$ is the coefficient that depicts the influence of the CBOE Skewness, at five days before the trading day, to the conditional variance related to the OPEN arithmetic values of the CBOE VIX; $E(1,2)$ is the coefficient that depicts the influence of the CBOE Skewness, at five days before the trading day, to the conditional covariance between the CLOSE and the OPEN arithmetic values of the CBOE VIX. The disturbances are distributed according to a t-student distribution, with a parameter equal to t. The last column of the panels reports the standard errors.

Panel 2.1: Estimation results based on ARITHMETIC values

Coefficient	Estimated value	Standard Errors
α_0	-0.002	0.001
α_1	-0.011	0.014
α_2	-0.047	0.013
α_3	-0.001	0.001
α_4	-0.265	0.013
α_5	0.746	0.019
$M(1,1)$	-0.001	0.000
$A(1,1)$	0.240	0.020
$B(1,1)$	0.929	0.009
$E(1,1)$ x 1000	0.009	0.000
$M(2,2)$ x 1000	-0.038	0.000
$A(2,2)$	0.445	0.107
$B(2,2)$	0.796	0.134
$E(2,2)$ x 1000	0.005	0.000
$E(1,2)$ x 1000	0.002	0.000
t	4.942	0.213

Panel 2.2: Estimation results based on LOGARITHMIC values

Coefficient	Estimated value	Standard Errors
α_0	-0.001	0.000
α_1	-0.015	0.014
α_2	-0.043	0.012
α_3 x 10	-0.005	0.000
α_4	-0.272	0.012
α_5	0.746	0.016
$M(1,1)$ x 10	-0.001	0.000
$A(1,1)$	0.252	0.020
$B(1,1)$	0.930	0.009
$E(1,1)$ x 1000	0.002	0.000
$M(2,2)$ x 100	-0.001	0.000
$A(2,2)$	0.400	0.091
$B(2,2)$	0.844	0.092
$E(2,2)$ x 10000	0.007	0.000
$E(1,2)$ x 10000	0.002	0.000
t	5.591	0.258

Table 3.

The Predicted OPEN and CLOSE values of the CBOE VIX

The table reports the descriptive statistics for the predicted OPEN and CLOSE values of the CBOE VIX as well as the realized ones, considering the following sub-periods: (i) The financial crisis (Q3 2007 – Q1 2009); (ii) the European Sovereign Debt crisis (Q3-2009 until Q4-2013); (iii) the 2020 stock market crash or Coronavirus crash (February 20th, 2020 – April 7th, 2020).

Panel 3.1: Realized vs. Predicted CLOSE values of the CBOE VIX

Statistics	Realized			Predicted					
				ARITHMETIC			LOGARITHMIC		
	FC	SDC	CC	FC	SDC	CC	FC	SDC	CC
Mean	30.938	19.901	50.273	30.930	19.950	49.320	30.876	19.892	49.582
Median	24.520	18.080	52.225	24.457	18.131	52.504	24.588	18.100	50.643
Max.	80.860	48.000	82.690	80.637	47.175	83.210	79.533	44.024	79.674
Min.	14.720	11.300	15.560	14.761	11.333	14.431	14.910	11.300	14.881
Std. Dev.	14.664	6.319	17.376	14.677	6.314	18.458	14.559	6.243	17.465
Skewness	1.344	1.431	-0.145	1.346	1.415	-0.196	1.322	1.378	-0.288
Kurtosis	3.837	5.035	2.294	3.824	4.948	2.201	3.704	4.697	2.183

Panel 3.2: Realized vs. Predicted OPEN values of the CBOE VIX

Statistics	Realized			Predicted					
				ARITHMETIC			LOGARITHMIC		
	FC	SDC	CC	FC	SDC	CC	FC	SDC	CC
Mean	31.078	20.017	49.611	31.009	20.016	49.020	31.019	20.010	48.963
Median	24.800	18.210	51.675	24.787	18.173	50.953	24.633	18.152	51.877
Max.	80.740	47.660	82.690	77.468	48.723	79.241	77.008	45.078	76.617
Min.	14.960	11.520	14.540	14.790	11.423	14.401	15.137	11.621	14.608
Std. Dev.	14.728	6.345	18.354	14.695	6.383	18.588	14.645	6.280	18.370
Skewness	1.320	1.438	-0.163	1.316	1.468	-0.244	1.305	1.404	-0.310
Kurtosis	3.700	5.099	2.105	3.664	5.273	1.988	3.601	4.879	1.914

Table 4.
Empirical results for the conditional illiquidity

The table reports the estimation results of the methodology for modelling the conditional illiquidity of the CBOE VIX at the CLOSE and OPEN of a trading day, relying on arithmetic changes (Panel 4.1) rather than logarithmic ones (Panel 4.2). The period of estimation is from January 1st, 1992 to October 9th, 2020. The panels report the estimated values of the coefficients and the standard errors.

Panel 4.1: ARITHMETIC changes

Coefficient	Estimated value	Standard Errors
β_0	-0.001	0.001
β_1	-0.003	0.001
β_2	-0.002	0.001
β_3	-0.003	0.001
$N(1,1)$ x 1000	0.004	0.000
$N(2,2)$ x 1000	0.001	0.000
$N(3,3)$ x 1000	-0.001	0.000
$N(4,4)$ x 1000	0.003	0.000
$C(1,1)$	0.072	0.006
$C(2,2)$	0.093	0.007
$C(3,3)$	0.088	0.006
$C(4,4)$	0.084	0.008
$D(1,1)$	0.997	0.001
$D(2,2)$	0.996	0.001
$D(3,3)$	0.996	0.001
$D(4,4)$	0.996	0.001
t	4.524	0.139

Panel 4.2: LOGARITHMIC changes

Coefficient	Estimated value	Standard Errors
β_0	-0.001	0.000
β_1	-0.001	0.000
β_2	-0.001	0.000
β_3	-0.001	0.000
$N(1,1)$	0.001	0.000
$N(2,2)$ x 10000	0.002	0.000
$N(3,3)$ x 10000	-0.001	0.000
$N(4,4)$	0.001	0.000
$C(1,1)$	0.077	0.007
$C(2,2)$	0.101	0.008
$C(3,3)$	0.094	0.007
$C(4,4)$	0.093	0.010
$D(1,1)$	0.997	0.001
$D(2,2)$	0.995	0.001
$D(3,3)$	0.996	0.001
$D(4,4)$	0.996	0.001
t	5.011	0.161

Table 5.

Realized Overnight variations vs. Fitted Overnight variations: Quantile regressions

The table reports the quantile regressions related to the difference between the realized overnight rate and the predicted values as a function of the following quantities: (i) **Overnight var. (-1)** is the overnight variation at the previous trading day; (ii) **Daytime (-1)** is the difference between the close and the open values at the previous trading day; (iii) **Volatility Close (-1)** is the conditional volatility for the close values of the CBOE VIX at the previous trading day; (iv) **Volatility Open** is the conditional volatility for the open values of the CBOE VIX; (v) **Illiquidity Close (-1)** is the conditional illiquidity for the close values of the CBOE VIX at the previous trading day; (vi) **Illiquidity Open** is the conditional illiquidity for the open values of the CBOE VIX; (vii) **Skewness** is The CBOE Skew Index. The quantile regressions are computed at the 1st percentile and rely on the MCMB-A bootstrap method, where the sparsity estimation is Chamberlain as a bandwidth method with a level of bandwidth equal to 0.0050196 and Gumbel is the quantile method based on Epanechnikov kernel. The number of bootstrap replications is equal to 10000. The standard errors are reported in the brackets. The significance levels at 1%, 5% and 10% are respectively represented in the following way: ***, **, *.

	(1)		(2)		(3)	
	ARITH	LOG	ARITH	LOG	ARITH	LOG
Overnight variation (-1)	-1.656*** (0.051)	-0.643*** (0.024)	-1.605*** (0.056)	-0.650*** (0.038)	-1.682*** (0.049)	-0.678*** (0.035)
Daytime (-1)	-0.897*** (0.036)	-0.081*** (0.017)	-0.907*** (0.051)	-0.105*** (0.022)	-0.942*** (0.044)	-0.099*** (0.016)
Volatility Close (-1)			-33.891*** (6.114)	-53.736*** (8.831)	-45.796*** (5.235)	-60.988*** (8.253)
Volatility Open			-19.117*** (5.772)	-20.352* (10.446)	-16.568*** (4.394)	-19.075** (9.041)
Illiquidity Close (-1)			-171.789 (196.022)	-519.041 (651.348)	-87.174 (150.430)	145.558 (481.590)
Illiquidity Open			-284.870*** (100.363)	-824.130*** (313.973)	-266.363*** (84.527)	-1052.547*** (253.621)
Skewness					0.047*** (0.009)	0.025*** (0.004)
Monday	-2.478*** (0.302)	-1.472*** (0.151)	1.428*** (0.392)	0.902*** (0.211)	-3.832*** (1.012)	-1.915*** (0.455)
Tuesday	-2.490*** (0.267)	-1.318*** (0.096)	1.451*** (0.360)	0.980*** (0.199)	-3.499*** (0.982)	-1.750*** (0.441)
Wednesday	-2.454*** (0.182)	-1.232*** (0.106)	1.530*** (0.378)	0.981*** (0.198)	-3.590*** (0.989)	-1.730*** (0.433)
Thursday	-2.317*** (0.263)	-1.075*** (0.064)	1.461*** (0.359)	0.978*** (0.194)	-3.665*** (0.980)	-1.814*** (0.447)
Friday	-2.171*** (0.160)	-1.343*** (0.124)	1.202*** (0.341)	0.769*** (0.189)	-3.881*** (0.980)	-1.921*** (0.435)
Adjusted R^2	**66.61%**	**47.82%**	**72.34%**	**57.69%**	**72.96%**	**59.19%**

Table 6.

Realized Overnight variations vs. Fitted Overnight variations: Robust Regressions

The table reports some robust regressions for the difference between the realized overnight rate and the predicted values as a function of the following quantities: (i) **Overnight variation (-1)** is the overnight variation at the previous trading day; (ii) **Daytime (-1)** is the difference between the close and the open values at the previous trading day; (iii) **Volatility Close (-1)** is the conditional volatility for the close values of the CBOE VIX, at the previous trading day; (iv) **Volatility Open** is the conditional volatility for the open values of the CBOE VIX; (v) **Illiquidity Close (-1)** is the conditional illiquidity for the close values of the CBOE VIX, at the previous trading day; (vi) **Illiquidity Open** is the conditional illiquidity for the open values of the CBOE VIX; (vii) **Skewness** is The CBOE Skew Index. The robust regressions are based on the **MM-estimation technique**. The standard errors are reported in the brackets. The significance levels at 1%, 5% and 10% are respectively represented in the following way: ***, **, *.

	(1)		(2)		(3)	
	ARITH	LOG	ARITH	LOG	ARITH	LOG
Overnight var. (-1)	-1.507*** (0.018)	-0.583*** (0.008)	-1.522*** (0.019)	-0.591*** (0.008)	-1.521*** (0.019)	-0.591*** (0.008)
Daytime (-1)	-0.768*** (0.012)	-0.009** (0.004)	-0.767*** (0.012)	-0.009** (0.004)	-0.767*** (0.012)	-0.009** (0.004)
Volatility Close (-1)			-3.051*** (0.962)	-3.424*** (1.009)	-2.815** (1.131)	-3.583*** (1.173)
Volatility Open			-2.910*** (1.023)	-3.505*** (1.191)	-2.950*** (1.026)	-3.448*** (1.194)
Illiquidity Close (-1)			14.673 (25.929)	86.812 (66.523)	14.045 (25.959)	89.107 (66.893)
Illiquidity Open			-79.662*** (13.324)	-297.622*** (35.582)	-81.231*** (13.537)	-296.980*** (36.124)
Skewness (x10)					-0.007 (0.001)	0.001 (0.001)
Monday	0.481*** (0.019)	0.241*** (0.010)	0.853*** (0.061)	0.432*** (0.029)	0.926*** (0.124)	0.415*** (0.060)
Tuesday	0.030* (0.018)	0.004*** (0.009)	0.414*** (0.062)	0.202*** (0.030)	0.487*** (0.125)	0.184*** (0.060)
Wednesday	-0.035** (0.016)	-0.017*** (0.008)	0.342*** (0.061)	0.178*** (0.029)	0.415*** (0.124)	0.160*** (0.060)
Thursday	0.019 (0.016)	-0.009*** (0.008)	0.396*** (0.061)	0.185*** (0.029)	0.469*** (0.124)	0.167*** (0.060)
Friday	-0.091*** (0.017)	-0.068*** (0.009)	0.280*** (0.061)	0.123*** (0.029)	0.353*** (0.124)	0.105*** (0.060)
Adjusted R^2	**54.78%**	**40.33%**	**55.01%**	**40.95%**	**55.00%**	**40.95%**
Adjusted – Rw^2	**87.41%**	**76.30%**	**87.68%**	**77.16%**	**87.68%**	**77.16%**

Figure 1.

The figure shows the dynamic of the conditional illiquidity for the OPEN values of the CBOE VIX, based on arithmetic and logarithmic changes as well as the dynamic of the OPEN values for the CBOE VIX, from January 1st, 1992 to October 9th, 2020.

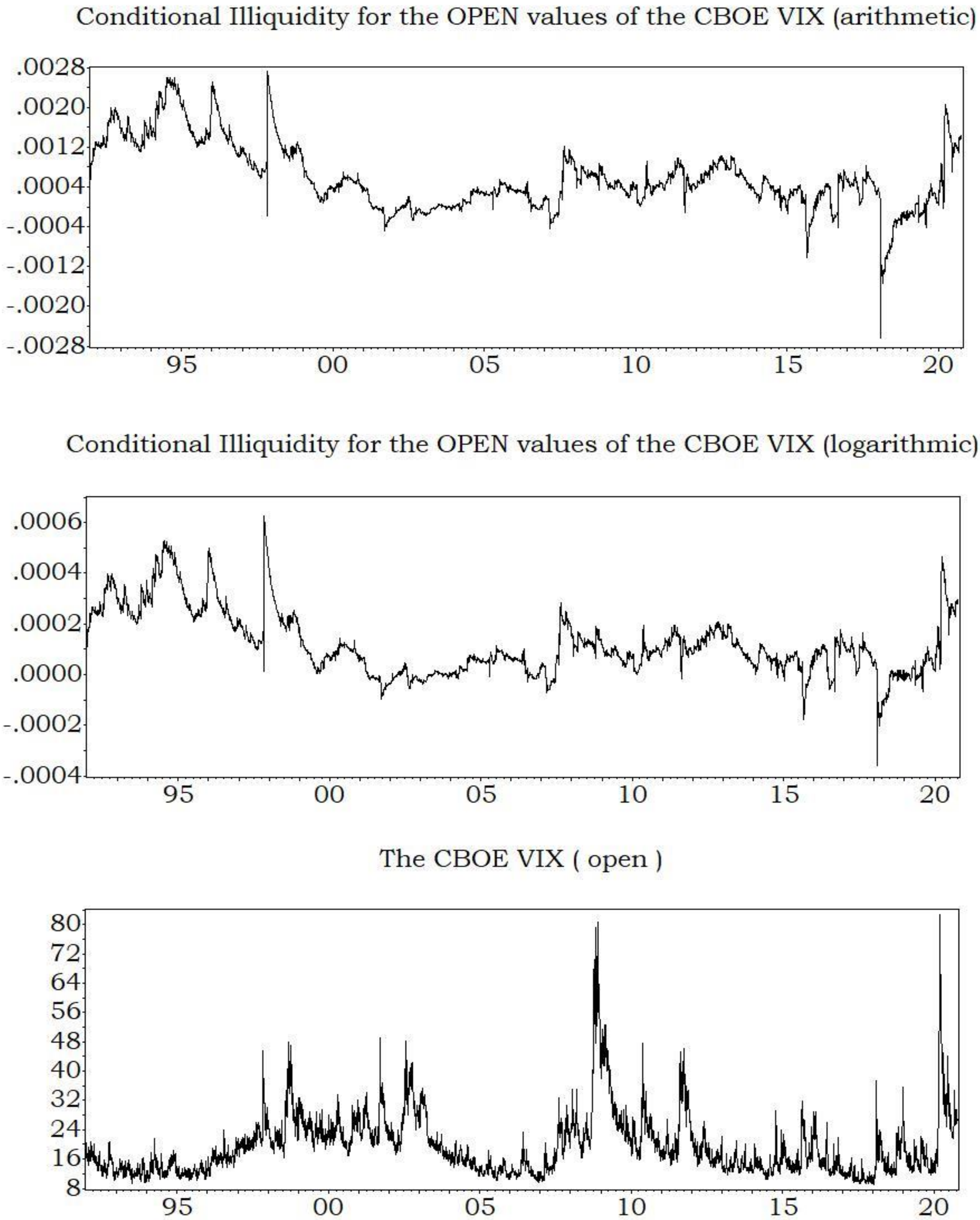

Figure 2.

The figure shows the dynamic of the conditional illiquidity for the CLOSE values of the CBOE VIX, based on arithmetic and logarithmic changes as well as the dynamic of the CLOSE values for the CBOE VIX, from January 1st, 1992 to October 9th, 2020.

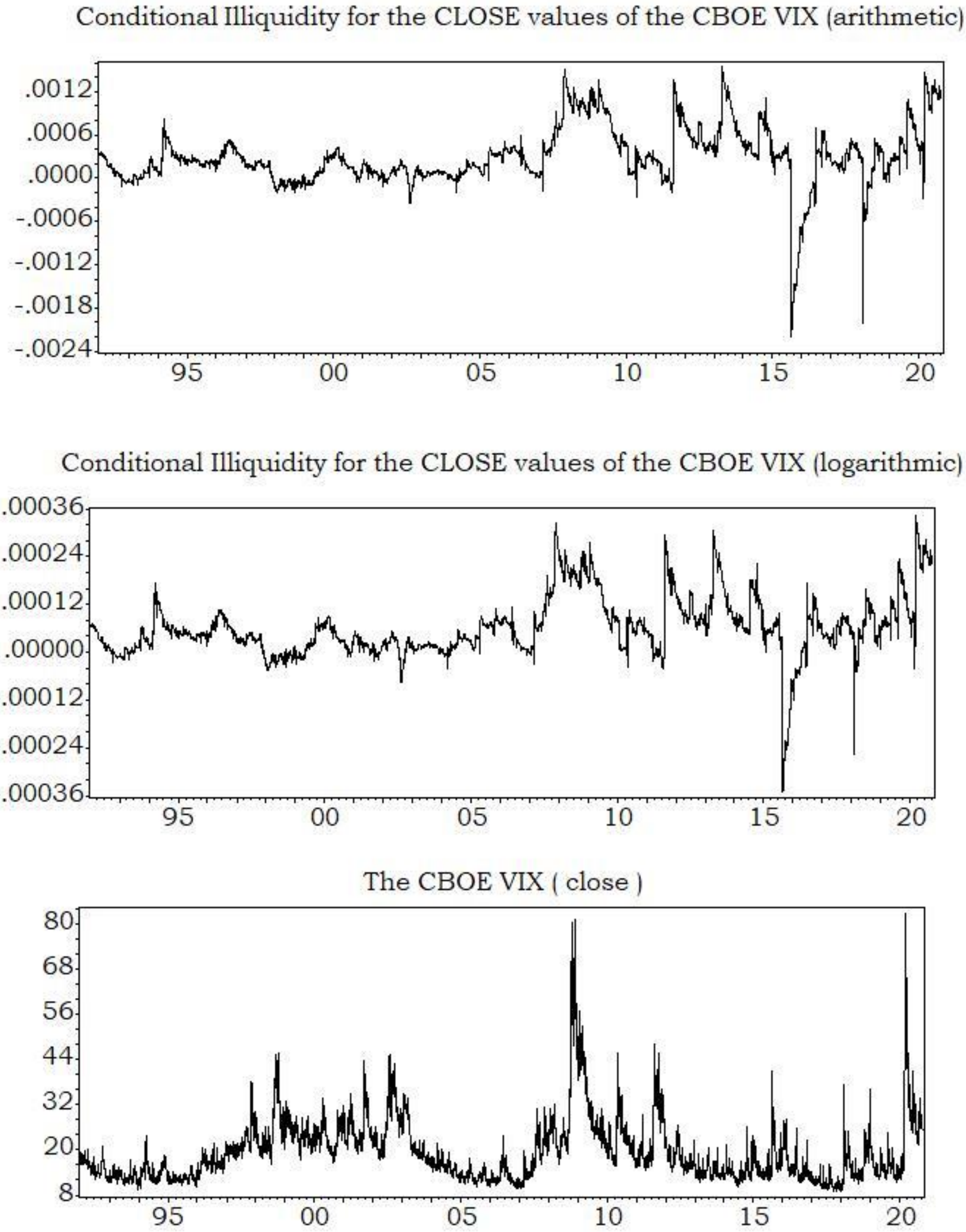

Figure 3.

The figure shows the change of the conditional illiquidity between the CBOE VIX at the open and the CBOE VIX at the close of the previous trading day for the period between January 1st, 1992 and October 9th, 2020. It also includes the dynamic of the overnight variations or rates.

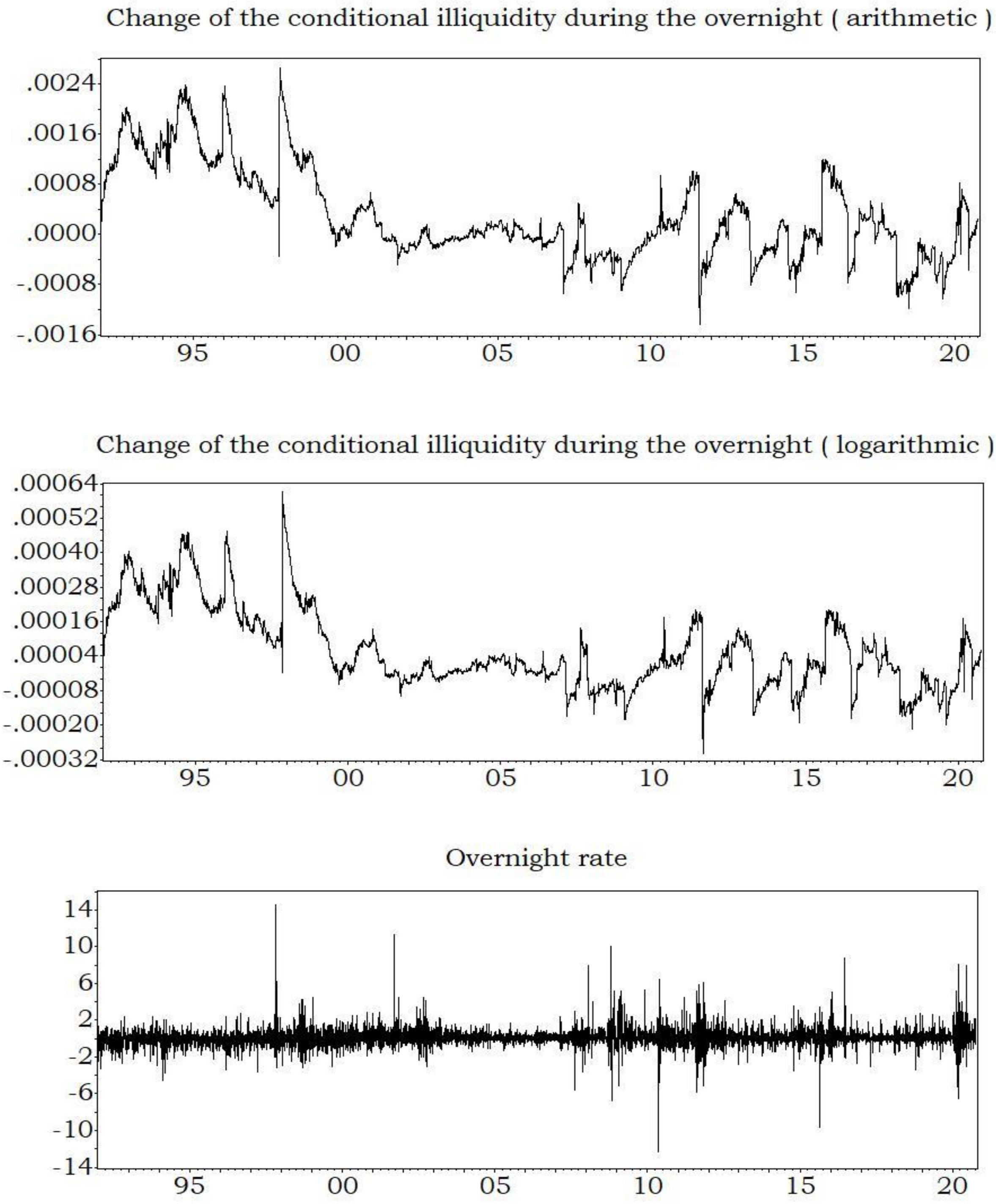

Figure 4.

The figure shows the change of the conditional illiquidity between the CBOE VIX at the close and the CBOE VIX at the open of a trading day for the period between January 1st, 1992 and October 9th, 2020. It also includes the dynamic of the daytime variations.