ESG, Risk, and (tail) dependence

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Abstract

While environmental, social, and governance (ESG) trading activity has been a distinctive feature of financial markets, the debate if ESG scores can also convey information regarding a company’s riskiness remains open. Regulatory authorities, such as the European Banking Authority (EBA), have acknowledged that ESG factors can contribute to risk. Therefore, it is important to model such risks and quantify what part of a company’s riskiness can be attributed to the ESG ratings. This paper aims to question whether ESG scores can be used to provide information on (tail) riskiness. By analyzing the (tail) dependence structure of companies with a range of ESG scores, using high-dimensional vine copula modelling, we are able to show that risk can also depend on and be directly associated with a specific ESG rating class. Empirical findings on real-world data show positive not negligible dependencies between clusters determined by ESG scores, especially during the 2008 crisis.

Keywords: ESG scores, Risk, Dependence, Tail dependence, Vine Copula models

JEL classification: G32, C51, C58

1. Introduction

After the 2007-2009 financial crisis, many models which used to capture the dependence between a large number of financial assets were revealed as being inadequate during crisis. Moreover, research has shown that the dependence structure of global financial markets has grown in importance in areas of optimal asset allocation, multivariate asset pricing, and portfolio tail risk measures (Xu and Li, 2009). Consequently, the enormous losses and the increased volatility in the global financial market elicited calls for an even more diligent risk management.

Over the past decade, the interest in socially responsible investments has grown exponentially (Auer and Schuhmacher, 2016). The availability of non-financial data, including corporate social responsibility (CSR) or environmental, social, and governance (ESG) data has skyrocketed, and has gained great interest from investors.

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for various reasons. These ESG scores indicate a level of ESG performances and are based on several criteria, given by a rating institution (Bhattacharya and Sharma, 2019). The rating institutions use quantitative and qualitative methods to assign an ESG score to a company (Berg and Lange, 2020). In brief, companies are awarded positive scores for ESG responsible behavior, but are awarded negative scores for ESG irresponsible behavior.

The increase in demand in socially responsible investments stems from investors and asset managers pressured by stakeholders to push companies to behave responsible and improve their ESG strategy (Henriksson et al., 2019). The biennial 2018 Global Sustainable Investment Review states that over $30 trillion had been invested with explicit ESG goals (Global Sustainable Investment Alliance (GSIA), 2018). According to a 2018 global survey, more than 50% of the international asset owners are currently considering or already implementing ESG scores in their investment strategy (Consolandi et al., 2020). Furthermore, Eccles and Klimenko (2019) state that this interest in ESG assets is driven by the growing evidence of the positive impact of ESG materiality on financial performance. Also, the European Banking Authority (EBA) stressed the importance of including ESG information into the regulatory and supervisory framework of EU credit institutions (European Banking Authority, 2018). Nevertheless, using ESG factors has made the investment process noticeably more complex (Berg and Lange, 2020).

While more than 2000 empirical studies have been done analysing ESG factors and financial performance, little is known about the dependence structure and associated risks (Friede, 2019; Shafer and Szado, 2018). This is especially important as ESG scores are often linked to investment risk. The EBA define ESG risks as “the risks of any negative financial impact to the institution stemming, from the current or prospective impacts of ESG factors on its counterparties” (p. 28) (EBA, 2020). While regulators finally acknowledge the role played by ESG factors in determining part of a companies risk, so far no guidelines are provided in how to best capture and quantify such risk. If this is accurate, then ESG scores should possibly deliver some information on a company's ESG risk and its riskiness as a whole. This also means that companies that have the same ESG rating should share similar risk characteristics and properties. This is especially important in times of increased volatility as understanding the (tail) dependence structure among similarly rated assets is essential in order to set in place effective risk management and diversification strategies (Ane and Kharoubi, 2003; Frahm et al., 2005; Malevergne et al., 2005; Berg and Lange, 2020).

This research aims to contribute to the understanding of the dependence structure of assets belonging to the same ESG class and provide some new indicators to capture such (tail) dependence. To model dependence we rely on a high-dimensional vine copula modelling (Bedford and Cooke, 2001, 2002; Aas et al., 2009; Joe, 2014; Czado, 2019). This allows us to show to our knowledge for the first time that risk can also depend on ESG rating class. Furthermore, by quantifying the overall dependence and tail dependence among assets that belong to the same
ESG class, we show that these dependencies are not negligible, especially in times of crisis.

The paper is structured as follows; Section 2 summarizes the literature and creates an understanding of ESG scores and the occurrence of (tail) risk and dependence structures. while Section 3 introduces dependence modelling and vine copulas. Section 4 then proposes the R-vine Copula ESG Risk model. The S&P 500 data is described in Section 5. Then Section 6 reports the empirical results. Lastly, Section 7 provides an overall summary and outlook for future research.

2. ESG Scores, Dependence, and Risk

Overall, ESG scores try to capture the amount of positive ESG disclosure and performance of a company (Bhat-tacharya and Sharma, 2019). In general, companies are awarded grades measuring their ESG responsible behavior: the higher the grade, the better the ESG performance. Acting as complementary non-financial information, ESG scores can have the potential to increase the accuracy in performance forecasts and risk assessments (Achim and Borlea, 2015).

In 2018, the European Commission published its Action Plan on Financing Sustainable Growth which provided the European Union (EU) with a roadmap on sustainable finance and for future work across financial systems (European Banking Authority, 2018). Furthermore, the European Banking Authority (EBA) stressed the importance of including ESG information into the regulatory and supervisory framework of EU credit institutions (European Banking Authority, 2018). It has also been shown that ESG factors may impact financial performance by substantiating themselves in “financial or non-financial prudential risks, such as credit, market, operational, liquidity and funding risks” (p.27) (EBA, 2020). According to the EBA, ESG risks are defined to materialize when ESG factors have a negative impact on the financial performance or solvency (EBA, 2020). Furthermore, it is argued that the materiality of ESG risks depends on the risks posed by ESG factors over different time frames (EBA, 2020). If this is accurate, the ESG ratings should contain information about the company's risk. Even though ESG has mostly been defined in terms of risk by the regulator, and there is an ongoing debate on the effects of using ESG scores on the financial performance research, there is no consensus on the (tail) dependence structure of assets within and between each ESG rating class. Understanding the (tail) dependence structure of several assets, however, is necessary in order to access inherent risks in the financial market.

Generally, market returns are assumed to follow a multivariate normal distribution; however, research has shown that this is found to be not accurate in reality, and left tails are often heavier than right tails (Cont, 2001; Jondeau and Rockinger, 2003). In the last years, dependencies between all financial asset returns have increased due to globalization effects (Frahm et al., 2005). Therefore, modelling the dependence among assets and under-
standing the appearance of joint (tail) risk is especially important for asset pricing and risk management. Tail risk or tail dependence is characterized as the probability of an extremely large negative (positive) return of an asset given that the other asset yields an extremely large negative (positive) return, and is commonly quantified by the so-called tail-dependence coefficient (Embrechts et al., 2001; Frahm et al., 2005; Xu and Li, 2009). Tail risk arises when the likelihood of an extreme event that is more than three standard deviations away from the mean is more likely to occur than shown by a normal distribution (Kelly and Jiang, 2014). Generally it is accepted that tail dependence can be used as a proxy of systemic risk and tail risk has been linked to negative consequences for corporate investment and risk-taking (Gormley and Matsa, 2011; Gormley et al., 2013; Shirvani and Volchenkov, 2019).

In the last years, companies with high ESG scores, so called ESG leaders, have experienced an intense influx in investments from asset managers and institutional investors, possibly leading to an increased valuation (Berg and Lange, 2020). This could lead to potential valuation risk which should be considered before any investment decision (Dunn et al., 2018; Lioui, 2018; Pollard et al., 2018; Berg and Lange, 2020). Some researchers argue that responsible ESG practices might mitigate the market's perception of a company's tail risk and, therefore, reduce ex-ante expectations of a left-tail event (Shafer and Szado, 2018). In brief, they state that considering responsible business practices when creating an equity portfolio can act as insurance against left-tail risk and, with that, protect company value. This is also in line with Wamba et al. (2020) who add that especially positive environmental performance can act as an insurance for companies, reducing the probability of a negative event occurring and with that reducing the company's systematic risk. Ashwin Kumar et al. (2016) add that positive ESG practices can make a company less vulnerable to reputation, political and regulatory risk and thus leading to lower volatility of cash flows and profitability (p. 292). Furthermore, positive ESG performance may generate more loyalty from customers and employees plus, through that, protect companies from unforeseen harmful events, resulting in reduced tail risk (Shafer and Szado, 2018). Besides, better ESG performance allows companies to experience adverse events less often and lose less value if they do occur (Minor, 2011).

Others recognize that improving ESG performance can also help with risk exposure (Giese et al., 2019). Maiti (2020) finds that overall portfolios developed using the overall ESG as well as the individual E (Environment), So (Social) and G (Governance) factors show generally better investment performance, implying policy modifications, while Breedt et al. (2019) show that ESG-tilted portfolios do not necessarily have higher risk-adjusted returns. Other scholars have observed a mitigating effect of ESG performance on stock price crash risk if a company has less effective governance (Kim et al., 2014). Hoepner et al. (2016) found supporting evidence that ESG engagements can be associated with subsequent reductions in downside risk. However, their findings are limited as they are based on one single institutional investor. Additionally, Sherwood and Pollard (2017) finds that inte-
grating ESG emerging market equities into institutional portfolios can create higher returns and lower downside risks compared to non-ESG equity investments, while Shafer and Szado (2018) add that better ESG performance significantly reduces the perceived one-month ahead tail risk.

Looking at a country's creditworthiness, Capelle-Blancard et al. (2019) show that countries with above-average ESG scores are linked to reduced default risk and smaller sovereign bond yield spreads. Additionally, Breuer et al. (2018) show that the cost of equity is reduced when a company invests in CSR and vice versa, given that it is located in a country with high investor protection. Moreover, Li et al. (2017) find that companies in regions of high social trust tend to have lower tail risk. A possible reason why environmental practices have the power to control tail risk could be that a company's good social records will be more valuable in the long run due to a lower frequency of litigation (Goldreyer and Diltz, 1999). Additionally, higher environmental standards are valuable to shareholders due to companies avoiding litigation costs, reputation losses, and environmental hazards (Chan and Walter, 2014). Furthermore, ESG can drive an asymmetric return pattern in which social responsible investment (SRI) funds (using positive rather than negative screenings) outperform conventional funds in times of crisis but underperform in calm periods (Nofsinger and Varma, 2014). Moreover, Bae et al. (2019) find that CSR reduces the costs of high leverage and decreases losses in market share when firms are highly leveraged. However, it is possible that ESG risk is non-diversifiable. Here Dorfleitner et al. (2016) provide evidence that ESG ratings are subject to a non-diversifiable risk component as it depends on the overall market.

Despite these extensive efforts, the effect of ESG scores on dependence and (tail) risks has not yet been clearly understood. While investors have been aware of the need to use more sophisticated models to assess the dependence behavior of assets since the financial crisis, such models, allowing investors to quantify dependence and (tail) risk among ESG-based classes of assets, have yet to be introduced. In this paper, we focus on dependence modelling via vine copulas (Czado, 2019), which are described in the next sections.

3. Dependence Modelling and Vine Copulas

Rarely a distribution follows the strict spherical assumptions with the same type of dependence structure across the components of the distribution as implied by correlation (Dorey and Joubert, 2005). While Pearson correlation has been used as a measure of pairwise dependence, modern risk management requires a thorough stochastic understanding beyond linear correlation. The copula approach accounts for complex dependency patterns, such as non-symmetry and dependence in the extremes.

A copula \( C \) is a cumulative distribution function (cdf) with uniform marginals on the unit interval. Sklar's theorem (1959) states that if \( F \) is a continuous \( d \)-dimensional distribution function for \( \mathbf{X} = (X_1, \ldots, X_d)^\top \) with a
univariate cdf $F_p(x_p)$ of a continuous random variable $X_p$ for $p = 1, \ldots, d$ with its realizations $x_p$, the joint distribution function $F$ can be written as

$$F(x_1, \ldots, x_d) = F_1(x_1), \ldots, F_d(x_d). \tag{1}$$

The corresponding density is

$$f(x_1, \ldots, x_d) = c(F_1(x_1), \ldots, F_d(x_d)) \cdot \prod_{p=1}^d f_p(x_p), \tag{2}$$

where $c$ is the $d$-dimensional copula density of the random vector $\mathbf{F} = (F_1(X_1), \ldots, F_d(X_d))^\top \in [0, 1]^d$ and $f_p(x_p)$ is the associated univariate marginal density of $F_p(x_p)$ for $p = 1, \ldots, d$.

Different copula types can accommodate flexible dependence patterns in the bivariate case ($d = 2$), as shown in Table 1. Nevertheless, the existing parametric families of multivariate copulas are not as flexible as the bivariate copula families to represent complex dependence patterns. For instance, the multivariate Gaussian copula does not accommodate any tail dependence and has been criticized immensely after the 2008 financial crisis (Li, 2000; Salmon, 2012; Puccetti and Scherer, 2018; Czado, 2019). The multivariate Student’s t copula apprehends tail dependence but does not capture any asymmetry in the tails. Furthermore, these multivariate exchangeable Archimedean copulas become inflexible as the dimension increases since they model the dependence between a large number of pairs of variables using not more than two parameters. If no dependence is found, and the random variables are independent, the independence copula best models their behavior.

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Table 1: Parametric copula families and their properties without rotations and reflections. Notation of copula families: $t =$ Student’s t, $F =$ Frank, $N =$ Gaussian, $C =$ Clayton, $J =$ Joe, $G =$ Gumbel, $I =$ Independence, $\text{BB1} =$ Clayton-Gumbel, $\text{BB7} =$ Joe-Clayton, $\text{BB8} =$ Extended Joe

To accommodate a great variety of dependence structures in higher dimensions and overcome the issues of multivariate elliptical, Archimedean, and even other copulas; vine copulas (so-called pair copula constructions) were made operational for data analysis by Aas et al. (2009). They are a class of copulas which are based on conditioning ideas first proposed by Joe (1996) and further more developed by Bedford and Cooke (2001, 2002). The approach allows to construct any $d$-dimensional copula and its density by $\frac{d(d-1)}{2}$ bivariate copulas and their den-

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1. To extend the range of dependence, counterclockwise rotations and reflections of the copula density are included. This allows the Gumbel, Clayton, Joe, BB1, BB7 and BB8 to also accommodate negative dependence ($\tau < 0$). For more details, we refer to Chapter 3 of Czado (2019).
sities. Furthermore, the expression can be represented by an undirected graphical structure involving a set of linked trees, i.e., regular vine (R-vine) (Bedford and Cooke, 2001).

A $d$-dimensional R-vine consists of $d - 1$ nested trees; each tree $T_m$ has a node set $V_m$ and an edge set $E_m$ for $m = 1, \ldots, d - 1$. The nodes in the tree level one and $m + 1$ are given by $V_1 = \{1, \ldots, d\}$ and $V_{m+1} = E_m$ for $m = 1, \ldots, d - 2$, respectively. Thus, the edges in the tree level $m$ are the nodes in the tree level $m + 1$. The proximity condition requires that if an edge connects two nodes in $T_{m+1}$, their associated edges in $T_m$ must have a shared node in $T_m$.

To express a vine copula’s density, each edge in a R-vine is linked to a bivariate copula. Therefore, it is computationally tractable and uniquely determined (Kurowicka and Cooke, 2006). Let $c_{e_a,e_b;D_e}$ be a bivariate copula density associated with an edge $e = \{a, b\}$ and $F_p$ denoting the marginal distribution function for $p = 1, \ldots, d$. Then a $d$-dimensional vine copula density $c$ under the simplifying assumption can be constructed as follows:

$$c(F_1(x_1), \ldots, F_d(x_d)) = \prod_{m=1}^{d-1} \prod_{e \in E_m} c_{e_a,e_b;D_e}(F_{e_a|D_e}(x_{e_a}|x_{D_e}), F_{e_b|D_e}(x_{e_b}|x_{D_e})), \quad (3)$$

where $x_{D_e} = (x_2)_{z \in D_e}$ is a subvector of $x = (x_1, \ldots, x_d)^\top \in \mathbb{R}^d$, $F_{e_a|D_e}$ is the conditional distribution function of the random variable $X_{e_a}|X_{D_e} = x_{D_e}$, which can be calculated recursively (Joe, 1996). The set $D_e$ is called the conditioning set, and the indices $e_a, e_b$ form the conditioned set. With the simplifying assumption, a bivariate copula density does not depend on the specific value of its conditioning variables but only on the conditioning values through its arguments. For more details, we refer to Stoeber et al. (2013).

Figure 1 shows a 6-dimensional R-vine with its edge labels. It has five tree levels, and the node set in the first tree level is $V_1 = \{1,2,3,4,5,6\}$. The edge set in the second tree is $E_2 = \{(1,4;2), (1,5;2), (2,3;1), (1,6;3)\}$. In the higher tree levels, it holds $V_2 = E_1$, $V_3 = E_2$, $V_4 = E_3$, $V_5 = E_4$ and $V_6 = E_5$. The tree levels are selected sequentially by a top-down approach, i.e., the selection starts with the first tree $T_1$ and continues tree by tree up to the last tree $T_{d-1}$. The chosen trees satisfy the proximity condition. For instance, in $T_2$ the nodes $\{1,3\}$ and $\{3,6\}$ are joined since they share a common node $\{3\}$ as shown in Figure 1. After the association of each edge of the R-vine with a bivariate copula density, we can write its vine copula density as in Equation (4), where the edge $e = \{1,4;2\}$ in $E_2$ is
associated with a bivariate copula density \( c_{1,4,2} \), for example.

\[
\begin{align*}
&c(F_1(x_1), \ldots, F_6(x_6)) = c_{13}(F_1(x_1), F_3(x_3)) c_{12}(F_1(x_1), F_2(x_2)) c_{56}(F_5(x_5), F_6(x_6)) c_{24}(F_2(x_2), F_4(x_4)) c_{25}(F_2(x_2), F_5(x_5)) \\
&c_{14,2}(F_{14}(x_1, x_2), F_{26}(x_2, x_6)) c_{35,2}(F_{35}(x_3, x_5), F_{25}(x_2, x_5)) c_{33,1}(F_{33}(x_3, x_3), F_{31}(x_3, x_1)) \\
&c_{16,3}(F_{16}(x_1, x_3), F_{63}(x_6, x_3)) c_{26,1,3}(F_{26,1,3}(x_2, x_6, x_1, x_3), F_{16,3}(x_1, x_3, x_1)) \\
&c_{35,1,2}(F_{35,1,2}(x_3, x_5, x_1, x_2), F_{51,2}(x_5, x_1, x_2)) c_{46,1,2,3}(F_{46,1,2,3}(x_4, x_1, x_2, x_3), F_{12,3}(x_1, x_2, x_3)) \\
&c_{56,1,2,3,4}(F_{56,1,2,3,4}(x_5, x_1, x_2, x_3, x_4), F_{12,3,4}(x_1, x_2, x_3, x_4)).
\end{align*}
\]

Figure 1: Example of a 6-dimensional regular vine with edge labels.

To specify a vine copula model, a vine tree structure and the associated bivariate copula families with corresponding parameters are needed. For the selection of vine tree structures, we follow the sequential top down approach proposed by Dißmann (2013) and Dißmann et al. (2013). It starts with the tree level one and finds the maximum spanning tree, where each edge has a predefined weight, e.g., the absolute value of the empirical Kendall’s \( \tau \) between the nodes forming the edge. Then, from the set of bivariate copula families in Table 1, we select the optimal pair copula families using the Akaike Information Criterion (AIC) (Akaike, 1998), which has been shown to have good copula family selection properties and high accuracy (Brechmann, 2010). Since each pair copula family can be specified sequentially and independently at the same tree level, an alternative selection criteria like the BIC would not be needed to induce sparsity at this step. In the next step, we compute the pseudo copula data for the second tree level \( T_2 \), using the estimated pair copulas in the first tree level \( T_1 \). These input data is similarly used to select the second tree level \( T_2 \) of the vine and corresponding pair copula families with their parameters. The approach continues sequentially up to the last tree level. Following this method, we estimate the vine tree structure together with its pair copula families and parameter values. As we choose a pair copula family corresponding to each edge separately, the parameters estimation step is at most a two-dimensional optimization, which is computationally efficient. Moreover, the performance of this method is satisfactory compared...
4. The R-Vine Copula ESG Risk Model

In the following, we introduce the ESG class dependence indicator $D_{jq}^{S,q}(\tau)$ with $\tau$ as our dependence measure and the ESG class lower tail dependence indicator $D_{jq}^{S,q}(\lambda)$ with $\lambda$ as the lower tail dependence measure to capture the overall and left-tail ESG class dependence for each asset $j$ within each industrial sector $S$ and period $q$. Inspiration for these indicators arises from Brechmann and Czado (2013), who propose to model dependence across geographical equity sectors. As ESG score values are weighted differently within industrial sectors, we fit a separate R-vine copula ESG risk model for any set of assets belonging to a specific industrial sector $S$ in period $q$. Within each industrial sector $S$ and period $q$, depending on the ESG scores, assets can be attributed to four different ESG class groups ($k \in A, B, C, D$) following the thresholds given by Refinitiv (2019). Assets are given a $D$ grade for an ESG score lower than 25, $C$ for an ESG score between 25 and lower than 50, $B$ for an ESG score between 50 and lower than 75 and finally $A$ for an ESG score between 75 and 100. The lower the ESG score, the poorer the relative ESG performance. We compute the mean ESG score ($\overline{ESG}_{jq}^{S,q}$) for each asset $j$ in sector $S$ over a specific time period $q$. We then use these values to group each asset $j$ within each sector $S$ in period $q$ into ESG class groups ($R_{jq}^{S,q}$) and compute the ESG class indices per ESG class $k$ ($I_{jq,1,k}^{S,q}$ i.e. the ESG class index $A$ in sector $S$ in period $q$ on trading day $t$ is $I_{jq,A}^{S,q}$) by linearly combining asset returns ($Y_{jq,t}^{S,q}$ defined as the return of an asset $j$ in sector $S$ in period $q$ on trading day $t$ belonging to ESG class $k$) in the same mean ESG class ($R_{jq}^{S,q}$) using the market capitalization weights. Later, in the R-vine model, we connect the assets to their belonging ESG class index ($I_{jq,1,k}^{S,q}$) and industrial sector index ($I_{jq}^{S,q}$) at time $t$. As shown in Figure 2 and 3 we use only the variable names such as $I_{jq}^{S,q}$, $I_{jq,1,k}^{S,q}$, $Y_{jq,t}^{S,q}$ etc. to denote the nodes. This association data is given by $I_{jq}^{S,q}$, $I_{jq,1,k}^{S,q}$, $Y_{jq,t}^{S,q}$ etc.

As can be seen, we fix the first two tree levels of the R-vine. The first tree level specification allows us to capture the behaviour of an asset in a period compared to its ESG peers within the same sectors as well as the the dependence between ESG class index and industrial sector index. In the second tree level we capture the dependence of an asset with its sector after the effect of its ESG class index is removed. Thus, fixing the first two trees is a necessary step in our R-vine copula ESG risk model. The R-vine model is structured as follows. For the first tree level $T_1$ shown in Figure 2, the central node is always chosen to be the sector index ($I_{jq}^{S,q}$) while the adjacent nodes are the ESG class indices: Index A ($I_{jq}^{A}$), Index B ($I_{jq}^{B}$), Index C ($I_{jq}^{C}$), Index D ($I_{jq}^{D}$). Then assets $\{A_1^{S,q}, ..., A_{n_A}^{S,q}\}$, $\{B_1^{S,q}, ..., B_{n_B}^{S,q}\}$, $\{C_1^{S,q}, ..., C_{n_C}^{S,q}\}$, and $\{D_1^{S,q}, ..., D_{n_D}^{S,q}\}$ within the sector $S$ in period $q$ are linked to their
belonging ESG class index, where \( n_k^S \) denotes the number of assets in sector \( S \) which are in ESG class \( k \) in period \( q \). The number of assets can differ overall across sectors \( S \) and also across ESG class in different time periods \( q \) within sectors \( S \) as assets can change ESG class over time. Thus, the assets are in contiguous groups based on the mean ESG score in time period \( q \). In the second tree level \( T_2 \), the nodes \( \{ T^S_A, I^S_A \}, \{ T^S_B, I^S_B \}, \{ T^S_C, I^S_C \}, \{ T^S_D, I^S_D \} \), and \( \{ I^S_D, I^S_A \} \), which are the edges in \( T_1 \), are sequentially ordered. The new order is motivated by the ESG Reuters thresholds which divides assets in contiguous groups based on the mean ESG scores. Additionally, the other edges of tree \( T_1 \) are connected to an edge with a common ESG class index. For example, \( \{ A^S_A, I^S_A \}, \ldots, \{ A^S_A, I^S_A \} \) is connected to \( \{ T^S_A, I^S_A \}, \{ B^S_A, I^S_A \}, \ldots, \{ B^S_D, I^S_D \} \) to \( \{ I^S_D, I^S_A \}, \{ I^S_D, I^S_B \} \) to \( \{ I^S_D, I^S_A \}, \{ I^S_D, I^S_B \} \) to \( \{ I^S_D, I^S_A \}, \{ I^S_D, I^S_B \} \) to \( \{ I^S_D, I^S_A \}, \{ I^S_D, I^S_B \} \) to \( \{ I^S_D, I^S_A \}, \{ I^S_D, I^S_B \} \).

![Figure 2: The first tree level \( T_1 \) of the regular vine used in the vine copula risk model for sector \( S \) in time period \( q \).](https://ssrn.com/abstract=3846739)

We then let the additional trees be chosen by the Dißmann Algorithm (Dißmann, 2013) as explained in Section 3. Here, the most important dependencies are measured by absolute weights \(|w|\) and modelled first in the maximum spanning tree. We choose the empirical Kendall’s \( \tau \) between the copula data associated with an edge as the weight measure \( w \) for the edge as it gives a bivariate monotone association and its values ranges from \([-1, 1]\) (Joe, 2014). Kendall’s \( \tau \) is a ranked based dependence measure robust to outliers that fits the aim of our analysis and can be defined in terms of copulas for two continuous random variables (\( X_1, X_2 \)) with copula \( C \) as

\[
\tau = 4 \int_{[0,1]^2} C(u_1, u_2) dC(u_1, u_2) - 1.
\]

An alternative overall weight measure could be Spearman’s \( \rho \). Furthermore, we estimate a lower tail dependence coefficient \( \lambda \). When Student’s \( t \) and Clayton (and BB1 and BB7) copulas are chosen as best fit among the bivariate copula families given in Table 1, we can use this information to also estimate the
lower tail dependence coefficient $\lambda$ which ranges from 0 to 1 and is defined for a bivariate distribution with copula $C$ as $\lambda^{lower} = \lim_{x \to 0^+} P(X_2 \leq F_2^{-1}(x) \mid X_1 \leq F_1^{-1}(x)) = \lim_{x \to 0^+} \frac{C(t,t)}{t}$. Other methods could include estimating a specific distribution or a family of distributions; or working with a non-parametric model. We refer to Frahm et al. (2005) for further reading on estimation methods and Czado et al. (2013) for additional weight measures. Fixing the first two trees and using these dependence measures allow us to capture the (tail) dependence of each asset with its belonging ESG class in order to understand joint dependence behaviour.

4.1. Dependence Indicators

First, we fit the R-vine tree structure, described in the previous section, with the specified first and second tree level to each sector $S$ and time period $q$. 3 Using the fitted copula parameters and families, we can estimate the Kendall’s $\tau$ and the implied lower tail dependence coefficient $\lambda$ of an asset $j$ with its ESG class $k$ and that of an ESG class index $I_{k,q}$ with its sector index $I_{S,q}$.

We now introduce a new ESG class dependence indicator $D_{j,S,q}(\tau)$, which we can compute for each asset $j$ with its ESG class $k$ in period $q$ within each sector $S$. It accounts for the conditional dependencies of the asset $j$ to other assets $o_{j_i}^{S,q}, ..., o_{j_{j_1}}^{S,q}$ given by the fitted vine copula model, where $j_1$ is the number of assets which occur in

\[2\text{The lower tail dependence coefficient is also non-zero when the fitted bivariate copula class is Student’s}$ t$, Clayton, 180° Joe, 180° Gumbel, BB1, BB7, or 180° BB8 in our model. For other bivariate copula families, we have zero lower tail dependence coefficient. 3We fit two different R-vine copula model classes. In the first, we allow only for the bivariate itau copulas, for which the estimation by Kendall’s $\tau$ inversion is available (Student’s $t$, Frank, Gaussian, Clayton, Joe, Gumbel, and Independence copula). Using these, we also allow for asymmetry in the upper and lower tails, as shown in Table 1. These copula families have a one-to-one relationship between their copula parameter and Kendall’s $\tau$. In the second model, we include additional copulas with two parameters. Including more bivariate copula families such as BB1 (a combination between both extreme cases of Clayton copula and Gumbel copula), BB7 (combination of Joe copula and Clayton copula), and the extreme-value copula BB8 (extended Joe) does not necessarily improve our model fits. To choose our optimal model we use the Bayesian Information Criteria (BIC) as it tends to select a parsimonious model that reasonably approximates the density (Schwarz, 1978). The selection of BB copula families is very rare. The information criteria are given for one sector in the Appendix C, all other sectors are available from the authors upon request. Applying the Vuong Test by (Vuong, 1989) with the parsimonious BIC correction and considering BIC values of the model fits indicate that itau bivariate copula family set is preferred in our data, except for the Real Estate sector in period $q_1 = 2006$–2010 and $q_2 = 2011$–2015. Therefore, we allowed for BB copula fits in these two cases. Since the number of parameters differ in the bivariate copula families that we considered, we used the parsimonious BIC correction in the Vuong test to favor more parsimonious models.
The conditioning set, when \( j \) is in the conditioned set with the fitted vine in sector \( S \) and period \( q \). We use the associated Kendall’s \( \tau \) and its estimate \( \hat{\tau} \) as a dependence measure. Our ESG class dependence indicator of asset \( j \) in sector \( S \) and period \( q \) is defined by:

\[
D_j^{S,q}(\tau) = \frac{|\hat{\tau}_{j,i_k}^{S,q}|}{|\hat{\tau}_{j,i_k}^{S,q}| + |\hat{\tau}_{j,o_l}^{S,q}| + |\hat{\tau}_{j,o_l}^{S,q}| + \ldots + |\hat{\tau}_{j,o_l}^{S,q}|}, \quad \forall S,q,j. \tag{5}
\]

To exclude unrealistic assumptions, we assume that asset \( j \) is not independent of all other assets and include all Kendall’s \( \tau \) only in absolute value. Similarly, we can also define the following ESG class lower tail dependence indicator \( D_j^{S,q}(\lambda) \) for each asset \( j \) with its ESG class \( k \) within each sector \( S \) and period \( q \). Here \( \lambda \) denotes the lower tail dependence coefficient, with estimate \( \hat{\lambda} \). Thus, we have

\[
D_j^{S,q}(\lambda) = \frac{|\hat{\lambda}_{j,i_k}^{S,q}|}{|\hat{\lambda}_{j,i_k}^{S,q}| + |\hat{\lambda}_{j,i_k}^{S,q}| + |\hat{\lambda}_{j,i_k}^{S,q}| + \ldots + |\hat{\lambda}_{j,o_l}^{S,q}|}, \quad \forall S,q,j. \tag{6}
\]

The values for both indicators are in the interval \([0, 1]\). Therefore, we obtain an indication on how strong the (tail) dependence of the asset \( j \) in sector \( S \) of period \( q \) to its ESG class \( k \), given its (conditional) dependence to its sector \( I^{S,q} \) and to other assets \( o_{j_l}^{S,q}, \ldots, o_{l-1}^{S,q} \). Fitting the vine copula models allows to capture these complex dependency structures, especially, as they allow for asymmetric and tail dependence. To get a more comprehensive understanding, instead of only looking at the dependence of an asset \( j \) with its ESG class index alone, the (tail) dependence indicators are standardized as ratios as given in Equation (5) and Equation (6).

The ESG class dependence indicator \( D_j^{S,q}(\tau) \) as shown in Equation (5) gives an indication of how strong the absolute dependence between an asset and its belonging ESG class in relation to all other assets in the same ESG class is. The ESG class lower tail dependence indicator \( D_j^{S,q}(\lambda) \) defined in Equation (6) quantifies the strength of the dependence within the lower-left-quadrant tail of an asset’s return and its associated ESG class index in relation to all other assets within the fitted R-vine copula model. The design of the indicators allows to capture the share of dependencies an asset has with other assets within the same ESG class and sector. This helps to understand if there are common behaviors - especially for tail dependence. Common dependencies could indicate that a time series (e.g., returns) exhibit comovement and, therefore, could share some risk properties. As many investors tilt their portfolio towards comparable large ESG score, using an inclusion or exclusion approach, understanding the dependence between similarly ESG rated assets is necessary in order to be able to capture comovements and promote diversification.
5. The Data

We consider daily prices and yearly environmental, social, and governance (ESG) data of 334 US companies $j$, constituents of S&P 500 index, for which the yearly ESG scores $\overline{ESG}_{j}^{S,q}$ from Refinitiv 4, which have values between 0 and 100, are available in the period from 3 January 2006 to 31 December 2018 (i.e. a total of $t = 3271$ trading days). This time period is chosen in order to have the largest possible time period for the largest number of assets to get a comprehensive analysis of assets belonging to the S&P 500. The overall time frame is split into three different time periods $q$: $q_1 = 2006 – 2010, q_2 = 2011 – 2015, and q_3 = 2016 – 2018$. The first two intervals are made of 5 years of data each, where the first interval includes the 2008 financial crisis. The last interval consists of 3 years of data and their ESG scores, according to Refinitiv are to be considered not definitive as they were not yet five years old when downloading the data (in November 2020). This means that the ESG scores can be modified from the provider post-publication. Furthermore, the S&P 500 market capitalization weights ($M_j^S$) from 1 January 2015 are used in combination with the ESG classes ($R_{j}^{S,q}$) created by the mean ESG scores ($\overline{ESG}_{j}^{S,q}$) of each asset $j$ and the Refinitiv ESG thresholds $k$, explained in the prior section, to compute ESG threshold indices ($I_{t,q}^{S,q}$), which correspond to the nodes Index A ($I_A^{S,q}$), Index B ($I_B^{S,q}$), Index C ($I_C^{S,q}$), and Index D ($I_D^{S,q}$) for trading day $t$ in period $q$ in Figure 2.5 These indices are computed for each of the 10 sectors $S$ and 3 time periods $q$ individually. Additionally, for each industrial sector $S$ the S&P 500 sector index ($I_{s}^{S,q}$) per period $q$ is also included. As the constituents of the ESG scores are weighted differently within each sector and it has been shown by Ashwin Kumar et al. (2016) that ESG scores affect various sectors to a different degree, we estimate a R-vine model for each industrial sector $S$ and time period $q$ separately (Eikon, 2020). In particular, we have 50 assets for Consumer Cyclicals, 35 for Healthcare, 23 for Utilities, 51 for Technology, 19 for Basic Material, 21 for Real Estate, 49 for Financials, 41 for Industrial, 16 for Energy and 31 for Consumer Non-Cyclicals. These assets are grouped given their mean ESG score ($\overline{ESG}_{j}^{S,q}$) into four different ESG class groups ($R_{j}^{S,q}$) per time period $q$ as shown in Table B.7 in Appendix B. Figure 4 shows the distribution of ESG scores for the different sectors during the different times periods, from the oldest at the bottom to the most recent ones at the top. It is clear that overall ESG scores have improved in time, possibly also due to the increasing pressure faced by companies to disclose information as well as the implementation of new policies according to ESG criteria.

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4The ESG scores considered are the ones available from Refinitiv, the financial and risk business unit of Thomas Reuters. Refinitiv offers a comprehensive ESG database and covers 70 percent of global market capitalization, across more than 450 different ESG metrics (Eikon, 2020). These percentile-ranked ESG scores are designed to measure the company's relative ESG performance, commitment, and effectiveness across ten main topics, including emissions, environmental product innovation, and human rights, all of which are based publicly-reported data (Eikon, 2020).

5All notation are reported in the Appendix A.5.
To get a preliminary understanding if ESG scores can provide additional information on the riskiness of a company, the empirical 95% Value at Risk (VaR), computed as the empirical quantile of the asset return distribution based on daily data for each asset $j$ in the period $q$ grouped according to their mean ESG scores across all sectors $S$ is presented in Figure 5. Notice VaR values are smaller and exhibit larger variability during the time period 2006-2010, due to the 2008 financial crisis, while estimates becomes larger and, therefore, less extreme in the other time periods. It is also quite evident, especially for the time periods 2006-2010 and 2011-2016 that ESG scores seem to be capable to provide information on the tail risk of an asset, as better rated companies tend to have also less negative VaR values. Remember also that ESG scores for 2016-2018 are yet not definitive. Differences tend to diminish.
in the last two periods for classes A, B, and C as also companies tend to improve in their ESG scores, as shown in Figure 4, and markets recover from the 2008 financial crisis. Still, ESG class D, which typically contain companies which have not yet fully disclosed ESG information and are also lower rated than other companies exhibit worst VaR levels also for the time interval 2016-2018.

![Figure 5: Empirical 95% Value at Risk classified by ESG class and time period](image)

Summing up, empirical data suggest that ESG scores can provide information on tail riskiness of companies and allow to group assets which might share similar tail-risk characteristics, at least in the first two time periods. The difference diminishes in time but this could be due to the fact that companies tend to overall improve their ratings reducing the number of assets in ESG class D and differences across ESG classes, except the ESG class D which tend to become smaller, especially in period of lower market turmoil. Plots for 99% VaR and 95% and 99% Expected Shortfall show similar behaviour.

6. Empirical Results

In this section, we report the main results utilizing the proposed R-vine copula ESG risk model. After estimating the marginal distribution according to the IFM – see for details Appendix A.5, we check the normalized
contour and pair plots of the pseudo-copula data to identify deviations from a Gaussian dependence structure. Then, two R-vine models for each sector $S$ and time period $q$ are fitted. Overall we fit $2 \cdot 10 \cdot 3 = 60$ (2 different R-vine model classes, 10 sectors $S$, and 3 time periods $q$) individual R-vine models following the structure shown in Figures 2 and 3. The first R-vine model is fitted allowing only for the itau copula families and their rotations and reflections, while in the second R-vine model we allow for all parametric copula families and their rotations and reflections (all copula families are presented in Table 1). When possible (as described in Section 4.1) the simpler model including only the itau copula families is used in the analysis, if not the R-vine model using all parametric copula families is chosen. A decision table for each sector $S$ and time period $q$ can be found in Table C.10, Appendix C.2. We find that only for the first two time periods $q_1$ and $q_2$ of the Real Estate sector, the model including all parametric copulas fits better.

The data fitted in each R-vine model includes the daily log-return data $(Y_{t,j,k}^{S,q})$ for each sector $S$, the daily sector index $(I_{t}^{S,q})$, as well as the daily ESG class indices $(I_{k}^{S,q})$ in period $q$ and ESG class $k$. In terms of complexity, this means that depending on the sector $S$, we are dealing with a 20-dimensional data to 55-dimensional data set. For example, the Energy sector $S$ includes $j = 16$ assets and the mean ESG scores of these assets $(ESG_{j}^{S,q})$, where $S$ belongs to the Energy sector, belong to all four threshold ESG scores $(k \in A, B, C, D)$, in the time period $q_1 = 2006–2010$, therefore, we have to compute four ESG class indices $(I_{t,A}^{S,q}, I_{t,B}^{S,q}, I_{t,C}^{S,q}, I_{t,D}^{S,q})$, where $S$ is the Energy sector) for each trading day $t$ in period $q_1$ using the market capitalization weights $(M_{t}^{S})$ for the Energy sector $S$.

Furthermore, we also include the Energy sector index $(I_{t}^{S})$, where $S$ belongs to the Energy sector). Then together with the logarithmic returns of these assets $(Y_{t,j}^{S})$, where $S$ belongs to the Energy sector) we create a 21-dimensional dataset for this period $q_1$ to fit both R-vine models. Figures 6 and 7 show the tree level $T_1$ and $T_2$ of the itau based R-vine model of the Energy sector. The same node notation as in Figures 2 and 3 are used.

For all sectors, we find that for the first tree $T_1$ we have mostly Student’s t copulas fitted. This indicates the presence of both, upper and lower tail dependence. Rarely the Gaussian and Frank copula families are chosen which do not model tail dependence. The independence copula is also chosen infrequently in the first tree level indicating the existence of dependence. In the second tree $T_2$ the number of independence copulas, indicating a decrease in model complexity, and Frank, Gaussian, and Clayton copulas increase, indicating less pronounced tail dependence. Looking at the example of the Energy sector in Figures 6 and 7, we illustrate these relationships. The exact number of itau copula families fitted for each sector $S$ can be found in the Table 2. The number parametric copula families fitted are available from the authors upon request.
Figure 6: The first tree level $T_1$ of the fitted regular vine using only the $\rho_{S}\tau$ copula families for the sector $S$ = Energy in period $q_1$ = 2006 – 2010. $\rho_{S}\tau$ denotes the Energy sector index, while $I_A^S$, $I_B^S$, $I_C^S$, $I_D^S$ indicate the ESG class index for the Energy sector. A letter at an edge with a number inside the parenthesis refers to its bivariate copula family with its estimated Kendall’s $\tau$. As we allow for rotations and reflections Kendall’s $\tau$ can also be negative. (See Table 1 for copula family abbreviations.)

Figure 7: The second tree level $T_2$ of the fitted regular vine using only the $\rho_{S}\tau$ copula families for the sector $S$ = Energy in period $q_1$ = 2006 – 2010. $\rho_{S}\tau$ denotes the Energy sector index, while $I_A^S$, $I_B^S$, $I_C^S$, $I_D^S$ indicate the ESG class index for the Energy sector. A letter at an edge with a number inside the parenthesis refers to its bivariate copula family with its estimated Kendall’s $\tau$. As we allow for rotations and reflections Kendall’s $\tau$ can also be negative. (See Table 1 for copula family abbreviations.)
Table 2: Bivariate \( \tau \) copula families and independence copula fitted for Tree 1 (\( T_1 \)). To simplify, no difference in the counts are made based on rotations and reflections. These are available on request by the authors. \((a)\) Basic Materials, \((b)\) Consumer Cyclicals, \((c)\) Consumer Non-Cyclicals, \((d)\) Energy, \((e)\) Financials, \((f)\) Healthcare, \((g)\) Industrials, \((h)\) Real Estate, \((i)\) Technology, \((j)\) Utility)

6.1. Dependence Estimation

We then compute both dependence indicators, \( D^{S,q}_{j}(\tau) \) and \( D^{S,q}_{j}(\lambda) \) of asset \( j \) in sector \( S \) in period \( q \) belonging to ESG class \( k \) using the optimal R-vine model.

\[
D^{S,q}_{k}(\tau) = \frac{1}{n^{S,q}_{k}} \sum_{j' \in [1,n_{S,q}]} D^{S,q}_{j'}(\tau), \quad \forall_{S,q,k}.
\]
\[
\hat{D}^{S,q}_k(\lambda) = \frac{1}{n^{S,q}_k} \sum_{j' \in [1,n_S]} D^{S,q}_{j'}(\lambda), \quad \forall S,q,k \tag{8}
\]

where \(n^{S,q}_k\) corresponds to the number of assets in sector \(S\), which are in ESG class \(k\) in period \(q\), i.e.,

\[
\sum_{j' \in [1,n_S]} 1, \quad \forall S,q,k
\]

and \(R^{S,q}_j\) is the ESG class group based on the mean ESG score of an asset \(j'\) in sector \(S\) in period \(q\).

The mean values for the overall and lower tail dependence indicator \(\hat{D}^{S,q}_k(\tau)\) and \(\hat{D}^{S,q}_k(\lambda)\) are reported in Table 3 and 4. Here the hyphen indicates that no dependence indicator can be computed due to missing values or we chose not to compute the dependence indicator for groups with only one asset as they dominate the dependence indicator. Boxplots for both dependence indicators are given in Figure D.8 and Figure D.9 in the Appendix D.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>**Sector</td>
<td><strong>ESG class</strong></td>
<td><strong>A</strong></td>
<td><strong>B</strong></td>
</tr>
<tr>
<td>Basic Materials</td>
<td>-</td>
<td>0.30</td>
<td>0.29</td>
</tr>
<tr>
<td>Consumer Cyclicals</td>
<td>0.25</td>
<td>0.22</td>
<td>0.18</td>
</tr>
<tr>
<td>Consumer Non-Cyclicals</td>
<td>0.22</td>
<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
<td>Energy</td>
<td>-</td>
<td>0.29</td>
<td>0.25</td>
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<td>Financials</td>
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<td>0.24</td>
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</tr>
<tr>
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<td>0.22</td>
<td>0.25</td>
</tr>
<tr>
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<td>-</td>
<td>0.38</td>
</tr>
<tr>
<td>Technology</td>
<td>0.19</td>
<td>0.17</td>
<td>0.23</td>
</tr>
<tr>
<td>Utility</td>
<td>-</td>
<td>0.32</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 3: \(\hat{D}^{S,q}_k(\tau)\) for each Sector and ESG Class using the \(\text{itau}\) copula families

The ESG class dependence indicators, \(D^{S,q}_{j'}(\tau)\) and \(\hat{D}^{S,q}_k(\tau)\), suggest a dependence of an assets with its ESG class given its belonging sector. Looking at Table 3, we find that the majority of assets has a dependence between 0.2 and 0.4, and is thereby not negligible. Finding dependence means that assets that belong to the same ESG class exhibit dependence among each other. We find that the largest ESG class dependence in \(q_1 = 2006 – 2010\) for ESG class \(A\) is found for Consumer Cyclicals as well as Healthcare assets, also we notice that many sector \(S\) in period \(q_1\) do not have enough assets in the ESG class \(A\) to compute a mean value. The largest ESG class dependence for ESG class \(B\) assets is found in the Utility sector. While the largest dependence for ESG class \(C\) is found in the Real Estate sector. The Energy sector shows to have the highest mean dependence in the ESG class \(D\). When looking

Electronic copy available at: https://ssrn.com/abstract=3846739
at the second time interval \( q_2 \) after the 2008 financial crisis one can see that for ESG score class \( A \), the Real Estate assets, for ESG score class \( B \), the Basic Material assets and for ESG class \( C \), the show the Energy assets show the highest ESG score class dependence. Now in ESG class \( D \) only two sectors have enough assets to compute the mean dependence.

In the last time interval \( q_3 \) one can see that the overall dependence among similarly rated asset is decreased compared to the earlier time periods. Furthermore, the Utility assets are found to have the highest ESG class dependence for ESG score \( A \) and \( B \). Here, the Energy assets show the highest dependence in ESG scores class \( C \). In the ESG class \( D \), only one sector is left. This is the result of the improvement of the ESG scores within each sector as shown in Figure 4.

Overall, we find overall dependence among assets which are similarly rated and across sector is present. During \( q_3 = 2016 - 2018 \), when ESG scores are not yet definitive, the dependence is found to be smaller. Such results might change in the case that ESG scores are updated to take into consideration further information or disclosure from the companies.

<table>
<thead>
<tr>
<th>Sector</th>
<th>ESG class</th>
<th>2006-2010</th>
<th>2011-2015</th>
<th>2016-2018</th>
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<tr>
<td></td>
<td>( D_{S,k}^{S,j}(\lambda) ) for each Sector and ESG Class using the ( \lambda ) copula families</td>
<td></td>
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<tbody>
<tr>
<td>Sector</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>Basic Materials</td>
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<td>0.82</td>
<td>0.68</td>
</tr>
<tr>
<td>Consumer Cyclicals</td>
<td>0.84</td>
<td>0.63</td>
<td>0.51</td>
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<tr>
<td>Consumer Non-Cyclicals</td>
<td>0.66</td>
<td>0.54</td>
<td>0.60</td>
</tr>
<tr>
<td>Energy</td>
<td>-</td>
<td>0.78</td>
<td>0.73</td>
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<tr>
<td>Financials</td>
<td>-</td>
<td>0.69</td>
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<td>Healthcare</td>
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<td>Industrials</td>
<td>-</td>
<td>0.69</td>
<td>0.69</td>
</tr>
<tr>
<td>Real Estate</td>
<td>-</td>
<td>-</td>
<td>0.85</td>
</tr>
<tr>
<td>Technology</td>
<td>0.75</td>
<td>0.55</td>
<td>0.61</td>
</tr>
<tr>
<td>Utility</td>
<td>-</td>
<td>0.81</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Furthermore, the ESG class tail dependence indicator \( D_{S,j}^{S,j}(\lambda) \) and its mean value \( \bar{D}_{S,k}^{S,j}(\lambda) \) are computed as stated in Equation (6) and Equation (8). Due to the design of the indicator and the number of copula families with zero tail dependence coefficients, the magnitude of the indicator is close to 1. Still, it is possible to compare the lower tail dependence across sectors \( S \) and ESG classes \( k \).

As expected due to the design of the indicator, we find that the tail dependence is higher than the overall dependence. Furthermore, for some sectors the tail dependence is explicitly lower than for other sectors. In the
first time interval for ESG class A and B, the dependence is lowest for the Consumer Non-Cyclicals. In the ESG class C, again this is true for the Consumer Non-Cyclicals as well as the Consumer Cyclicals, Financials, Healthcare, and Technology assets. Lastly, the lowest dependence is found in the Consumer Cyclicals assets for ESG class D.

In the second interval \( q_2 \), the lowest dependence is found for the Technology sector in ESG class A and ESG class B, and the the Consumer Cyclicals in ESG class C and D. Again not many sectors with enough assets are available in ESG class D.

In the final interval, \( q_3 \), we find the lowest tail dependence for Consumer Cyclicals assets for each ESG class. Overall, the tendency seems to be a decrease in tail dependence in the last time interval. This is in line with the findings of the overall dependence in Table 3. However, we find that some sectors including Utilities and the Basic Materials maintain their relative high score throughout all three intervals. As the ESG scores in the last interval are not yet definite, they might be adapted in the next years by the provider. Nevertheless, these ESG scores can still be used by investors for their decision making and dependence analysis.

Overall, by introducing an indicator to capture overall dependence among assets with the similar ESG scores and across sector, we are capable to quantify dependence. We see that such dependence is not negligible, with values often between 0.2 and 0.4, which tend to increase during crisis. Also, by quantifying explicitly tail dependence, we are able to provide a measure of it, and, therefore, show that tail dependence tends to be higher during crisis. Still, as the overall ESG dependence vary between 0.2 and 0.4, the idiosyncratic component for each stock as well as some other effects could still play a relevant role.

7. Conclusion

Starting from an empirical analysis on real-world financial data, we notice that that ESG rating classes can provide information on (tail) dependence and tail risk. In fact, as Figure 5 shows, especially in the time periods 2006-2010 and 2011-2015, assets with better ESG rating seem to exhibit less tail risk. However, as companies tend to improve their rating throughout time, these differences tend to diminish, especially in the interval 2016-2018, in which, however, Refinitiv ESG scores cannot yet be considered definitive. Based on such observations, we propose the R-vine copula ESG risk model to capture dependence and tail dependence among assets belonging to the same industrial sector based on their ESG class and among classes. Vine copula models are a flexible tool to capture financial times series characteristics including non-symmetry and dependence in the extremes, as it can rely on a pair copula construction. Furthermore, especially, the R-vine model is able to flexible model the dependence structures using the ESG scores and the given ESG thresholds as it also allows for different and not only recursive conditioning orders making the model less restrictive. Moreover, as for each level an arbitrary bivariate...
copula can be specified, every complex dependence structure can be captured effectively and optimally. After estimating the R-vine model, we can not only compute all the conditional dependencies among assets as well as specify their interactions as modelled by different copulas families, but we can also introduce two new ESG class dependence indicators that capture both overall dependence and also tail dependence. Furthermore, we are capable to quantify the dependence component that relies on belonging to a specific ESG class and industrial sector. Such components are not negligible as they vary between 0.2 and 0.4, with some even larger values during 2008 financial crisis. We expand these findings further by quantifying also tail dependence among assets belonging to the same ESG class and industrial sector. We notice that such tail dependence is also stronger during crisis.

The understanding and estimation of such dependence is of utmost importance for setting up adequate risk management and mitigation tools as well as building portfolios, ideally also ESG diversified and resilient to crises. Current popular ESG inclusion approaches that focus on picking only assets in the highest ESG rating classes could have indeed possibly benefit in the past from better VaR values but such behavior is not clear for the most recent interval, where ESG classes are overlapping. In fact, picking assets with the highest ESG scores does not lead to better VaR values necessarily and could instead results in applying too much pressure on a specific set of assets, without a clear benefit. The constant trend in improving ESG scores might be a factor behind the lack of VaR differentiation between the classes A, B, and C in the last time interval, joint to the fact that such ESG scores are yet not definitive. Still, we notice that ESG class D assets tend to exhibit poorer VaR values than other ESG classes, suggesting that ESG disclosure might also have some indirect and positive effect on the company risk management. High on the agenda, the current model could be used to develop new ESG investment strategies and ESG based risk mitigation and management modelling tools.
Appendix A. Notation and Computation

Appendix A.1. Data

We introduce the mathematical indices, data sets and their notations used in the paper.

<table>
<thead>
<tr>
<th>Type of Data</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector</td>
<td>$S = 1, \ldots, 10$</td>
</tr>
<tr>
<td>Trading day</td>
<td>$t = 1, \ldots, 3271$</td>
</tr>
<tr>
<td>Year</td>
<td>$y = 2006, \ldots, 2018$</td>
</tr>
<tr>
<td>Period</td>
<td>$q = 1, 2, 3$</td>
</tr>
<tr>
<td>Total number of assets in sector $S$</td>
<td>$n_S$ in Section Appendix B, Table B.7</td>
</tr>
<tr>
<td>Asset $j$ in sector $S$</td>
<td>$j = 1, \ldots, n_S$</td>
</tr>
<tr>
<td>ESG class</td>
<td>$k \in {A, B, C, D}$</td>
</tr>
<tr>
<td>ESG score of asset $j$ in sector $S$ in year $y$</td>
<td>$ESG^{S,y}_{j}$</td>
</tr>
<tr>
<td>Log return of asset $j$ in sector $S$ in period $q$ on trading day $t$ belonging to ESG class $k$</td>
<td>$\gamma_{t,j,k}^{S,q}$</td>
</tr>
<tr>
<td>S&amp;P 500 log return for sector $S$ on trading day $t$ in period $q$</td>
<td>$l_t^{S,q}$</td>
</tr>
<tr>
<td>Market capitalization weight of asset $j$ in sector $S$ (by 1.01.2015)</td>
<td>$M_j^S$</td>
</tr>
<tr>
<td>Mean ESG score of asset $j$ in sector $S$ and period $q$</td>
<td>$\overline{ESG}^{S,q}_{j}$</td>
</tr>
<tr>
<td>ESG class of asset $j$ in sector $S$ and period $q$</td>
<td>$\bar{a}_{j}^{S,q}$</td>
</tr>
<tr>
<td>ESG class weight of asset $j$ in sector $S$ in period $q$</td>
<td>$a_{j}^{S,q}$</td>
</tr>
<tr>
<td>Values of ESG class $k$ in sector $S$ and period $q$ at trading day $t$</td>
<td>$P^{S,q}_{t,k}$</td>
</tr>
</tbody>
</table>

Table A.5: Mathematical Indices, datasets, and their notation used in the paper.

As mentioned in Section 5, we worked with the mean ESG score and the corresponding ESG class of an asset $j$ in sector $S$ for period $q$. Accordingly, we calculated the assets’ ESG class weights, which were then used to calculate their corresponding ESG class values, as defined in Table A.5. Their computations are given as follows.

Appendix A.2. Mean ESG score of asset $j$ in sector $S$ and period $q$ ($\overline{ESG}_{j}^{S,q}$):

\[
\overline{ESG}_{j}^{S,q} = \frac{1}{|P_q|} \sum_{y \in P_q} ESG^{S,y}_{j} \quad \text{for} \quad \forall j, S, q, \quad (A.1)
\]


Electronic copy available at: https://ssrn.com/abstract=3846739
Appendix A.3. ESG class of asset j in sector S and period q ($R_{S,j}^{S,q}$):

We have $\forall j, S, q$:

$$R_{S,j}^{S,q} = \begin{cases} 
A, & \text{if } ESG_{S,j}^{S,q} \in [75, 100], \\
B, & \text{if } ESG_{S,j}^{S,q} \in [50, 75), \\
C, & \text{if } ESG_{S,j}^{S,q} \in [25, 50), \\
D, & \text{otherwise.}
\end{cases} \tag{A.2}$$

ESG class weight of asset j in sector S in period q ($\alpha_{S,j}^{S,q}$):

$$\alpha_{S,j}^{S,q} = \frac{M_{S,j}}{\sum_{j' \in [1,n_S]} \sum_{j':R_{j'}^{S,q}=R_{j}^{S,q}} M_{j'}} \text{ for } \forall j, S, q. \tag{A.3}$$

Appendix A.4. Values of ESG class k in sector S and period q at trading day t ($I_{S,q,t}^{S,q}$):

$$I_{S,q,t}^{S,q} = \sum_{j' \in [1,n_S]} \sum_{j':R_{j'}^{S,q}=k} \alpha_{S,j'}^{S,q} \cdot Y_{S,q,t,j'} \text{ for } \forall S, q, k \text{ and } t \in T_q. \tag{A.4}$$

where $T_1 = [1, 1260], T_2 = [1261, 2517], T_3 = [2518, 3271]$.

Appendix A.5. Two-Step Inference for Margins

As financial data are strongly dependent on past values and not uniformly distributed on $[0, 1]^d$, which is the necessary input for a copula, a two-step inference for margins (IFM) approach is followed. This approach as been investigated by Joe (2005). We follow a parametric marginal model and estimate the margins first, we then use the estimated marginal distributions to transform the data on the copula scale by defining the pseudo-copula data. This allows us to remove the marginal time dependence by utilizing standard univariate time series models and then proceed with standardized residuals obtained from these models. We fit a generalized autoregressive conditional heteroskedasticity (GARCH) model with Student $t$ innovations to our data, allowing for time-varying volatility and volatility clustering.
As an input of a R-vine model in sector $S$ and period $q$, we have a data matrix $X^{S,q}$ defined in Table A.6. Overall, we have $|S| \times |q| = 10 \times 3 = 30$ vine copula risk models.

In sector $S$ and period $q$, we fit a GARCH(1,1) models with appropriate error distribution for a marginal time series, $X_{t,d}^{S,q}$, and estimate the parameters of the following model:

$$
\varepsilon_{t,d}^{S,q} = \sigma_{t,d}^{S,q} \cdot z_t
$$

$$
(a_{t,d}^{S,q})^2 = \gamma_0 + \gamma_1 \cdot (\varepsilon_{t-1,d}^{S,q})^2 + \beta_1 \cdot (a_{t-1,d}^{S,q})^2
$$

(A.5)

where $(z_t)_{t>1}$ is a sequence of normal random independent and identically distributed random variables satisfying the standard assumptions $E[z_t] = 0$ and $\text{var}[z_t] = 1$ and follows a Student’s $t$ distribution. Then using the cumulative distribution function of the standardized Student’s $t$ distribution, we determine the pseudo-copula data as probability integral transformation (PIT), i.e.

$$
\hat{\theta}_{t,d}^{S,q} := F_{\hat{\theta}} \left( \frac{X_{t,d}^{S,q}}{\hat{\sigma}_{t,d}^{S,q}} ; \hat{\psi}_{S,j}^{d} \right),
$$

(A.6)

Following this two-step approach allows us to convert data to the copula scale, which can be used for estimation of the copula parameter of the chosen bivariate copula family.
Appendix B. Number of Assets for each ESG class per Sector and interval

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic Materials</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>2006-2010</td>
<td>1</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>2011-2015</td>
<td>2</td>
<td>15</td>
<td>23</td>
</tr>
<tr>
<td>2016-2018</td>
<td>6</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>Consumer Cyclicals</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2006-2010</td>
<td>1</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>2011-2015</td>
<td>1</td>
<td>17</td>
<td>30</td>
</tr>
<tr>
<td>2016-2018</td>
<td>2</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>Consumer Non-Cyclicals</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2006-2010</td>
<td>6</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>2011-2015</td>
<td>9</td>
<td>17</td>
<td>4</td>
</tr>
<tr>
<td>2016-2018</td>
<td>9</td>
<td>18</td>
<td>4</td>
</tr>
<tr>
<td>Energy</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>2006-2010</td>
<td>1</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>2011-2015</td>
<td>1</td>
<td>17</td>
<td>30</td>
</tr>
<tr>
<td>2016-2018</td>
<td>2</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>Healthcare</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2006-2010</td>
<td>1</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>2011-2015</td>
<td>1</td>
<td>14</td>
<td>4</td>
</tr>
<tr>
<td>2016-2018</td>
<td>6</td>
<td>16</td>
<td>21</td>
</tr>
<tr>
<td>Industrials</td>
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<td></td>
<td></td>
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<tr>
<td>2006-2010</td>
<td>1</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>2011-2015</td>
<td>1</td>
<td>14</td>
<td>4</td>
</tr>
<tr>
<td>2016-2018</td>
<td>6</td>
<td>16</td>
<td>21</td>
</tr>
<tr>
<td>Real Estate</td>
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<td></td>
</tr>
<tr>
<td>2006-2010</td>
<td>0</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>2011-2015</td>
<td>1</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>2016-2018</td>
<td>2</td>
<td>18</td>
<td>3</td>
</tr>
</tbody>
</table>

Table B.7: Number of Assets for each ESG class per sector S and time period q

Appendix C. Model Fit

Comparison of the *itau* and *parametric* R-vine models.

Appendix C.1. Energy Sector Model Fit

In the following the BIC are compared from three different vine models for the Energy sector and time period. In every model the preselect feature of *rvinecopula* is not activated. The output of all additional sectors and time intervals are available from the authors upon request. See Table 1 for copula family abbreviations.

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>loglik</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006-2010</td>
<td>32393.89</td>
<td>-63074.45</td>
</tr>
<tr>
<td>2011-2015</td>
<td>30135.86</td>
<td>-58637.47</td>
</tr>
<tr>
<td>2016-2018</td>
<td>11526.09</td>
<td>-21727.11</td>
</tr>
</tbody>
</table>

Table C.8: Model Fit - Energy
Table C.9: Vuong Test - Energy

<table>
<thead>
<tr>
<th>Interval</th>
<th>Statistic</th>
<th>Schwarz Statistic</th>
<th>Schwarz p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006-2010</td>
<td>2.012</td>
<td>5.806</td>
<td>6.383e-09</td>
</tr>
<tr>
<td>2011-2015</td>
<td>4.470</td>
<td>5.182</td>
<td>2.194e-07</td>
</tr>
</tbody>
</table>

Appendix C.2. Decision Table Model Fit

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2006-2010</td>
<td>Itau</td>
<td>Itau*</td>
<td>Itau*</td>
<td>Itau*</td>
<td>Itau*</td>
<td>Par*</td>
<td>Itau*</td>
<td>Itau*</td>
<td>Itau*</td>
<td>Itau*</td>
</tr>
</tbody>
</table>

Table C.10: Model Decision based on parsimonious Schwarz correction and BIC (Schwarz, 1978). The copula families corresponding to each *itau and parametric model are given in Table 1. (* Basic Materials, h Consumer Cyclicals, c Consumer Non-Cyclicals, d Energy, e Financials, f Healthcare, g Industrials, h Real Estate, i Technology, j Utility)

* Statistically Significant at $\alpha = 5\%$

Appendix D. Boxplots of Dependence Indicator

Similar to Tables 3 and 4, ESG classes which contain only one asset are not plotted below.

Appendix D.0.1. ESG class dependence indicator $D_{f}^{S,q}(\tau)$

Figure D.8: ESG class dependence indicator for each time interval
### Appendix D.0.2. ESG class lower tail dependence indicator $D_{ij}^{S,q}(\lambda)$

#### ESG class lower tail dependence indicator

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Utilities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Estate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrials</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financials</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Healthcare</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer Non-Cyclicals</td>
<td></td>
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</tr>
<tr>
<td>Consumer Cyclicals</td>
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<td></td>
</tr>
<tr>
<td>Basic Materials</td>
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<td></td>
</tr>
</tbody>
</table>

#### Dependence Measure

![Graph showing ESG class lower tail dependence indicator for different sectors over time intervals](https://ssrn.com/abstract=3846739)

**Figure D.9:** ESG class lower tail dependence indicator for each time interval
References


Joe, H., 1996. Families of m-variate distributions with given margins and m (m-1)/2 bivariate dependence parameters. Lecture Notes-Monograph Series, 120–141.


