

Stock market volatility: friend or foe?

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Abstract

Although a good deal of research effort has been allocated to understanding the time-series volatility of stock returns – as both market (or systematic) volatility and idiosyncratic (or non-systematic) volatility – the relationship of such volatility with cross-sectional volatility or dispersion of outcomes is sparse. Nevertheless, the quest to understand one must involve the quest to understand the other. In this paper, we investigate the dynamic of the dispersion of return outcomes in generating a portfolio's expected return outcome. We find that changes in the level of cross-sectional volatility have highly significant implications for portfolio performances and the notion of risk.

Key words: Investment horizon; Market return; Market; Risk; Stock performance; Volatility

JEL classification: G10, G11, G12

1. Introduction

Although a good deal of attention has been accorded to stock volatility as the time-variation of stock returns, much less attention has been accorded to their cross-sectional variation, which is to say, the dispersion, of stock returns. Nevertheless, the two variations are related. In which case, a satisfactory understanding of one requires an understanding of the other. The present paper highlights how a dispersion of return outcomes, *of its statistical nature*, implies a relation between inter-temporal volatility and higher returns. In this case, the higher portfolio returns so generated are not the outcome of investors “setting prices” in accordance with their required expectations in relation to risk (the fundamental assumption of academic asset pricing), but, rather, are forthcoming as *a mathematical or statistical consequence* of the risky nature of growth itself. Our results have implications for professional investment strategies as well as for a fuller understanding of risk and volatility.

Campbell, Lettau and Malkiel (2001) have reported how a higher level of inter-temporal sector volatility and a reduction in cross-sector correlations contribute to an increased cross-sectional dispersion of outcomes (Ankrum and Ding, 2002). Cross-sectional dispersion is thereby related to inter-temporal firm volatility. In a similar vein, Solnik and Roulet (2000) show how the cross-sectional dispersion of stockmarket returns is related to global levels of inter-temporal stockmarket correlations. Consistently, for Australian stocks, Alaganar and Whiteoak (2004) find that the cross-sectional volatility of stock returns is associated with increases in inter-temporal sector volatility and reductions in cross-sector correlations. As an outcome of a bootstrapping exercise whereby historical returns are assumed to represent unbiased estimates of the range of market outcomes over investment horizons, Fama and French (2018) have also demonstrated a relationship between market performance and the cross-sectional dispersion of outcomes.

Consistent with the above, we advance a statistical model that recognizes that the individual time-series volatility of asset returns and the cross-sectional volatility of their returns combined are necessarily related.¹ Consistent with the model, we determine that - as opposed to market outcomes representing the outcome of investors *setting prices* (so as to generate higher expected outcomes) in response to higher volatility - the dynamic of higher market rewards can be related to the merely *statistical* outcome of volatility. The idea that “excess growth” can emerge as the statistical outcome of stock return variance is not new (Fernholz and Shay, 1982; Booth and Fama, 1992). Nevertheless, the impact of such papers to date appears to have been marginal. We demonstrate the proposed statistical model in the context of both the US stockmarket (over long-term investment periods), as well as in the context of historical Australian share prices (over monthly investment periods).

To these ends, the rest of the paper is as follows. The following section clarifies the fundamental statistical model applied to asset growth. Section 3 applies the model to the long-term performances of the US stockmarket and Section 4 applies the model to the monthly stockmarket performances for Australian shares. Section 5 concludes.

2. The model

When we apply a randomly selected exponential growth rate from a normal distribution with mean μ per period with standard deviation σ about such mean, to an asset price $\$P_0$ at the commencement

¹ We can recognize that in the particular case that asset returns are perfectly *uncorrelated*, but share an identical time-series volatility, the time-series volatility and the cross-sectional volatility of the asset individual returns are one and the same.

of a period, we can represent the (unknown) outcome asset price at the end of N periods, $\$P_{t+N}$, as

$$\$P_{t+N} = \$P_t e^{N\mu + \sqrt{N}Z\sigma} \quad (1)$$

where Z represents a selection from $-\infty$ to $+\infty$ with the probability of selection determined by the “unit normal” distribution (with mean zero and standard deviation equal to one).

The assumption that stockmarket growth might be modeled at least approximately by such a process is, to some extent, justified by the evidence of past stock price performance. In addition, we have the argument of the “central limit theorem” (the “theorem of large numbers”) that reveals that even when exponential growth rates are *not* normally distributed in the short-term, provided the outcomes over successive periods are drawn *independently* from the *same* distribution, the outcome longer-term growth rate over a large number of N sub-time intervals will tend to be a *normal* distribution with mean (μ) and standard deviation (σ) determined as

$$\mu = N\mu^* \quad (2)$$

and

$$\sigma = (\sqrt{N})\sigma^* \quad (3)$$

where μ^* is the mean of the exponential growth rates with standard deviation, σ^* , over each sub-time interval (for example, Fama, 1976).

The exponential growth rate, R , that provides the “expected” (which is to say, probability-weighted) price outcome at the end of the period, is defined by

$\$P_0 e^R \equiv$ the "expected" price outcome of investment $\$P_0$ for a single period,

which is to say as

$$R \equiv \text{Ln}[\text{expected wealth outcome of investing } \$1 \text{ for a single period}] \quad (4)$$

where Ln represents the natural log function. As a statistical outcome, R is determined as

$$R = \mu + \frac{1}{2}\sigma^2 \quad (5)$$

where the asset or portfolio volatility (σ) in Equation (5) may comprise both market and idiosyncratic volatility.²

Investors, of course, assess stock prices not in terms of their potential exponential growth characteristics, but in terms of their "expected discrete periodic return", by which we mean the expected percentage increase in wealth generated over some *discrete* period (a month, a year).

Such return (r) is expressed:

² As a point of interest, Equation (5) is applied in the Black-Scholes model. Thus, the assumption of "risk neutrality" leads to the risk-free rate, $r_f = \mu + \frac{1}{2}\sigma^2$, so that μ is then replaced by $r_f - \frac{1}{2}\sigma^2$ in the model's familiar expressions. See, for example, Chapter 8 in the student textbook: "Financial Risk Management and Derivative Instruments" (2021) by Michael Dempsey (Routledge, Oxon, England).

$$r \equiv \frac{[\text{expected price outcome of investment } \$P_0 \text{ for a single period} - \$P_0]}{\$P_0} \quad (6)$$

where $\$P_0$ is the investment at commencement of the investment period. In order to generalize the relationship between the above *periodic* return – which is to say, the “surface” return that market participants seek to measure – and the “sub-surface” *continuously applied* parameters μ and σ (which generate the surface periodic return), we observe that the expected price outcome can be expressed equally as either

$$\$P_0 e^R \text{ (with the definition of Equation (4))}$$

or as

$$\$P_0(1 + r) \text{ (with the definition of Equation (6)).}$$

Thus, we have

$$r = e^R - 1 \quad (7)$$

which with Equation 5 provides

$$r = e^{\mu + \frac{1}{2}\sigma^2} - 1 \quad (8)$$

so that provided R is a “smallish fraction” (allowing that we are considering rates on an annual

basis, or less), we can express e^R as approximately $1 + R$, so that we have

$$r \text{ approximately} = R = \mu + \frac{1}{2} \sigma^2 \quad (9)$$

Since $\frac{1}{2}\sigma^2$ is necessarily positive, Equation (9) is the statement that the volatility of returns (σ) in an exponential growth process acts to *increase* the expected (probability-weighted) periodic return.³ Nevertheless, the theoretical return outcome remains probabilistic. That is to say, although the expected or probability-weighted wealth outcome is *increased* by outcome volatility (σ) as Equation (9), the ultimate outcome is nevertheless a single outcome return from a distribution made wider (more risky) by virtue of the self-same return volatility (σ).⁴

An additional dynamic of volatility emerges when we consider Equation (9) applied to the *individual assets* that comprise a portfolio. To see this, we must consider how the individual returns of assets generated by volatility contribute to an overall portfolio return, as well as how the individual asset volatilities combine to determine the overall volatility of the portfolio of such assets. Thus, firstly, we observe that the discrete expected return for a portfolio of remains as the asset value weighted-average of the expected returns of the individual assets in the portfolio assets (notwithstanding that the individual asset returns are augmented by asset volatility). Secondly, we observe that the *market* volatility of the portfolio returns is also the asset value weighted-average of the market volatilities of the individual assets. Thirdly, however, we observe that the

³ This observation is in accord with findings to date. For example, Malkiel and Xu (1997), Goyal and Santa-Clara (2003) and Fu (2009) report that idiosyncratic volatility is significantly and positively related to stock returns in the US. Findings by Ang, Hodrick, Xing and Zhang (2006) who report a *negative* relationship between *lagged* idiosyncratic volatility and future average returns are often referenced as contradicting the findings of a positive relation. Nevertheless, their findings may be interpreted in relation to the behavioral role of idiosyncratic volatility as the “messenger” of declining market confidence with consequent lower asset prices.

⁴ The implications of Equation 9 for investments and portfolio construction are developed in the student textbook: “Investment Analysis: An Introduction to Portfolio Theory and Management” (2020) by Michael Dempsey (Routledge, Oxon, England).

idiosyncratic volatility of the portfolio is necessarily *less* than the asset value weighted-average of the idiosyncratic volatilities of the individual assets, and approaches zero with many assets. We therefore observe that with independently performing assets, we *retain* the asset value weighted return generated by the individual asset volatilities (which benefits from the individual asset volatilities), while simultaneously *reducing* the portfolio volatility of the assets combined.

In the following section, we demonstrate the contribution of *market* volatility to the expected return of the market portfolio over long-term periods. In the section thereafter, we observe the impact of *individual* asset volatilities (allowing both market and idiosyncratic volatility) on portfolio outcomes.

3. The long-term implications of the cross-sectional dispersion of outcomes: the results of Fama and French (2018)

Fama and French (2018) examine the average stockmarket returns (above the *T*-bill rate) for a range of investment time horizons. The results are the outcome of a bootstrapping exercise whereby the monthly returns for the stockmarket period 1963-2016 are treated as the population of return possibilities from which 100,000 trials are drawn randomly with associated *T*-bill returns. The results are presented in the first numerical column of Table 1 (annualized in parentheses). Thus, we observe a dramatic increase in the “expected” (average) premium return, *RET* (over the *T*-bill rate), as the distributions become more disperse with a longer investment time horizon. As observed by Fama and French (2018), a contributing factor to the higher values for the equity premiums is the increasing skew to higher outcome returns combined with a higher kurtosis of both higher positive and negative outcome returns than is predicted by a normal distribution. The

second numerical column is the standard deviation (SD) of the returns over the same investment time horizon as reported by Fama and French.

Table 1 about here.

The third and fourth columns impose a “best fit” normal distribution on the Fama and French data by transposing the expected return (*RET*) and standard deviation (*SD*) of the Fama and French outcomes to the expected return (μ) and the standard deviation (σ) consistent with a normal distribution of *exponential* growth rates. Thus, in numerical column 3 of the table, we apply the formula⁵:

$$\sigma^2 = Ln \left[1 + \frac{SD^2}{(1+RET)^2} \right] \quad (10)$$

where the expected return (*RET*) and standard deviation of returns (*SD*), respectively, are as numerical columns 1 and 2. For example, for a 10-year investment horizon, we have the expected return (*RET*) = 133% and standard deviation (*SD*) = 149%, so that

$$\sigma^2 = Ln \left[1 + \frac{1.49^2}{(1+1.33)^2} \right] = 0.343,$$

which determines $\sigma = \sqrt{0.343} = 0.585$ (58.5%), as in the table, or as $0.585/\sqrt{10} = 0.185$ (18.5%) annualized. We note the reasonably consistent value of the annualized standard deviation so

⁵ See the Appendix of Mehra (2003) for a proof of Equation (10).

calculated (average = 17.5% per year), which is consistent with returns generated over successive periods by a normal distribution. With σ so defined, we transpose the Fama and French expected return (RET) as the mean exponential growth rate (μ) assuming a normal distribution (consistent with Equation (5) as

$$\mu = \text{Ln}(1 + RET) - \frac{1}{2}\sigma^2 \quad (11)$$

as in the fourth numerical column of the table. As confirmation, column 5 re-constructs the Fama and French expected (average) returns of column 1 assuming a normal distribution of outcomes with μ and σ as in columns 3 and 4.

With a view to revealing the impact of skew and kurtosis with an increasing investment horizon, numerical column 6 provides the expected (average) returns assuming a normal distribution with μ and σ as their one-year values ($\mu = 5.5\%$ per annum and $\sigma = 15.8\%$ per annum, first numerical row), which is to say, imposing skew and kurtosis equal to zero following a one-year investment horizon. The implications of skew and kurtosis appear to be highly significant. Thus, we observe that with an increasing investment time horizon, the skew to a higher distributed outcome return has a compounding effect, with the outcome that the expected (average) outcomes calculated by Fama and French (2018) are increasingly higher than the fixed annualized rate (7.0%) predicted by a normal distribution. After 10-years, the Fama and French annualized expected (average) return outcome is 8.8% compared with 7.0% predicted by a normal distribution; and after 30 years, the Fama and French annualized return is 10.9% compared with 7.0% predicted by a normal distribution (numerical columns 1 and 6).

The final column represents the outcome of maintaining $\mu = 5.5\%$ per annum while setting $\sigma = 0$, which is to say, assuming a normal distribution with skew, kurtosis and volatility each equal to zero. The column reveals a consistent 1.35% return per annum (7.0% - 5.65%) as the outcome of market volatility.

In summary, the table reveals that stock volatility combined with a positive skew or tilt (to the higher returns) and kurtosis leads *statistically* to increasingly higher return outcomes with longer investment horizons than is predicted by risk-free growth. The overall addition to the expected return over a 10-year period as an outcome of diverse return outcomes appears to be close to 3.15% annualized (8.8%-5.65%), and over a 30-year period, closer to 5.25% annualized (10.9%-5.65%).

In Table 2, we present the outcomes calculated by Fama and French from their 1963-2016 returns (the F&F columns) for (a) the probability of underscoring the *T*-bill rate over a range of investment time horizons (as “Neg”) and (b) the dispersion of the equity premium returns at the 5% and 95% probability boundaries (as 5% percentile and 95% percentile respectively). The table shows how with an increasingly longer investment horizon, excess returns become more disperse but move faster to the right. We also observe that as the return horizon increases, the probability of underscoring the *T*-bill rate decreases monotonically. In addition, we observe that the 95% percentile increases monotonically, whereas the 5% percentile becomes more negative (indicating the loss at this probability) as the investment horizon increases – but, intriguingly, only to a point – as somewhere above the 10-year investment horizon, the loss at this probability actually begins to decrease before turning positive. At a 30-year investment horizon, the anticipated outcome at the lower 5% percentile is a positive 37% return.

Table 2 about here.

Retaining a normal distribution with μ and σ as their one-year values ($\mu = 5.5\%$ per annum and $\sigma = 15.8\%$ per annum) for each investment time horizon, we recalculate the outcomes for the 5% and 95% probability boundaries, as well as the probability that the premium is less than zero. The outcomes are as presented alongside those of Fama and French in Table 2. In the table, we observe that the normal distribution has preserved the essential features noted by Fama and French, even the observation that the 5% percentile becomes more negative (a greater loss at this probability) as the investment horizon increases, but only to a point, beyond which the loss at this probability actually begins to decrease. Thus, consistent with the Fama and French results, at a 30-year investment horizon, the predictions have been transformed from a negative return outcome to a (25%) gain. For the 95% percentile, the outcomes increase monotonically.

The essential dynamic underlying the outcomes for a normal distribution is that whereas the mean μ of the outcome returns increases linearly with time, the standard deviation σ of the return outcomes increases only as the square-root of time (Equations 2 and 3), so that with increasing investment horizon, the impact of increasing μ eventually dominates the impact of increasing σ . Notwithstanding, in the absence of skew and kurtosis, the outcome returns with an increasing investment horizon for a normal distribution, fall increasingly behind those of Fama and French. As revealed in Table 1, at the 20-year investment horizon, the normal distribution (column 6) predicts only $\frac{1}{2}$ of the return indicated by the empirically-based result of Fama and French (column 1); and at the 30-year investment horizon, the normal distribution predicts only $\frac{1}{3}$ of the return indicated by Fama and French.

The above observations motivate us to examine the empirical relation between the inter-temporal monthly return performance for a portfolio of stocks in relation to the cross-sectional dispersion of the portfolio's stock returns, to which we turn in the following section.

4. The implications of monthly cross-sectional dispersion of outcomes: the evidence of Australian data

We avail of the data from Datastream for the Australian Securities Exchange (ASX) monthly stock prices for the period January 2006 – December 2017. We calculate the monthly discrete return $r_{i,t} = \frac{\$P_{i,t}}{\$P_{i,t-1}} - 1$ and monthly exponential growth rate $g_{i,t} = \ln \frac{\$P_{i,t}}{\$P_{i,t-1}}$ for each individual stock, where $\$P_{i,t}$ is the price of stock i at the end of month t . Thereafter, for each calendar month, t , we calculate the following:

- (a) the *discrete* monthly return for portfolio P represented by the average of the individual monthly *discrete* returns of all the stocks in the portfolio:

$$RET_{P,t} = \frac{1}{n} \sum_{i=1}^n r_{i,t},$$

- (b) the average monthly *exponential* growth rate of the stocks in portfolio P as:

$$\mu_{P,t} = \frac{1}{n} \sum_{i=1}^n g_{i,t},$$

- (c) the *actual* exponential growth rate for portfolio P :

$$R_{P,t} = \ln \frac{\$V_{P,t}}{\$V_{P,t-1}},$$

(where $\$V_{P,t}$ is the dollar value of the portfolio at the end of month t),

- (d) the cross-sectional dispersion of the *discrete* returns for portfolio P represented by the standard deviation of the discrete returns of all the stocks in the portfolio:

$$SD_{P,t} = \sqrt{\frac{\sum_{i=1}^n (r_{i,t} - RET_{P,t})^2}{n-1}},$$

(e) the cross-sectional dispersion of the *exponential* growth rates for portfolio P represented by the standard deviation of the exponential growth rates of all the stocks in the portfolio:

$$\sigma_{P,t} = \sqrt{\frac{\sum_{i=1}^n (g_{i,t} - \mu_{P,t})^2}{n-1}}, \text{ and}$$

(f) the time-series volatility of portfolio discrete returns represented by the standard deviation of portfolio discrete returns for the most recent twelve months:

$$VOL_{P,t} = \sqrt{\frac{\sum_{t=1}^{12} (RET_{P,t} - \overline{RET}_P)^2}{11}},$$

where \overline{RET}_P is the average monthly discrete return for portfolio P in the twelve-month period.⁶

To begin, we measure both the time-series volatility and cross-sectional standard deviation for a portfolio of (1) “all” stocks, (2) stocks of “large” firms and (3) stocks of “small” firms. “Large” firms are the largest 25% of firms by market capitalization in the ASX, and “small” firms are the remaining 75% of firms in the sample for each month. Table 3 presents the average time-series monthly volatility ($VOL_{P,t}$) of the stock discrete returns, and the average monthly cross-sectional standard deviation ($SD_{P,t}$) of the stock discrete returns for the above three portfolios. We observe that a portfolio of stocks of small firms has both a higher time-series volatility (VOL) and a higher cross-sectional dispersion (SD) compared with stocks of large companies and the entire market.

Table 3 about here.

⁶ We focus on the time-series volatility of discrete returns (as opposed to exponential returns) because the concept of stockmarket “risk” in the literature is generally identified as the time-series volatility of discrete returns.

We then consider the relation between the monthly cross-sectional dispersion of stock returns ($SD_{P,t}$) and the inter-temporal (time-series) monthly volatility ($VOL_{P,t}$) of a portfolio of such stocks as

$$SD_{P,t} = \alpha + \beta(VOL_{P,t}) + \varepsilon_{P,t} \quad (12)$$

where $SD_{P,t}$ and $VOL_{P,t}$ are as defined above and $\varepsilon_{P,t}$ is the error term. The results are summarized in Table 4. We note that for stocks of small firms, the coefficient of the $VOL_{P,t}$ variable is insignificant. Thus, although both the inter-temporal volatility (VOL) and dispersion for stock returns (SD) of small firms are higher than for stocks of large firms (Table 3), the cross-sectional dispersion of returns for small firms appears unrelated to the inter-temporal volatility of their portfolio returns. For stocks of small firms, it therefore appears that a higher inter-temporal portfolio volatility is the outcome of alternating but nevertheless fairly uniform shifts to either higher or lower stock returns in the portfolio through time with little impact on their cross-sectional distribution. For stocks of large firms, however, the $VOL_{P,t}$ variable enters the regression model with a positive coefficient significant at the 1 percent level (coefficient = 0.82; t -statistic = 3.20), implying a positive relation between inter-temporal portfolio volatility and the cross-sectional dispersion of its constituent asset returns. The correlation coefficient is 71% with $R^2 = 51\%$. The findings here are broadly consistent with Alaganar and Whiteoak (2004) for Australian firms.

Table 4 about here.

We wish to test the contribution of individual stock volatilities to a portfolio's overall return as predicted by Equation (5). To this end, the actual monthly exponential return for a portfolio, $R_{P,t} = \ln \frac{\$V_{P,t}}{\$V_{P,t-1}}$, as above, over and above the mean monthly exponential return for the stocks in the portfolio, $\mu_{P,t}$, as $R_{P,t} - \mu_{P,t}$, is regressed against the dispersion of the portfolio's exponential returns in that particular month ($\sigma_{P,t}$), as

$$R_{P,t} - \mu_{P,t} = \alpha + \gamma \left(\frac{1}{2} \sigma_{P,t}^2 \right) + \varepsilon_{P,t} \quad (13)$$

The results are summarized in Table 5. For all stocks, as well as for stocks of both large and small firms separately, the findings reveal a clear positive relation between the average inter-temporal return for a portfolio over and above the median of the portfolio's individual stock returns, as $R - \mu$, and the cross-sectional dispersion of the stock returns as $\frac{1}{2}\sigma^2$ for the period. For small firms, we observe an effective alignment of outcomes with the statistical relation of Equation (5): correlation = 75% and $R^2 = 56\%$, and a coefficient on $\frac{1}{2}\sigma^2 = 0.98$ (t -statistic 13.47) with an insignificant intercept. For large firms, we have an even greater correlation = 88% and $R^2 = 77\%$, with a coefficient on $\frac{1}{2}\sigma^2 = 0.39$ (t -statistic 21.68) with a significant intercept = 0.0042 (t -statistic 14.53) together accounting for the robust correlation and R^2 .

Table 5 about here.

With an average annual cross-sectional standard deviation (σ) for the exponential returns over the period of study as 21.8%, Equation (5) predicts that the return generated by such standard

deviation is $\frac{1}{2}(0.218)^2$ or 2.4% per annum. For stocks of larger firms, the annual standard deviations are lower, at an average 13.6%, implying a contribution to performance as $\frac{1}{2}(0.136)^2$ or 0.9% per annum; while for stocks of smaller firms, the standard deviations are higher, at an average 23.7%, implying a contribution to performance as $\frac{1}{2}(0.237)^2$ or 2.8% per annum. The outcomes appear broadly consistent with the result derived for US stocks in the previous section, namely a 1.35% return per annum (7.0% - 5.65%) as the outcome of market volatility.

As for the US findings in the previous section, the impact of volatility is significantly augmented by skew and kurtosis. Thus, the average *median* monthly returns (of $\frac{\$P_t}{\$P_{t-1}} - 1$) for the stocks are close to zero for all firms, large firms and small firms, whereas the average monthly *mean* returns (of $\frac{\$P_t}{\$P_{t-1}} - 1$) are, respectively, 1.3% per month (all firms), 0.27% per month (large firms) and 1.6% per month (small firms). In other words, the positive performance of stockmarket returns is *dependent* on the volatility and outcome dispersion of the return outcomes, whereby the mean return of the stock returns exceeds their median return. Conclusion: the investor needs volatility!

Finally, we repeated the regression Equation (13) by restricting the sample in each month to (a) stocks with positive outcome returns only and (b) stocks with negative outcome returns only. The results are displayed in Table 6. For positive outcome returns, the table reveals the additional contribution of dispersion (σ) to the overall return R in that month as a coefficient = 2.0 on $\frac{1}{2}\sigma^2$, while for the negative returns sample, the additional contribution of dispersion (σ) to the return R is a coefficient = 0.45. Thus, although the benefits of cross-sectional dispersion are most effectively materialized during up-market periods, the effect remains overall positive even in declining markets.

Table 6 about here.

5. Discussion and Conclusion

For a normal distribution of return outcomes, we have the outcome that a greater dispersion of stock returns implies both a higher expectation of return and a lower probability of underperforming the risk-free rate. Although the probability of a very substantial loss (while remaining fairly low) increases up to quite a long investment period, the calculated loss at the 5% percentile level decreases beyond an investment horizon of approximately 10 years. These observations are the statistical outcome that as excess returns become more disperse with increasingly longer investment horizons, they move faster to the right. The results of Fama and French (2018) indicate that the impact of additional skew and kurtosis in the market is a significant increase in dispersion of stock return outcomes with increasing investment horizon, leading thereby to an even greater role of stock dispersion in the determination of portfolio performances.

For Australian stocks, our analysis again suggests important implications for the cross-sectional dispersion of stock returns. As for US stocks, we have the statistical outcome that a greater dispersion for a portfolio of stock returns implies a higher expectation of portfolio return. We find that the cross-sectional dispersion of stock returns and the time-series volatility of their portfolio are related (particularly for a portfolio of stocks of large firms). Allowing a positive statistical relation between cross-sectional standard deviation and return performance, we then have a positive relation between portfolio inter-temporal volatility and return performance.

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Table 1. Expected returns and standard deviations for the US stockmarket from July 1963 to December 2016 from Fama & French (2018) compared with a constructed normal distribution

Investment Horizon	Fama & French		Normal distribution				
	1	2	3	4	5	6	7
	Fama & French expected return, <i>RET</i> (annualized)	Fama & French standard deviation, <i>SD</i>	Exponential standard deviation (σ) (annualized)	Exponential return (μ) (annualized)	$Exp[\mu + \frac{1}{2}\sigma^2] - 1$ $\mu = \text{col. 4}$ $\sigma = \text{col. 3}$	$Exp[\mu + \frac{1}{2}\sigma^2] - 1$ $\mu = 5.5\%/year$ $\sigma = 15.8\%/year$	$Exp[\mu] - 1$ $\mu = 5.5\%/year$
1 year	7.0%	17%	15.8%	5.5%	7.0%	7.0%	5.65%
3 years	23% (7.1%)	37%	29% (17%)	5.5%	23% (7.1%)	22% (7.0%)	18% (5.65%)
5 years	45% (7.7%)	59%	39.0% (17.5%)	5.9%	45% (7.7%)	40% (7.0%)	32% (5.65%)
10 years	133% (8.8%)	149%	58.5% (18.5%)	6.7%	133% (8.8%)	96% (7.0%)	73% (5.65%)
20 years	610% (10.3%)	658%	79.0% (17.6%)	8.25%	610% (10.3%)	286% (7.0%)	200% (5.65%)
30 years	2,139% (10.9%)	2,551%	91.0% (16.6%)	9.0%	2,139% (10.9%)	657% (7.0%)	421% (5.65%)

Note: Numerical columns 1 and 2, respectively, are the expected returns and standard deviations for the US stockmarket from July 1963 to December 2016 from Fama and French (2018). Numerical columns 3 and 4 are the implied expected standard deviation (σ) and mean return (μ) of exponential returns assuming a normal distribution of exponential returns. Numerical column 5 presents the predicted outcomes assuming a normal distribution of exponential returns with the outcomes for standard deviation (σ) and mean return (μ) as columns 3 and 4, respectively. Column 6 presents the predicted outcomes with the one year outcomes: $\mu = 5.5\%$ and $\sigma = 15.8\%$, while column 7 presents the predicted outcomes with $\mu = 5.5\%$ and $\sigma = 0$.

Table 2: Equity premiums from Fama & French (2018) for July 1963 to December 2016 compared with a normal distribution of exponential growth outcomes.

Investment Horizon	Neg		5% percentile		95% percentile	
	1	2	3	4	5	6
	Fama & French	Normal distribution	Fama & French	Normal distribution	Fama & French	Normal distribution
Monthly	41.3%	46.0%	-7.0%	-7.0%	7.0%	8.3%
1 year	36.0%	36.4%	-20%	-18.5%	36%	37%
3 years	28.5	27.3%	-30%	-25%	90%	85%
5 years	23.4	21.8%	-35%	-26%	154%	135%
10 years	15.6%	13.5%	-44%	-24%	414%	294%
20 years	7.9%	6.0%	-38%	-6.0%	1,857%	860%
30 years	4.1%	2.8%	+37%	+25.5%	6,673%	2,062%

Note: The F&F columns are from Fama and French (2018). “Neg” denotes the probability of underscoring the T-bill rate, and the 5% and 95% percentiles are the boundary outcomes at this level of probability for their simulations of historical market outcomes. The “Normal distribution” columns are the corresponding outcomes assuming a normal distribution of exponential returns ($\mu = 5.5\%$ and $\sigma = 15.8\%$ per annum).

Table 3: Market volatility versus dispersion of discrete monthly outcome returns

Category	Time-series monthly volatility (<i>VOL</i>)	Cross-sectional monthly standard deviation (<i>SD</i>)
ALL	5.1%	28%
LARGE	3.9%	12%
SMALL	5.65%	32%

Note: The relation between average market monthly inter-temporal (time-series) volatility (*VOL*) and the average dispersion (standard deviation) (*SD*) of monthly discrete returns: for stocks of all firms, large firms and small firms. Large firms are the largest 25% of firms by market capitalization in the ASX following Datastream and small firms are the remaining 75% of firms.

Table 4: Dispersion of monthly outcome discrete returns versus market volatility

Category	Correlation	R ²	Coefficient (β)	Intercept (α)
ALL	34%	12%	0.48 (1.15)	0.19*** (8.77)
LARGE	71%	51%	0.82*** (3.20)	0.10*** (9.60)
SMALL	12%	1.5%	-0.70 (-0.39)	0.24** (2.28)

Note: The table reports regression output for Equation (12): $SD_{P,t} = \alpha + \beta(VOL_{P,t}) + \varepsilon_{P,t}$. Monthly observations for the period January 2006 – December 2017 are used to estimate the model. In the table, *Correlation* is the correlation coefficient between two variables, R^2 is the coefficient of determination, and *Coefficient* and *Intercept* are the β and α parameters generated by the model. The first row is for all firms; the second row for large firms and the final row for small firms. Large firms are the largest 25% of firms by market capitalization in the ASX, and small firms are the remaining 75% of firms. The *t-statics* are reported in parentheses. The ***, **, * indicate statistical significance at the 99%, 95%, 90% level, respectively.

Table 5: Monthly portfolio performance versus dispersion of monthly outcomes

Category	Correlation	R ²	Coefficient (γ)	Intercept (α)
ALL	75%	57%	0.88*** (13.72)	0.0031** (1.92)
LARGE	88%	77%	0.39*** (21.68)	0.0042*** (14.53)
SMALL	75%	56%	0.98*** (13.47)	0.0018 (0.820)

Note: The table reports regression output for Equation (13): $R_{p,t} - \mu_{p,t} = \alpha + \gamma \left(\frac{1}{2} \sigma_{p,t}^2 \right) + \varepsilon_{p,t}$. Monthly observations for the period January 2006 – December 2017 are used to estimate the model. In the table, *Correlation* is the correlation coefficient between two variables, R^2 is the coefficient of determination, and *Coefficient* and *Intercept* are the γ and α parameters generated by the model. The rows are for all firms, large firms, and small firms. Large firms are the largest 25% of firms by market capitalization in the ASX, and small firms are the remaining 75% of firms. The *t-statistics* are reported in parentheses. The ***, **, * indicate statistical significance at the 99%, 95%, 90% level, respectively.

Table 6: Monthly portfolio performance vs dispersion of monthly outcomes for stocks with positive returns and negative returns separately

Category	Correlation	R ²	Coefficient (γ)	Intercept (α)
Returns +ve	80%	64%	2.0*** (15.78)	-0.009** (2.02)
Returns -ve	90%	81%	0.45*** (24.76)	0.005*** (11.64)

Note: The table reports regression output for Equation (13): $R_{p,t} - \mu_{p,t} = \alpha + \gamma \left(\frac{1}{2} \sigma_{p,t}^2 \right) + \varepsilon_{p,t}$. Monthly observations for the period January 2006 – December 2017 are used to estimate the model. For each month, we create two samples of stocks: (i) a sample with positive returns only (first numerical row) and (ii) a sample with negative returns only (second numerical row). In the table, *Correlation* is the correlation coefficient between two variables, *R²* is the coefficient of determination, and *Coefficient* and *Intercept* are the γ and α parameters generated by the model. The *t-statistics* are reported in parentheses. The ***, **, * indicate statistical significance at the 99%, 95%, 90% level, respectively.