Where’s My Change?

CECL Allowance Change Attribution

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On a quarterly basis, US insurance companies establish a forward-looking Credit Loss Allowance as part of CECL (Current Expected Credit Loss) regulations (see “References”, FASB, 2016). A method to attribute the quarterly change in the Allowance to the underlying risk factors of Exposure at Default (EAD), Probability of Default (PD), and Loss Given Default (LGD) will be presented in this paper. We will show that the Allowance Change Attribution can be expressed as simple functions of the changes in EAD, PD, and LGD, which can easily be implemented in any spreadsheet, accounting system or credit risk management platform.

CECL regulations prescribe that the Credit Loss Allowance should be calculated as the product of Probability of Default (PD) of the borrowers, counterparties or issuers, the Loss Given Default (LGD) and the Exposure At Default (EAD):

\[
\text{Credit Loss Allowance} = \text{PD} \times \text{LGD} \times \text{EAD}
\]

In this paper we will use the following hypothetical example to illustrate and illuminate the proposed Allowance Change Attribution methodology:

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>EAD</td>
</tr>
<tr>
<td>Q1</td>
</tr>
<tr>
<td>Q2</td>
</tr>
<tr>
<td>Delta</td>
</tr>
</tbody>
</table>

In the example, in the quarter, the credit loss Allowance for the portfolio increased by $2.55M or 17% from $15M to $17.55M as the net result of the impact of a 10% decrease of EAD from $500M to $450M, a 20% increase in PD from 5% to 6%, and an 8.3% increase in LGD from 60% to 65%.

In order to attribute the 17% increase in the allowance to the changes in the underlying risk factors of PD, LGD and EAD, we will apply a simple waterfall approach. Starting from the $15M allowance at the beginning of the quarter, we could attribute an allowance decline of $1.5M coming from the 10% decrease in EAD, resulting in a $13.5M intermediate result. Applying the PD increase of 20% to this
intermediate result adds $2.7M and produces a second intermediate result of $16.2M. As a final step we incorporate the 8.3% LGD increase which adds $1.35M to the second intermediate result, producing the final result of $17.55M, which is equal to the allowance at the end of the quarter. These attribution results are shown in the following Figure 1:

![Simple Allowance Change Attribution](image)

*Figure 1*

The problem with this approach is that if we change the attribution order, e.g. if rather than considering the EAD impact first, PD impact second, and LGD impact third, we consider the PD impact first, EAD impact second, and LGD impact third, we obtain different attribution results. In fact, the attribution results for all six permutations of the triad of EAD, PD, LGD changes are shown in the following Table 2:

<table>
<thead>
<tr>
<th>Order</th>
<th>First Factor</th>
<th>Second Factor</th>
<th>Third Factor</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>EAD, PD, LGD</td>
<td>-1,500,000</td>
<td>2,700,000</td>
<td>1,350,000</td>
<td>2,550,000</td>
</tr>
<tr>
<td>EAD, LGD, PD</td>
<td>-1,500,000</td>
<td>1,125,000</td>
<td>2,925,000</td>
<td>2,550,000</td>
</tr>
<tr>
<td>PD, EAD, LGD</td>
<td>3,000,000</td>
<td>-1,800,000</td>
<td>1,350,000</td>
<td>2,550,000</td>
</tr>
<tr>
<td>PD, LGD, EAD</td>
<td>3,000,000</td>
<td>1,500,000</td>
<td>-1,950,000</td>
<td>2,550,000</td>
</tr>
<tr>
<td>LGD, EAD, PD</td>
<td>1,250,000</td>
<td>-1,625,000</td>
<td>2,925,000</td>
<td>2,550,000</td>
</tr>
<tr>
<td>LGD, PD, EAD</td>
<td>1,250,000</td>
<td>3,250,000</td>
<td>-1,950,000</td>
<td>2,550,000</td>
</tr>
</tbody>
</table>

The EAD change attributions to the allowance change are -$1.5M, -$1.5M, -$1.8M, -$1.95M, -$1.625M and -$1.95M for the six different permutations. We suggest taking the average value of these six
To obtain an attribution that is order invariant, i.e. the attribution does not depend on the attribution order, which is -$1.72M or -11.5% of the initial allowance of $15M. Following the same logic, we obtain allowance attributions of $2.97M for the change in PD (19.8% of the initial allowance) and $1.3M for the change in LGD (8.7% of the initial allowance). Note that the aggregate of these three attribution values equals the quarterly allowance change of $2.55M.

The following Figure 2 shows the resulting attribution of the allowance change to the underlying EAD, PD and LGD impacts:

![Figure 2](image)

The quarterly allowance change of $2.55M can therefore be attributed to the change in EAD for -$1.72M, to the change in PD for $2.97M and to the change in LGD for $1.3M.

In order to avoid having to go through this procedure, it might be worth finding analytical expressions for the EAD, PD and LGD change attributions, expressed as (hopefully) simple functions of the changes in EAD, PD and LGD. To achieve this, we utilize the implicit attribution formulae underlying the above example in Table 2, expressed in terms of the relative changes in EAD, PD and LGD, which we will denote as ΔEAD, ΔPD and ΔLGD in Table 3:
From Table 3 it follows that the allowance change attribution for the change in EAD is:

\[
\text{Equation 1}
\]

\[
\frac{1}{6} \times ( (1+\Delta\text{EAD})^{-1} + (1+\Delta\text{EAD})^{-1} + (1+\Delta\text{PD})(1+\Delta\text{EAD})^{-1}(1+\Delta\text{PD}) + (1+\Delta\text{EAD})(1+\Delta\text{PD})(1+\Delta\text{LGD})^{-1}(1+\Delta\text{PD}) + (1+\Delta\text{LGD})(1+\Delta\text{EAD})^{-1}(1+\Delta\text{LGD}) + (1+\Delta\text{EAD})(1+\Delta\text{PD})(1+\Delta\text{LGD})^{-1}(1+\Delta\text{LGD})(1+\Delta\text{PD}) )
\]

Note that Equation 1 is based on selecting and aggregating the appropriate EAD change attributions from Table 3, i.e. we select the first factor from the first two permutations, the second factor from the third and fifth permutations, and the third factor from the fourth and sixth permutations. To obtain the average value we use a multiplication factor of 1/6.

If we expand and simplify Equation 1 (see Annex 1 for details) we obtain the following result for the allowance change attribution of the change in EAD:

\[
\text{Equation 2: Allowance Change Attribution for EAD Impact}
\]

\[
\Delta\text{EAD} + \Delta\text{EAD} \times \Delta\text{PD} / 2 + \Delta\text{EAD} \times \Delta\text{LGD} / 2 + \Delta\text{EAD} \times \Delta\text{PD} \times \Delta\text{LGD} / 3
\]

Likewise, the allowance change attribution for the change in PD and for the change in LGD are as follows:

\[
\text{Equation 3: Allowance Change Attribution for PD Impact}
\]

\[
\Delta\text{PD} + \Delta\text{PD} \times \Delta\text{EAD} / 2 + \Delta\text{PD} \times \Delta\text{LGD} / 2 + \Delta\text{EAD} \times \Delta\text{PD} \times \Delta\text{LGD} / 3
\]

\[
\text{Equation 4: Allowance Change Attribution for LGD Impact}
\]

\[
\Delta\text{LGD} + \Delta\text{LGD} \times \Delta\text{EAD} / 2 + \Delta\text{LGD} \times \Delta\text{PD} / 2 + \Delta\text{EAD} \times \Delta\text{PD} \times \Delta\text{LGD} / 3
\]
Analyzing the attribution results provided in Equation 2, Equation 3 and Equation 4, it is convenient that most of the factors in Equation 1 cancel each other out in the process of simplifying the equation.

It is also insightful to observe that most of the attribution is related to the individual changes of the factors, i.e. ΔEAD in Equation 2, ΔPD in Equation 3, and ΔLGD in Equation 4. The other elements in the equations are multiplications of two or three factor changes, the results of which are typically much smaller than the individual factor changes. This is intuitive, because one would expect that ΔEAD, ΔPD and ΔLGD drive most of the allowance changes without a similarly large contribution from other factors.

Also note that the sum of equations 2, 3, and 4 is equal to the allowance change (see Annex 2 for details), which is obviously a requirement of an allowance change attribution system.

Continuing our example to demonstrate how this works, the allowance change attribution coming from EAD, according to Equation 2 is:

\[-10\% + -10\% \times 20\% / 2 + -10\% \times 8.3\% / 2 + -10\% \times 20\% \times 8.3\% / 3 = -11.5\%\]

The dollar equivalent is -11.5\% \times \$15M = -\$1.72M.

The allowance change attribution from PD, according to Equation 3 is equal to:

\[20\% + 20\% \times -10\% / 2 + 20\% \times 8.3\% / 2 + -10\% \times 20\% \times 8.3\% / 3 = 19.8\%\]

The dollar equivalent is: 19.8\% \times \$15M = \$2.97M.

For LGD we obtain the following allowance change attribution, using Equation 4:

\[8.3\% + 8.3\% \times -10\% / 2 + 8.3\% \times 20\% / 2 + -10\% \times 20\% \times 8.3\% / 3 = 8.7\%\]

The dollar equivalent is: 8.7\% \times \$15M = \$1.30M.

Concluding, in this paper we have provided a method for CECL Allowance Change Attribution which is based on simple functions of changes in EAD, PD, and LGD that can easily be implemented in most systems that can perform additions and multiplications.

Disclaimer: The views and opinions expressed in this article are those of the author and do not necessarily reflect the official policy or position of Voya Financial or any of its affiliated companies.
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ANNEX 1

The allowance change allocation for the EAD impact according to Equation 1 is:

\[
\frac{1}{6} \times \left( \frac{1}{1+\Delta EAD} + \frac{1}{1+\Delta EAD} + \frac{1+\Delta PD}{1+\Delta EAD} \left( \frac{1}{1+\Delta PD} + \frac{1+\Delta PD}{1+\Delta LGD} \right) + \frac{1+\Delta PD}{1+\Delta EAD} \left( \frac{1}{1+\Delta LGD} + \frac{1}{1+\Delta LGD} \right) \right) =
\]

Simplify the notation with \( E = \Delta EAD \), \( P = \Delta PD \) and \( L = \Delta LGD \), we will expand the equation to simplify the result as follows:

\[
\frac{1}{6} \times \left( \frac{1}{1+E} + \frac{1}{1+E} + \frac{1+P}{1+E} \left( \frac{1}{1+P} + \frac{1+P}{1+L} + \frac{1+P}{1+L} \right) + \frac{1+P}{1+E} \left( \frac{1}{1+L} + \frac{1}{1+L} \right) \right) =
\]

\[
\frac{1}{6} \times \left( \frac{2E+EP}{1+E} + \frac{EP}{1+P+L+PL} \right) =
\]

\[
\frac{1}{6} \times \left( 3E+EP+3E+EP \right) =
\]

\[
\frac{1}{6} \times \left( 6E+3E+EP+3E+EP \right) =
\]

\[
E + EP / 2 + EL / 2 + EPL / 3
\]

This result is also shown in Equation 2 while substituting back \( \Delta EAD \) for \( E \), \( \Delta PD \) for \( P \) and \( \Delta LGD \) for \( L \).
In this annex we will show that the allowance change attributions of EAD, PD and LGD as derived and shown in Equations 2, 3, and 4, sum to the allowance change.

The change in allowance is given by the following equation:

\[(1+\Delta EAD) \times (1+\Delta PD) \times (1+\Delta LGD) - 1\]

In the example we used in the article, this resulted in an allowance change of \((1-10\%) \times (1+20\%) \times (1+8.3\%) - 1 = 17\%\).

Simplifying the notation using \(E = \Delta EAD\), \(P = \Delta PD\) and \(L = \Delta LGD\), we will expand the equation to simplify the result as follows:

\[(1+E) \times (1+P) \times (1+L) - 1 = (1+E) \times (1+P+L+PL) = 1+P+L+P+P+L+PE+EL+EPL-1 = E+P+L+PE+EL+PL+EPL\]

This result is also what we should get when adding Equations 2, 3 and 4, as follows:

\[E + EP / 2 + EL / 2 + EPL / 3 + P + PE / 2 + PL / 2 + EPL / 3 + L + LE / 2 + LP / 2 + EPL / 3 = E+P+L+EP+EL+PL+EPL\]