HOW TO MAKE ECONOMIC TIME-SERIES-BASED RISK MODELS MORE ACCURATE AND TIME SENSITIVE

BY:
Dr. Vikash Kumar Sharma and Animesh Mandal

PROBLEM STATEMENT

Migration of flat capital requirements of Basel I to more risk-sensitive and granular capital requirements of Basel II enabled a bottom-up exposure level risk assessment through internal estimates of the probability of default (PD), the loss given default (LGD) and the exposure at default (EAD). This internal ratings-based (IRB) approaches allows banks to compute capital charges for each exposure and subsequently roll up to various aggregated levels. The risk associated with business cycle fluctuations is captured by procyclic nature of these internal estimates. The embedment of pro-cyclicality is managed by through-the-cycle adjustment of economic fluctuations.

$PD_{it_{TTC}} = F_1\ (\beta_1 customer\ profile_{it} + \beta_2 product\ profile_{it} + \ \beta_3 economic\ fluctuations_t)$ ...... (1)

$LGD_{it_{TTC}} = F_2\ (\gamma_1 facility\ profile_{it} + \gamma_2 recovery\ trend_{it} + \ \gamma_3 economic\ fluctuations_t)$ ...... (2)

$EAD_{it_{TTC}} = F_3\ (\delta_1 facility\ profile_{it} + \delta_2 product\ profile_{it} + \ \delta_3 economic\ fluctuations_t)$ ...... (3)

Where, ‘i’ refers to i-th exposure/customer and ‘t’ refers to current time.

Under the IRB approach of Basel II, capital requirements are an increasing function of the PD, LGD and EAD parameters estimated for each borrower, and these inputs are likely to rise in downturns. An increase in the PD from 1% to 3% increases the capital requirement for corporate asset class exposures from 6.21% to 9.32%. Clearly, a jump of 50% in required capital would need to be accommodated in the middle of a recession. The worsening of borrowers' creditworthiness in recession will increase the requirement of capital for banks and lead to severe contraction in the supply of credit. At the same time, there can be a complementary concern of lower than normal capital requirements during economic booms, which may contribute to emergence of inflated credit and asset pricing and bubbles.

Unusual fluctuations in capital requirements under IRB approach of Basel II through economic boom, and bust cycle are direct contributions of the economic fluctuation component embedded in internal risk parameter estimates (refer to equation 1, 2, and 3). This calls for a procedure that corrects the bias towards exacerbating the inherent cyclicality of economic performance. The question is therefore, how should the pro-cyclicality problem be addressed without discounting the cyclical risk-sensitivity of Basel II IRB approach capital computation standard?

APPROACH

The point-in-time estimates of the probability of default (PD), the loss given default (LGD), and the exposure at default (EAD) are specific to the idiosyncratic nature of risk and derived from interpretations of historical data. A point-in-time estimate assesses the likelihood of an event over a future period, most often as 1 year. Accurate point-in-time estimates of these risk parameters describe an expectation of the future, starting from the current situation and integrating all relevant cyclical changes and all components of idiosyncratic risks. Through-the-cycle estimates, in contrast, reflect circumstances anticipated over a long period in which effects of the credit cycle would average close to zero. Point-in-time estimates are essential to the day-to-day management of credit risk. However, Basel II calls for long-run average (analogous to through-the-cycle) estimates of risk parameters in calculation of regulatory capital. In practice there are several methods of determining through-the-cycle risk estimate, namely:

- The variable scalar approach
- Structural through-the-cycle model
- Hybrid model

The variable scalar approach consists of converting the PIT risk estimates to TTC risk estimates via a scalar that varies throughout the credit cycle. In a benign period with low credit losses, the scalar will adjust the PIT estimates upwards to match the TTC estimates. In a downturn period with high credit losses, the adjustment will be downwards.

The second approach consists of building a separate TTC estimates rather than adjusting existing PIT estimates. Under this approach, TTC estimates are determined by utilizing macroeconomic variables and non-cyclical risk drivers to predict default rates over an economic cycle.

A hybrid approach considers a separate TTC estimates by considering macroeconomic variables and applies a static scalar on PIT estimates to bring in credit cycle adjustment.

Variable scalar approach requires TTC estimates to be recalibrated once every year. Structural modeling approach fails to establish connection between PIT and TTC estimates. Although hybrid approach requires data covering a full economic cycle, it’s more prevalent in the context of through-the-cycle adjustments.

Macroeconomic variables are cyclical indicators, e.g. GDP growth or interest rates, and financial market indicators, e.g. stock market prices and stock market volatility. The time series macroeconomic data have four components:

1. Secular trend – Upward or downturn trend for a period of years and this may be due to factors like increase in population, change in technological progress, large scale shift in consumer demands etc.
2. Seasonal variation – Short-term fluctuation in a time series which occurs periodically every year. The major factors that are responsible for the repetitive pattern of seasonal variations are weather conditions, customs of people etc.
3. Cyclical variation – Recurrent upwards or downward movements in a time series but the period of cycle is more than one year. Also these variations aren't as regular as seasonal variations. There can be different type of cycles of varying length and size.
4. Irregular variation – Fluctuations in time series that are short in duration, erratic in nature and follow no regularity in the occurrence pattern. These variations are also referred to as residual variations since by definition they represent what is left out in a time series after trend, cyclical, and seasonal variations.

If it can be assumed that, irregular variation is infrequent, macroeconomic time series can be defined as:

Time series = trend (or growth) component + cycle component ...... (4)

Before using macroeconomic time series data in through-the-cycle modeling, it is essential to separate the cyclical component of the time series from the raw data. Appropriate procedure is to be applied to obtain a smoothed-curve representation of macroeconomic time series, one that is more accurate and sensitive to long-term than to short-term fluctuations, i.e. de-trending the data and represent the underlying trend. Without any consensus about which model represents the trend best, a popular alternative to model-based de-trending is to use smoothening filters.

In capitalist economies, aggregate economic variables experience repeated fluctuations about their long-term growth paths. One of the maintained hypotheses in macroeconomics, based upon growth theory considerations, is that the growth component of aggregate macroeconomic time series varies smoothly over time.

- Growth is characterized by roughly proportional growth in (per-capita) output, investment, consumption, capital shock and productivity, and little change in the hours of employment per capita or household
- The cyclical variations in output arise principally as the result of changes in cyclical hours of employment and not as the result of changes in cyclical productivity or capital shocks

HP (HODRICK PRESCOTT) FILTER

INTRODUCTION

The goal of the filter is to serve as a means to separate the cyclical component in the output from the growth component. Assuming time series for an aggregate macroeconomic variable, y_t which is composed of both a growth component (non-stationary trend), g_t and a cyclical component c_t, so that

$$y_t = g_t + c_t \qquad \text{for t = 1, ..., T.} \quad \text{...... (5)}$$

The measure of the smoothness of the $g_t$series is the sum of squares of its second difference.

The c_t series represents a deviation from the g_t series but under the framework considered, the long-run average of the deviation is zero.

The actual filtering technique is therefore represented as an optimization problem where g_t, the filtered “growth” component, is chosen solve

$$\min_{\{g_t\}_{t=-1}^{T}} \{\sum_{t=1}^{T} c_t{}^2 + \lambda \sum_{t=1}^{T} [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2\} \text{...... (6)}$$

Where λ is a positive number which penalizes variability in the growth component g_t. The larger the value of λ chosen prior to optimization, the smoother is the g_t series provided as the result of filtering. Conversely, a smaller λ chosen will provide a relatively more volatile $g_t$filtrate.

The typical λ recommended by both Hodrick and Prescott is $\lambda = 1600$. Ravn and Uhlig (2002) researched the important question of his this parameter should be adjusted for data with varied time intervals. It has been concluded that, for quarterly time series data $\lambda = 1600$ should be considered and $\lambda = 6.25$ for annual times series data, and $\lambda = 129{,}600$ for monthly time series data should be considered.

APPLICATION

HP filter smoothening procedure has been applied on real time data to analyze the benefit. This paper explains the application steps of HP filter smoothening procedure on through-the-cycle probability of default (PD) models across all retail assess classes i.e. credit card (revolving), home loan (mortgage), and personal loan (other). HP filter considers macroeconomic time series data and splits each time series into trend and cyclicality components and explains cyclicality in terms of trend.

Firstly, smoothening of macroeconomic time series has been performed by applying HP filter procedure and then macroeconomic models (linear only exogenous parameter based or dual-time dynamics models) are fit. HP filter application procedure involves following step-wise actions:

1. Consider raw time series data and compute the growth rate
2. Divide the growth rate time series data in to trend and cyclicality components. (HP filter smoothening procedure can also be applied on raw time series data without converting the same to growth rate)

   Trend = $(y_t - g_t)$ ...... (7)
   Cyclicality = $[(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]$ ...... (8)

   Where, y_t stands for actual growth rate and g_t stands for smoothened trend for all $t = 1, 2 \ldots \ldots T$. Trend captures the variability in the time series data whereas cyclicality can be of different periodicity to that of trend. HP filter captures cyclicality in terms of trend (g_t) by considering a second order derivative (equation 8). In order to ensure convergence of the objective function, squares of trend and cyclicality are considered in the objective function.

3. Develop the objective function as

$$\{\sum_{t=1}^{T} c_t^{\,2} + \lambda \sum_{t=1}^{T} [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2\} \text{ ...... (9)}$$

The target is to minimize the objective function with respect to g_t , with plan to minimize $(y_t - g_t)$ and $[(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]$ and compute g_t for all $t = 1, 2 \dots \dots T$

4. Since through-the-cycle probability of default (PD) models are developed on five or more years of macroeconomic time series data (corresponding to observed default rate data), usage of HP filter smoothened g_t in model development may result in over-fitting of the model. To counter it and to evaluate TTC $g_{t_{ij}}$, g_t is regressed against y_t, to determine

$$g_{t_{ij}} = \alpha_i + \beta_i \, y_{t_{ij}} \text{ ...... (10)}$$

$g_{t_{ij}}$ for all macroeconomic variables $i = 1, 2 \dots .. N$ are computed and the same are used for modeling fitting procedure. Where $j = 1, 2 \dots . M$ represents number of data points in a time series.

5. The final one dimensional only exogenous factor based through-the-cycle probability of default (PD) model would be as:

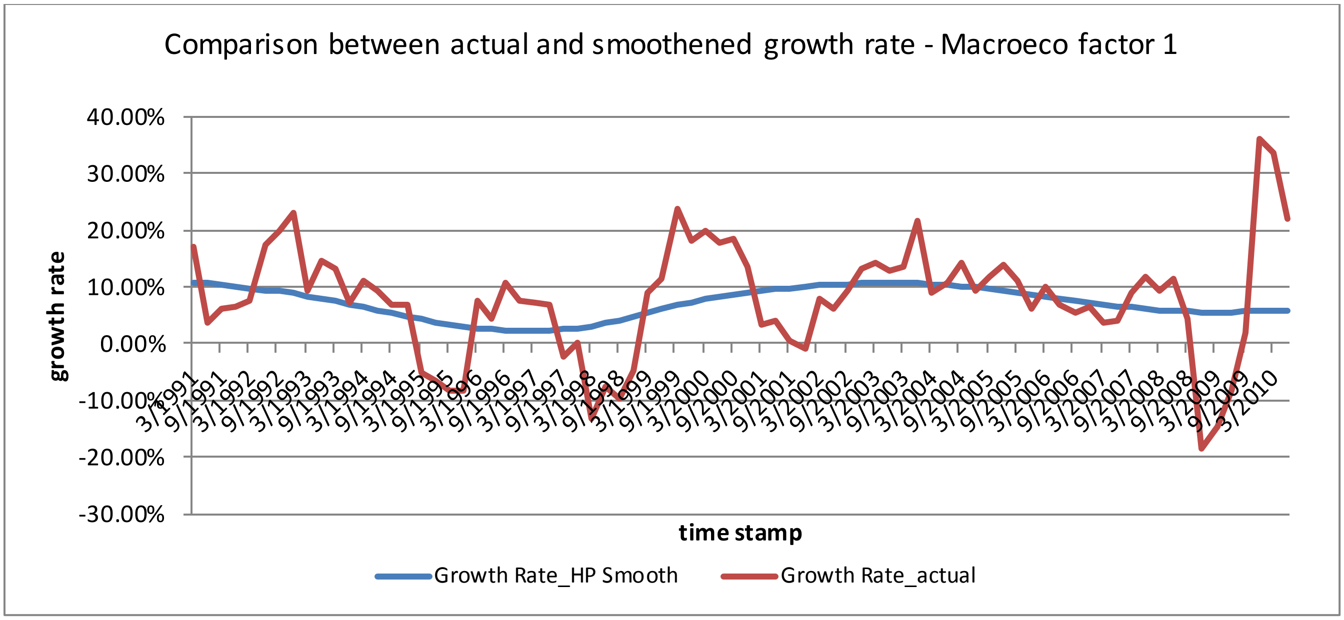
$$PD(TTC) = \gamma_0 + \gamma_1 g_{t_1} + \gamma_2 g_{t_2} + \cdots \text{ ...... (11)}$$

Where, $g_{t_1} = \alpha_1 + \beta_1 \, y_{t_1}$,
$g_{t_2} = \alpha_2 + \beta_2 \, y_{t_2}$ etc.

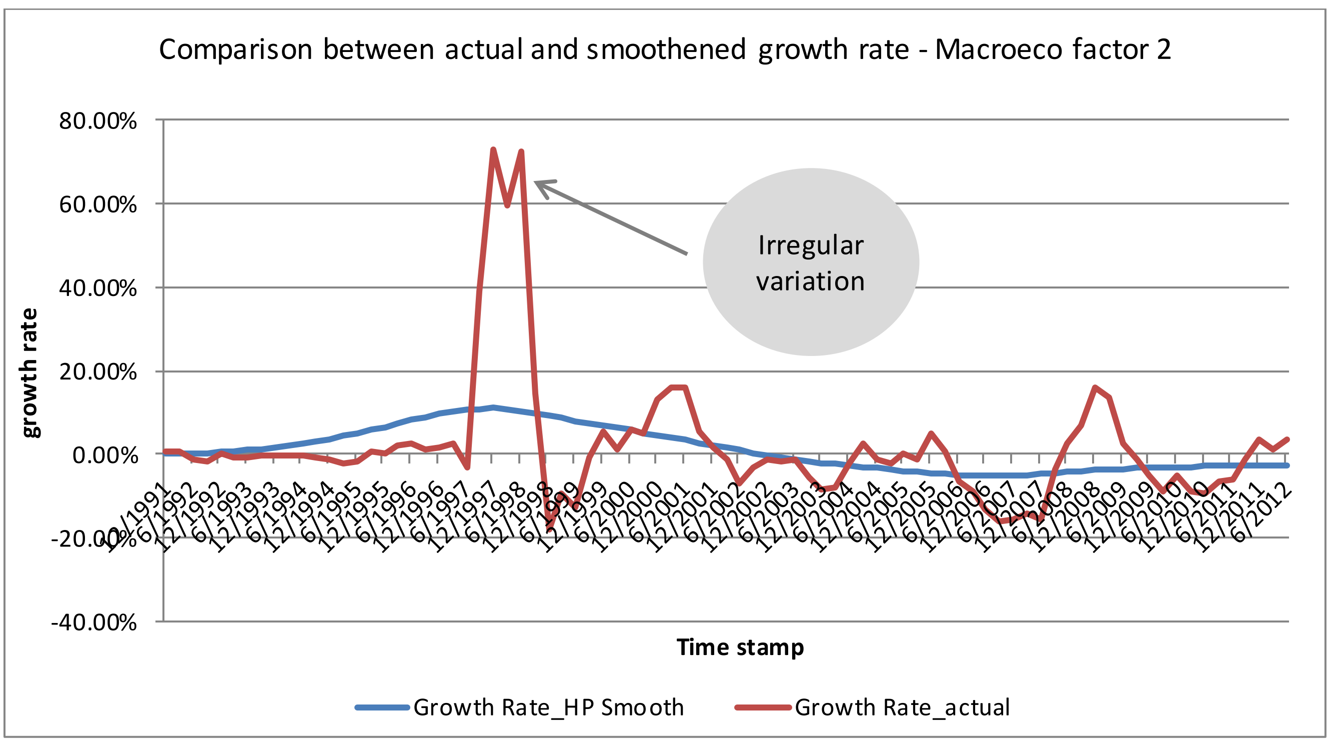
INTERPRETATION & RESULTS

Under this section the comparative analysis between existing through-the-cycle probability of default (PD) model results and HP filter smoothened existing through-the-cycle probability of default (PD) model results are demonstrated. In order to comprehensively demonstrate the benefit of HP filter smoothening process, comparative analyses covering all retail asset classes are provided

STEP 1: IMPACT OF TIME SMOOTHENING ON TIME SERIES DATA



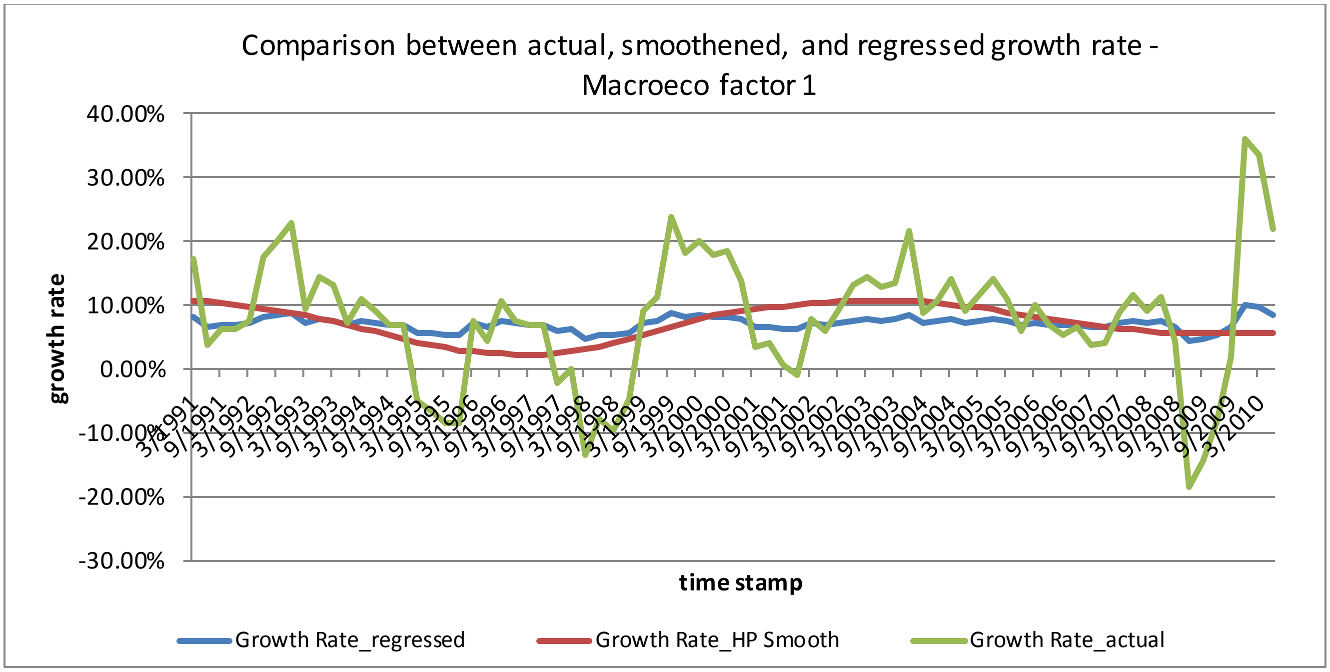
Picture 1: comparison between actual and smoothened growth rate of a macroeconomic factor



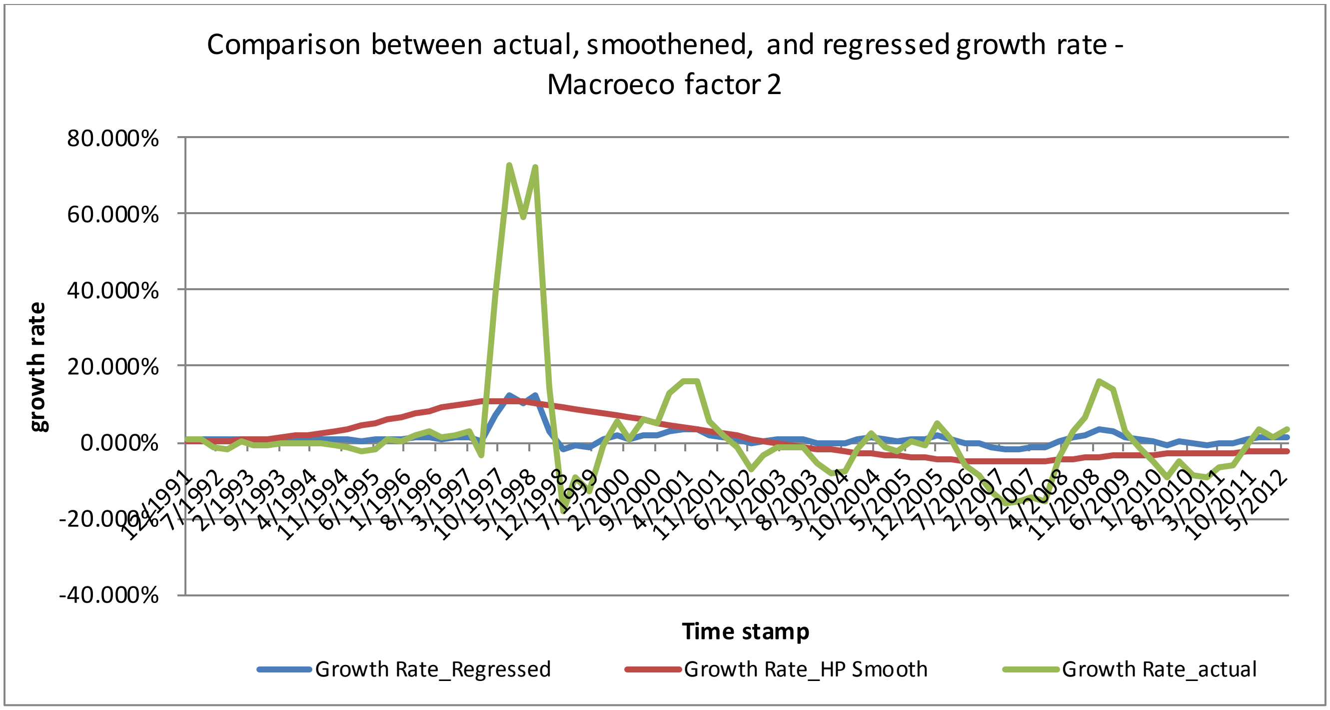
Picture 2: comparison between actual and smoothened growth rate of a macroeconomic factor

As pointed out in picture 2, HP filter smoothening process treats existing irregularity in macroeconomic time series yet maintains the trend.

Above two pictures demonstrates how application of HP filter smoothening procedure accounts for unusual cyclicality and at times irregular variations existing in macroeconomic time series data, to produce smoothened time series. However, usage of this smoothened time series in through-the-cycle model may result in over-fitting since the model fitting procedure only considers five to seven years of macroeconomic time series data. Hence smoothened time series is regressed against original time series to compute regression coefficients. These coefficients are then used to derive modified through-the-cycle macroeconomic time series, and the same is used in model developed process as independent variable.



Picture 3: comparison between actual, smoothened, and regressed growth rate of a macroeconomic factor



Picture 4: comparison between actual, smoothened, and regressed growth rate of a macroeconomic factor

Benefit of smoothened and regressed macroeconomic time series is that, it ensures the trend and cyclicality in the time series even after discounting all unusual and irregular variations.

STEP 2: IMPACT OF HP FILTER SMOOTHENED AND REGRESSED TIME SERIES IN MODEL FITMENT & OUTCOME

As specified earlier, four through-the-cycle probability of default (PD) models are refit by using HP filter smoothened and regressed macroeconomic time series data. These through-the-cycle models span across all retail asset classes. The model fitment comparison and accuracy analysis are as following:

Credit Card Portfolio – 1							
	Average Default rate	Computed TTC PD	Model R - square	Model Error - Intercept	Model Error – variable 1	Model Error – variable 2	Model Error – variable 3
Existing TTC model	2.28%	2.32%	90.52%	2.63%	2.71%	2.76%	2.76%
HP Filter applied TTC Model	2.28%	2.31%	90.57%	2.50%	2.58%	2.62%	2.62%

Table 1: Comparative accuracy analysis between existing TTC model and HP filter smoothened TTC model for a credit card portfolio

Credit Card Portfolio – 2							
	Average Default rate	Computed TTC PD	Model R - square	Model Error - Intercept	Model Error – variable 1	Model Error – variable 2	Model Error – variable 3
Existing TTC model	1.59%	2.02%	82.35%	3.17%	3.34%	3.37%	3.40%
HP Filter applied TTC Model	1.59%	1.93%	82.44%	2.53%	2.67%	2.69%	2.71%

Table 2: Comparative accuracy analysis between existing TTC model and HP filter smoothened TTC model for a credit card portfolio

Retail Home Loan Portfolio							
	Average Default rate	Computed TTC PD	Model R - square	Model Error - Intercept	Model Error – variable 1	Model Error – variable 2	Model Error – variable 3
Existing TTC model	3.34%	4.22%	92.48%	1.22%	1.37%	1.33%	2.76%
HP Filter applied TTC Model	3.34%	3.93%	92.47%	0.89%	0.96%	1.00%	0.97%

Table 3: Comparative accuracy analysis between existing TTC model and HP filter smoothened TTC model for a retail home loan portfolio

Personal Loan Portfolio							
	Average Default rate	Computed TTC PD	Model R - square	Model Error - Intercept	Model Error – variable 1	Model Error – variable 2	Model Error – variable 3
Existing TTC model	3.47%	3.63%	95.30%	1.20%	1.31%	1.55%	1.60%
HP Filter applied TTC Model	3.47%	3.55%	95.30%	1.00%	1.09%	1.29%	1.33%

Table 4: Comparative accuracy analysis between existing TTC model and HP filter smoothened TTC model for a personal loan portfolio

STEP 3: KEY TAKEAWAYS FROM ACCURACY ANALYSIS COMPARISON

Finding a smoothened trend of a time series have been notoriously difficult as result of a smoothened series can take an unexpected path. The comparative accuracy analysis between existing and HP filter smoothened through-the-cycle probability of default (PD) models across all retail classes demonstrate the following trend:

- Final projected through-the-cycle probability of default (PD) consolidates down towards portfolio average default rate. It opens up opportunity for banks to manage enhanced risk sensitive and lower capital requirements
- Model R-square values either remain unchanged or get better. It doesn't go too far to indicate any potential over-fitting challenge.
- The error terms of independent variables as well intercept go down significantly to demonstrate increased significance and predictive power of these

CONCLUSION

It has been widely observed that unusual fluctuations in capital requirements under IRB approach of Basel II through economic boom, and bust cycle is primarily contributed by untamed and irregular fluctuations in economic time series data. As demonstrated in this paper, HP filter smoothening technique has been able to successfully eliminate the superfluous and irregular fluctuations in macroeconomic time series data, yet maintain the trend and cyclicality of the data to foster long-run effects on internal risk estimates. Considering the results achieved by HP filter smoothening process, the same can be effectively applied on all type of time series modeling approach such as stress testing, credit pricing etc.

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