CVA capital charge under Basel III standardized approach

An explanation document

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By

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Abstract[2]

Since the 2007 – 2009, Counterparty Credit Risk (CCR) has become one of the biggest issues and challenges for financial institutions.

As the crisis revealed shortcomings and loopholes in managing CCR, and more specifically CVA risk, new regulations have been issued in the sole intent of capturing this risk and building an extra cushion of capital to absorb losses and consequently to strengthen the resilience of the banking industry.

Basel III framework proposes two ways for measuring CVA Risk: a standardized approach and an advanced approach.

In this paper, the standardized approach will be analyzed and studied. At first, an analysis will be provided to better understand why CCR became so important, what are its characteristics, etc…. Then a discussion around the CVA definition from the regulator's perspective will be presented. Finally, a paragraph will be dedicated to better understand what the standardized formula refers to, what is being computed, and for what purpose.

Our decision to focus on the treatment of counterparty risk in Basel III -*standard method only*- can be explained by three major observations:

1) A lot of literature already exists, and a certain number of very good specialists refer to the subject. We do not pretend to add other new elements, in all cases not herein;
2) Few banks actually are able to assess their counterparty risk under some advanced and internal methodologies. The application of the standard method is highly widespread among financial institutions subject to Basel III;
3) Few people, when they need to assess their risk using the standard approach, really take the time to analyze choices and specific assumptions according to this method.

Our main objective here is to help financial institutions better understand how their regulatory capital levels evolve under this approach and the impact on their day to day business.

Keywords: CVA, CCR, CVA capital requirement, VaR, Basel III
JEL Classification : C1, G18, G21, G32

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1. Counterparty Credit Risk, a major concern for today's financial institutions

During the last decade, the world has witnessed the worst financial crisis since the crash of 1929. Its magnitude was significant because it spread from a local financial crisis to a more worldwide economic crisis.

During this period, shortcomings were revealed in identifying and measuring correctly financial risks and what was once a total certainty is now overwhelmed by doubt. In fact, the crisis revealed that Counterparty Credit Risk (CCR) is not to be taken lightly and that in an environment of extreme volatility, losses due to bad CCR management can threaten the very existence of a financial institution and cause dramatic damages on the overall economy.

Consequently, understanding the nature of this risk, its impacts and ways of capturing it efficiently have become a major concern in today's banking activity. So, one can ask himself what CCR is about, why is it so important, and how banks, regulators and governments dealt with this issue?

1.1. What is Counterparty Credit Risk?

The first question one might raise concerning CCR is about its definition and how it defers from Credit Risk.

CCR is defined as the risk that a counterparty defaults before honoring its engagements. This definition is by far too close to Credit Risk definition, suggesting that CCR is a form of Credit Risk. Yet, many differences occur between these two types of risks.

First, while Credit Risk concerns classic instruments (such as mortgage loans, etc…), CCR deals with OTC derivative instruments, repo-style transactions and other securities financing transactions. By the very nature of these instruments, computing exposure is a difficult task. In fact, exposure on such instruments is uncertain because it depends on market factors (such as interest rates, exchange rates, etc…) which can't be predicted with a 100% certainty. For example, if we consider an Interest Rate Swap, its current value can be determined accurately whereas its future value is based on the evolution of floating rates.

Second, CCR is a bilateral risk type. In fact, from a bank's perspective, an OTC derivative can be an asset or a liability, depending on the sign of its value. For example, if we consider an Interest Rate Swap, where the bank pays the floating leg and receives the fixed leg, the instrument's value is positive if the floating rate is below the fixed rate. This means that the instrument is an asset and that the bank carries the CCR. On the opposite, if the floating rate is above the fixed rate, the instrument is a liability and the CCR is carried by the counterparty (no CCR from the bank's perspective).

Third, OTC derivatives are valued using the fair value (whereas instruments such as mortgage loans are valued using the discounted cash flows technique), which means that they are marked-to-market and their value is defined from market participants' perspective. This also means that in a context of market volatility, OTC derivatives values can also be volatile and induce P&L losses. This point will be discussed further.

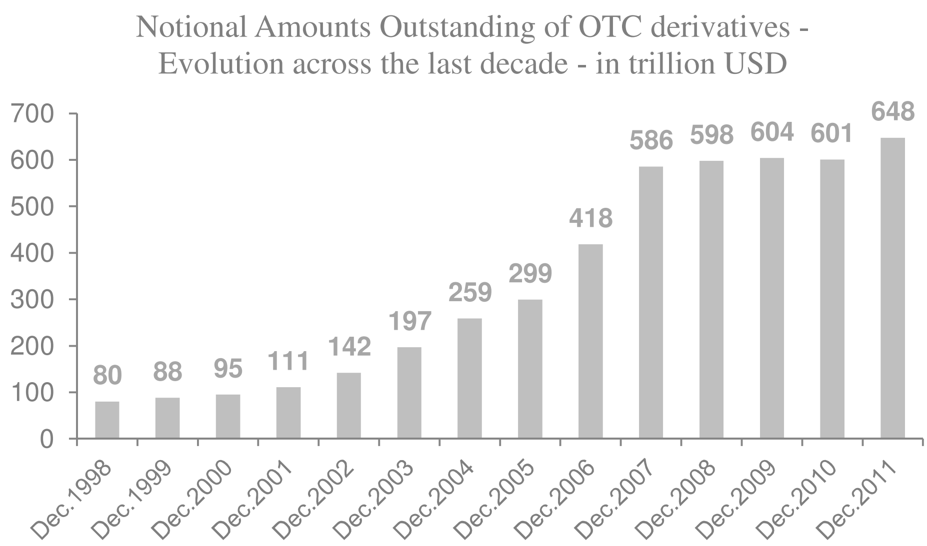
Considering what is said above, it is obvious that dealing with CCR is no easy task because it requires knowledge in Credit Risk and Market Risk as well as understanding synergies and dynamic of interactions between them.

1.2. Why Counterparty Credit Risk became so important?

It is obvious that CCR has become a major concern for banks and its importance has grown significantly since the last crisis. Yet, this doesn't mean that it never existed before. It was just not treated properly.

There are 4 main reasons that place CCR at the heart of today's banks issues.

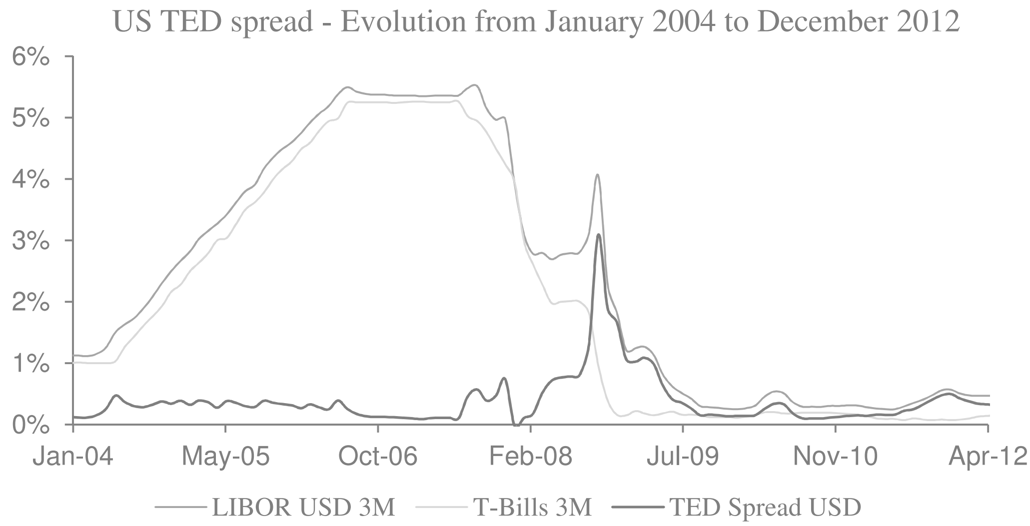
First, during the last decade, the volume of OTC derivative transactions has increased significantly. As shown above, notional amounts increased from 90 trillion USD in 2000 to 670 trillion USD in 2008. As the number and volume of transactions increased drastically, banks' exposure on OTC derivatives became naturally higher and potential losses sufficiently important not to be denied and left unhedged.



Source: Bank for International Settlements

Second, during the last crisis, the world witnessed an unseen confidence crisis with situations of extreme market volatility. Consequently, as OTC derivatives are valued on a fair value basis, the negative perception of market participants had an impact on instruments value where a major decline has been observed. In fact, as credit spreads increased significantly and a liquidity crisis occurred, banks recognized significant losses on OTC derivatives portfolio.

As an illustration of what happened during the crisis, the TED Spread is shown below. It is an indicator of the level of confidence within the financial system. In fact, if the TED Spread increases, this shows that banks are unwilling to lend money to each other and rather prefer to invest in governments bond (considered as risk free). During the crisis, the TED Spread increased significantly with high volatile trends as a sign of extreme turbulence and a lack of confidence. Consequently, Marked-to-Market instruments faced a decrease in their value, which led banks to recognize P&L losses, even without a counterparty default. In fact, the Basel Committee estimates that 75% of losses that occurred on OTC derivatives portfolios during the 2007 – 2009 were not due to counterparty defaults but by rating downgrades and credit spread volatility (which have an impact on Marked-to-market instruments, as discussed above).



Source: GRA analysis

Third, if OTC derivatives did not cause the 2007 – 2009 financial crisis, they are a potential source of systemic risk. In fact, most large financial institutions have huge portfolios of derivatives with other large financial institutions as counterparties. Consequently, if one of these financial institutions fails, other financial institutions will face huge losses. Moreover, the idea of a large financial institution to fail was considered impossible for too long. In fact, market participants always considered that such counterparties would never default or would never be allowed to default ("Too Big To Fail" concept). So, for this reason, CCR was neglected until the 2007 – 2009 crisis where high-profile bankruptcies occurred with the fall of Lehman Brothers and other pseudo-bankruptcies such as AIG, Meryl Lynch, Bear Sterns, Freddie Mac, Fannie Mae…

Fourth, the 2007 – 2009 crisis revealed that banks did not deal efficiently with CCR and neither did regulators. In fact, the Basel Committee for Banking Supervision identified loopholes in the Basel II framework concerning CCR measurement. Accounting rules did as well. Consequently, there was such a social and political pressure to come with more regulations and more constraints to avoid financial institutions from threatening the overall economy again.

1.3. How did the BCBS respond?

The result of such an intense environment was a global economic crisis with huge losses in the banking industry around the world. These losses have brought to light weaknesses in the Basel II global regulatory framework and "in house" poor risk management practices.

In response, the BCBS proposed a variety of measures to enhance the stability of the financial system and to better capture risks that were once neglected or not well-known and understood. In fact, tools were designed to capture a large variety of risks (such as leverage effect, systemic risk, geographic risk, correlation between financial institutions, liquidity issues…)

and more specifically the Counterparty Credit Risk. As CCR turned to be of a great importance and caused massive losses, the BCBS introduced a new capital charge for Credit Value Adjustments (CVA) losses. As discussed above, only 25% of losses on OTC derivatives were due to actual defaults and 75% were due to rating migration and credit spread volatility. Consequently, within the Basel III Framework, the BCBS designed a new tool (CVA) with the purpose of capturing risks and losses related to credit spread volatility. It aims to allocate more capital resources to CCR and rectify what was once neglected.

The CVA capital charge as presented within Basel III comes into two forms. The first form is a standardized formula to be used as such. The second form is a more advanced formula which is based on inputs from complex and robust models (EPE, etc…) and requires a high computational capacity.

If the advanced approach has been subject to reflection and understanding of how the formula was determined, the standardized approach is still less known from the public.

In the following paper, we discuss how to define the CVA from the regulator's perspective, what its intent is, and where the standardized formula comes from. The main purpose is to better understand the meaning and the origins of the CVA in its standardized approach.

2. Credit Value Adjustments, a new concept for a long lasting idea

CVA is a long lasting idea that has existed for many years. But most financial institutions neglected it because it was considered as highly improbable to generate massive losses or was considered as fully hedged because transactions were made with large financial institutions that could not fail.

As the "Too Big to Fail" myth has been forever shattered due to high-profile bankruptcies, CVA risks and losses could no longer be denied. For this reason, the BCBS developed a new concept for capturing CVA losses.

2.1. CVA definition under Basel III

To begin with, CVA under Basel III could be defined as the risk of loss on OTC derivatives and securities financing transactions due to changes in counterparties 'credit spread caused by a change in its creditworthiness. In other words, CVA reflects the market value of the cost of credit spread's volatility.

For instance, let's consider a bank A that has an Interest Rate Swap deal with a bank B (knowing that this derivative is an asset for bank A). If bank B's credit quality deteriorates without failing (no default), then the value of the IRS decreases and bank A will suffer a loss. In fact, knowing that the credit spread of bank B increased, market participants judge that it becomes riskier and that the chance of honoring its entire engagement less probable. Consequently, they are willing to pay less to purchase this deal than they would do when the credit spread of bank B was better. However, OTC derivatives are marked-to-market and valued on a fair value basis, which means that their value is defined by market participants.

This example shows that P&L losses on OTC derivatives could occur without counterparty default, but only due to credit spread volatility and rating migration.

It is important to point out that if the IRS comes to its expiration date without counterparty default, P&L losses recognized during the lifetime of the instrument are offset because all payments were received by the bank.

It is then obvious to debate concerning the scope of application of the CVA capital charge (Banking book vs. trading book). This point will be further discussed in part 2.3.

2.2. Basel III CVA purpose

CVA as defined within Basel III was designed in the sole intent of capturing and measuring losses on OTC derivatives and securities financing transactions due to credit spread volatility. As the 2007 – 2009 crisis showed, this risk was not captured accurately within Basel II.

Consequently, the BCBS introduced a new capital charge to be constituted in order to absorb CVA losses. In fact, as discussed above, OTC derivatives are a potential source of systemic risk which needs to be neutralized or at least contained. This new capital charge was designed in this state of mind.

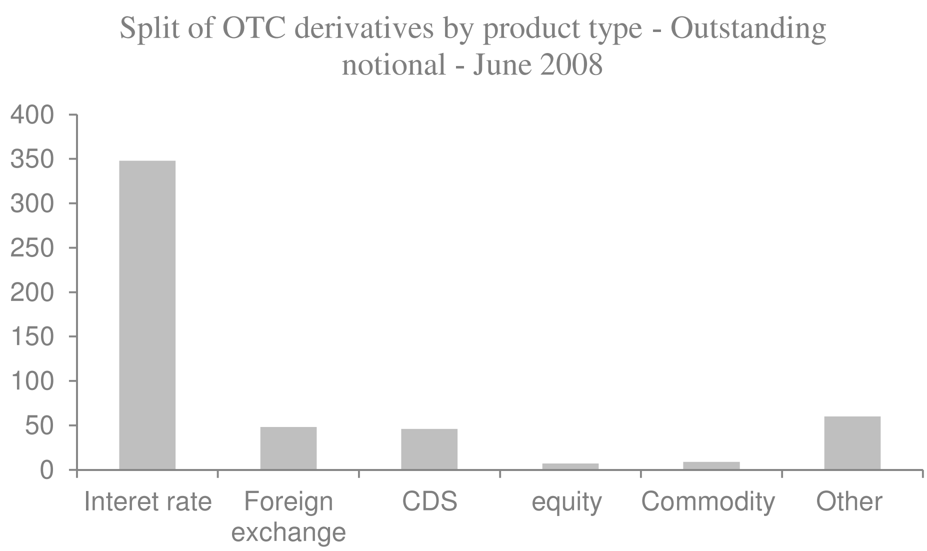
2.3. Basel III CVA scope of application

CVA under Basel III is a core component of Counterparty Credit Risk. It is then computed on OTC derivatives and securities financing transactions.

OTC derivatives are options, swaps, forwards and CDS and apply to underlying such as interest rate, currency, equity, commodities etc…

Securities financing transactions are repo-style transactions, securities lending/borrowings…

Yet, Interest rate products contribute to 67% of the outstanding notional:



Source: ISDA

Moreover, derivatives that are cleared by Central Counterparties are excluded from CVA's scope of application, because CCR risk between the seller and the buyer is neutralized by the CCP through margin calls and collateral deposits. Nonetheless, Basel III recognizes a CCR risk with CCPs. In fact, under Basel II, cleared transactions were considered as risk free (with a 0% risk weight). Under Basel III, these transactions have a 2% risk weight which means that an extra cushion of capital is to be constituted in order to cover losses due to CCPs default.
Securities financing transactions can be excluded if local regulators judge that exposures arising from these transactions are not material.

Nonetheless, all internal transactions within entities of the same financial institution are to be excluded.

Finally, all OTC derivatives from the banking and the trading book are to be taken into consideration while computing the CVA capital charge. Yet, one can ask himself why derivatives from the banking book are to be taken into consideration. In fact, these products are held to maturity, which means that at the expiration date, credit spread volatility (without default) has no impact on P&L and doesn't generate losses, and only default risk has an impact.

2.4. What about the bilateral nature of CCR?

As discussed above, CCR has a bilateral nature because OTC derivatives could be assets and liabilities as well. Consequently, if CVA risk exists, DVA risk exists as well.

DVA (Debt value adjustments) is the CVA risk from counterparty's perspective. If we consider a bank that has a derivative with a counterparty and that this instrument is a liability for the bank (and an asset for the counterparty), then if the value of the instrument decreases (due to an increase in the bank's credit spread), the counterparty faces CVA losses. Yet, the value of the bank's debt decreases as well which has a positive impact on the P&L (profit). This means that if the bank's default risk increases, its debt value decreases registering gains in the P&L.

Consequently, from the bank's perspective, total losses on OTC derivatives due to credit spread volatility could be determined as (CVA – DVA).

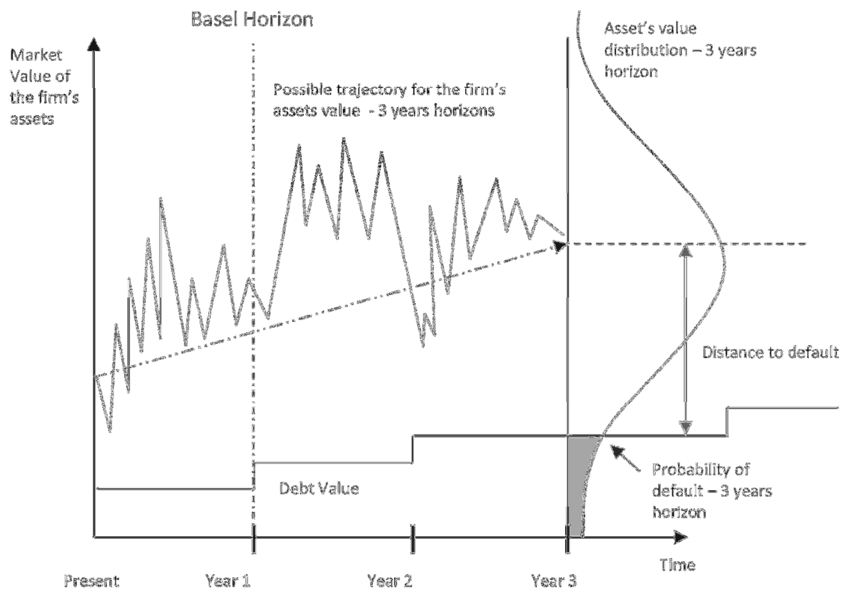
This is true in IFRS rules (accounting rules, that recognize CVA risk and DVA risk as well), but doesn't apply in Basel III. In fact, in article 371 (CRD IV) it is said that "*adjustment reflects the current market value of the credit risk of the counterparty to the institution, but does not reflect the current market value of the credit risk of the institution to the counterparty*." In other words, Basel III recognizes CVA risk but doesn't take DVA risk into consideration.

3. From Basel II to Basel III, CCR perception changed

The purpose of this paragraph is to address the differences between Basel II and Basel III and clarify why Basel II didn't capture accurately CVA losses.

3.1. CCR under Basel II

First, to understand how CCR is treated within Basel II, it is important to understand how "Risk Weight" formulas have been determined. Based on the Merton Structural Model, it is a firm's distance to default that is determined based on the value of its assets in comparison with the value of its debts. A firm will default if the value of its assets is underneath the value of its debts. This means that if the firm sells all its assets, it won't be sufficient to pay all its debts:



Consequently, Basel II captures losses due to default. Additionally, Basel II introduced a maturity adjustment coefficient which purpose is to take deal's maturity into consideration as well as rating migration. Yet, this coefficient proved insufficient to capture MtM losses due to credit spread volatility as precised by the BCBS (because it is a static coefficient with no connections to market participants perception) :

"*Under Basel II, the risk of counterparty default and credit migration risk were addressed but mark-to-market losses due to credit valuation adjustments (CVA) were not. During the financial crisis, however, roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one-third were due to actual defaults.*" (BCBS, June 2011)

Second, within the Basel II model, assumptions have been made. One of these assumptions is that the portfolio is perfectly granular which means that the contribution of an individual exposure to the portfolio is infinitesimal and that correlation between assets is not taken into consideration. Consequently, default risk capital charge is not computed on a counterparty or portfolio level, but rather is determined for each single asset.

CCR and more specifically CVA risk is treated differently within Basel III.

3.2. CCR under Basel III

As MtM losses due to credit spread volatility were not captured within Basel II, Basel III introduced a new capital charge to capture these losses. Consequently, we can write:

$$\text{CCR under Basel III} = \text{Default Risk (Basel II)} + \text{CVA Risk (New)}$$

Furthermore, CVA standardized approach under Basel III is computed on a portfolio level. In fact, exposures are aggregated by counterparty, after applying collateral and netting agreements, and CVA is computed on counterparties of the entire portfolio. After that, the contribution of each counterparty to the portfolio CVA is determined via reallocation methods. This procedure is different from what is done within Basel II for two reasons.

First, CVA is determined for the entire portfolio (or at least by counterparty) and not computed on individual exposure level.

Second, in opposition to Basel II, the CVA of the portfolio is not equal to the sum of each individual CVA (on a counterparty or exposure level) which means that compensation effects are taken into consideration which is not the case in Basel II where default risk is computed on exposure level. In fact, as the portfolio is considered as perfectly granular and because the risk is computed on individual exposure, the portfolio level of risk is equal to the sum of default risk level of each exposure.
Moreover, the CVA standardized approach is a Value-At-Risk model with 99% confidence level on a 1 year horizon (this will be demonstrated in part 4.) and is applied to the entire portfolio whereas the distance-to-default model in Basel II is applied on each exposure independently of other exposures.

3.3. What are the new challenges for banks?

To fully understand the challenges the banks might face, it is important to understand how the CVA in its standardized approach reacts to the different parameters.

The standardized formula for Basel III CVA is:

$$K_{CVA} = 2.33\sqrt{h}\sqrt{\left(\sum_i 0.5.w_i.(M_i.EAD_i^{total} - M_i^{hedge}.B_i) - \sum_{ind} w_{ind}.M_{ind}.B_{nd}\right)^2 + \sum_i 0.75.w_i^2.(M_i.EAD_i^{total} - M_i^{hedge}.B_i)^2}$$

Where,

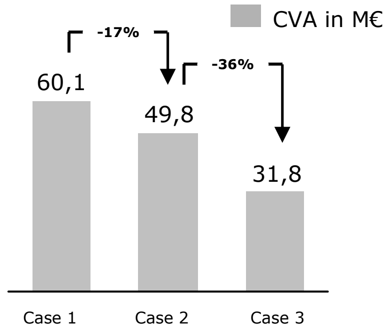
- h is the horizon (1 year) of the VaR
- i refers to the counterparty i
- w_i is the weight of the counterparty i determined via counterparty's rating
- M_i is the maturity of the exposure
- EAD_i is the aggregated exposure towards counterparty i
- M_i^{hedge} and M_{ind} are the maturities of CDS used to specifically hedge CVA risk
- B_i the notional of purchased single name credit default swap hedges (summed if more than one position) referencing counterparty "i" and used to hedge CVA risk
- B_{ind} is the full notional of one or more index credit default swap of purchased protection used to hedge CVA risk.

If we consider that there are no CVA hedge instruments in the portfolio (which is the case in a big part of OTC derivatives portfolio), then B_i and B_{ind} are equal to 0.

We study the CVA sensitivity towards the maturity, the exposure, the rating and the number of counterparties within the portfolio:

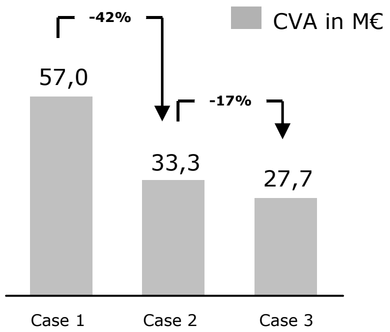
a) CVA sensitivity to maturity : (All parameters are fixed, only the maturity changes)

	Case 1	Case 2	Case 3
Nb of counterpaties	2	2	2
EAD by counterp.	500 M€	500 M€	500 M€
Portfolio EAD	1 Md€	1 Md€	1 Md€
Maturity	3y 5y	1y 5y	1y 3y
Weight(Rating)	0,8% (A)	0,8% (A)	0,8% (A)



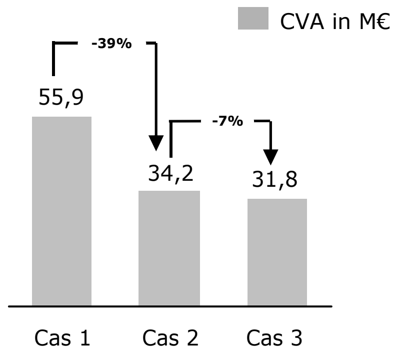
b) CVA sensitivity to rating : (All parameters are fixed, only the rating changes)

	Cas 1	Cas 2	Cas3
Nb of counterpaties	2	2	2
EAD by counterp.	500 M€	500 M€	500 M€
Portfolio EAD	1 Md€	1 Md€	1 Md€
Maturity	2y	2y	2y
Weight(Rating)	2% (BB) 1% (BBB)	0,8% (A) 1% (BBB)	0,8% (A) 0,7% (AA)



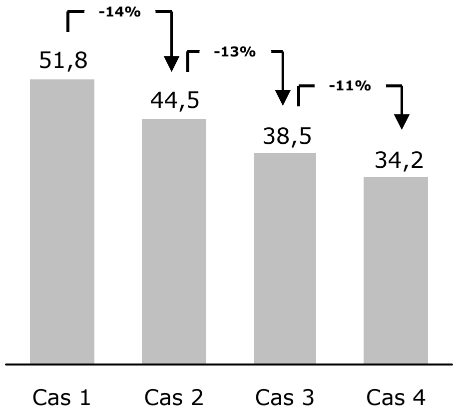
c) CVA sensitivity to the number of counterparties : (concentration – number of counterparties)

	Cas 1	Cas 2	Cas3
Nb of counterpaties	1	6	10
EAD by counterp.	1 Md€	167 M€	100M€
Portfolio EAD	1 Md€	1 Md€	1 Md€
Maturity	3y	3y	3y
Weight(Rating)	0,8% (A)	0,8% (A)	0,8% (A)



d) CVA sensitivity to EAD repartition : (concentration – exposure)

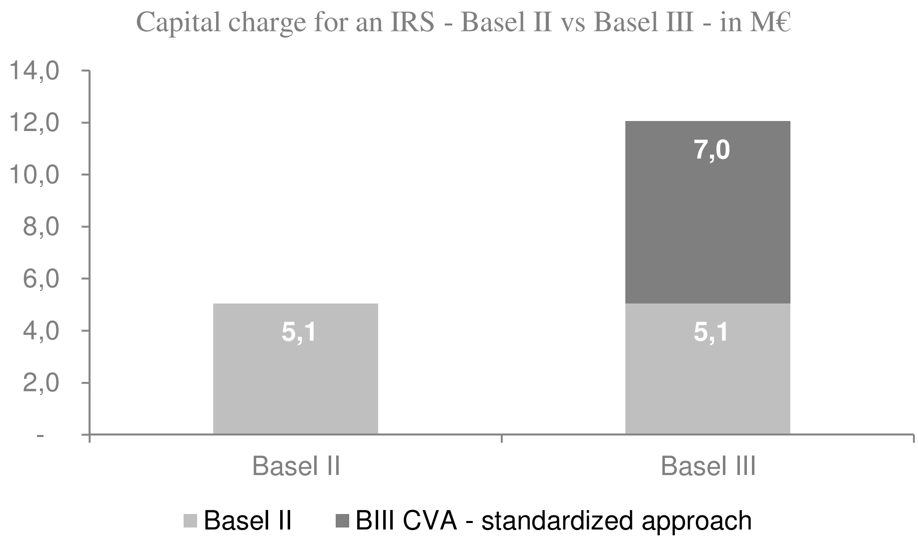
	Cas 1	Cas 2	Cas3	Cas4
Nb of counterparties	6	6	6	6
Maturity	3y	3y	3y	3y
Weight (ratings)	0,8% (A)	0,8% (A)	0,8% (A)	0,8% (A)
EAD counterp 1	0,9 Mds€	0,7 Mds€	0,5 Mds€	0,16 Mds€
EAD counterp 2	0,02 Mds€	0,1 Mds€	0,1 Mds€	0,16 Mds€
EAD counterp 3	0,02 Mds€	0,05 Mds€	0,1 Mds€	0,16 Mds€
EAD counterp 4	0,02 Mds€	0,05 Mds€	0,1 Mds€	0,16 Mds€
EAD counterp 5	0,02 Mds€	0,05 Mds€	0,1 Mds€	0,16 Mds€
EAD counterp 6	0,02 Mds€	0,05 Mds€	0,1 Mds€	0,16 Mds€
Portfolio EAD	1 Mds€	1 Mds€	1 Mds€	1 Mds€



Source: GRA

As we can notice, the sensitivity of the CVA towards these parameters is high. It is also interesting to point out the fact that concentration plays also an important role in explaining the behavior of the CVA standardized approach. The more the portfolio is diversified in terms of number of counterparties or EAD distribution, the less the CVA capital charge is.

Consequently, financial institutions will have to optimize their portfolio in order to reduce the capital charge. In fact, the additional capital charge due to CVA is significant. Below is shown a comparison of capital charge under Basel II and Basel III for an IRS, maturity 3 years, contracted with a BBB financial institution for 100 M€ of EAD :



Source: GRA

As shown above, the capital charge under Basel II for this transaction is 5.1 M€ whereas under Basel III it is 7 M€ + 5,1 M€ = 12,1 M€. This example shows that the impact of Basel III could be significant. Yet, it is important to consider the fact that the CVA capital charge for this example is maximized because we consider only one instrument in the portfolio (concentration effect of the VaR).

Considering the behavior and the impact of the CVA standardized approach, banks will face great challenges.

First, banks will have to optimize correctly their portfolio by measuring and identifying accurately counterparties that contribute the most to the CVA of the portfolio. This requires analyzing concentration risk, as well as analyzing the maturities of each deal and counterparty ratings. In fact, long maturity deals or deals with risky counterparties will increase the capital charge significantly. Banks will have to diversify more and more their portfolios in terms of number of counterparties they are dealing with as well as focusing on a homogenous EAD distribution.

Second, banks will have to hedge CVA risk which requires buying protection against credit spread volatility at a counterparty level (single name) or at a portfolio level. An arbitrage has to be made to provide the better protection with an optimized cost. Hedging CVA comes also by pricing correctly the CCR cost within MtM (this will not be discussed within this paper).

4. Where does the standardized formula for CVA come from?

As discussed above, the formula proposed by the BCBS for the standardized approach is a Value-At-Risk model with 99% confidence level and 1 year time horizon.

If the CVA advanced formula has been subject to many studies, the standardized approach is still not well known and understood. In this paragraph, a demonstration is proposed to better understand the model and its underlying assumptions.

4.1. General formula of the VaR

To determine the general formula of a VaR model, some assumptions are made:

- Loss distribution of the portfolio follows a normal law, with mean m and standard deviation σ. This comes from the Laplace's central limit theorem and the theory of large numbers;
- The mean m is considered equal to 0 which is an assumption frequently used.

First, we don't know the integral of the function e^{-x^2} because we don't know a primitive for this function. Consequently, the algebraic expression of the normal distribution function can't be determined. Which means that the values of N(m;σ) are unknown.

However, the values of the standard normal distribution have been empirically determined. Consequently, a transformation will be used to determine the values of N(m;σ) from those of N(0;1).

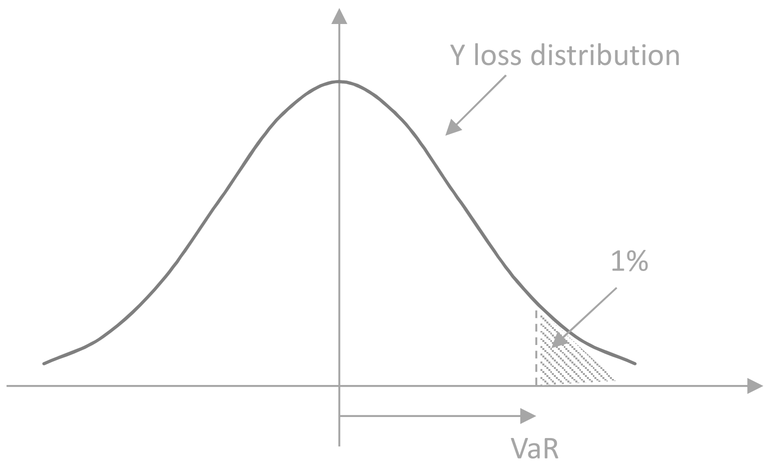
Let's consider the random variable Y who follows a normal law with mean m and standard deviation σ:

$$Y \rightarrow N(m;\sigma)$$

We define the random variable T as:

$$T = \frac{Y - m}{\sigma}$$

T follows a standard normal distribution.



By considering the distributions we deduce that:

$$N^{-1}(0;1) = \frac{VaR\ (Y) - m(Y)}{\sigma\ (Y)}$$

And

$$VaR\ (Y) = m(Y) + \sigma(Y).N^{-1}(0;1)$$

Finally, because m(Y) = 0:

$$\boldsymbol{VaR(Y) = \sigma(Y).N^{-1}(0;1)}$$

4.2. VaR calculus

Let's consider the δ of the portfolio:

Variation of the portfolio value

$$\delta = \frac{\Delta P}{\Delta S}$$

Variation of the underlying value

We do a second order Taylor expansion:

Gamma of the portfolio

$$\Delta P = \delta \Delta S + \frac{1}{2}\frac{\partial \delta}{\partial S}(\Delta S)^2$$

$$\Delta P = \delta \Delta S + \frac{1}{2}\gamma(\Delta S)^2$$

In the CVA context, portfolio loss is determined by MtM variations (because under Basel III, the CVA captures losses due to MtM variation caused by credit spread volatility) in accordance with the evolution of risk factors such as credit risk factors including credit spreads.
Consequently, we define MtM = M and risk factors = F.

As we try to capture MtM variations, we can write:

$$\Delta M = \delta \Delta F + \frac{1}{2}\gamma(\Delta F)^2$$

The variations of MtM define the losses of the portfolio. If we compute the Value-at-Risk of ΔM, we obtain:

$$VaR(\Delta M) = VaR\left[\delta\Delta F + \frac{1}{2}\gamma(\Delta F)^2\right]$$

From part 4.1 we know that:

$$VaR(\Delta M) = \sigma(\Delta M).N^{-1}(0;1)$$

$$VaR(\Delta M) = \sigma((\delta\Delta F + \frac{1}{2}\gamma(\Delta F)^2).N^{-1}(0;1)$$

We focus on the term:

$$\sigma((\delta\Delta F + \frac{1}{2}\gamma(\Delta F)^2)$$

In fact, instead of considering the standard deviation, we consider the variance because it is easier to compute the variance than the standard deviation as we eliminate the square root:

$$\sigma((\delta\Delta F + \frac{1}{2}\gamma(\Delta F)^2) \Leftrightarrow \sigma^2((\delta\Delta F + \frac{1}{2}\gamma(\Delta F)^2)$$

We develop the variance:

$$\sigma^2((\delta\Delta F + \frac{1}{2}\gamma(\Delta F)^2) = \sigma^2(\delta\Delta F) + \sigma^2(\frac{1}{2}\gamma(\Delta F)^2) + 2\operatorname{cov}(\delta\Delta F; \frac{1}{2}\gamma(\Delta F)^2)$$

$$= \delta^2\sigma^2(\Delta F) + (\frac{1}{2}\gamma)^2\sigma^2(\Delta F^2) + 2(\delta.\frac{1}{2}\gamma).\operatorname{cov}(\Delta F;(\Delta F)^2)$$

Because:

$$Variance(aX + bY) = a^2 Variance(X) + b^2 Variance(Y) + 2ab.CoVariance(X;Y)$$

But ΔF follows a normal law and the covariance is the first order moment of two random variables. Moreover, for a symmetric function (which is the case for normal distribution), the first order moment is null.

Consequently,

$$\sigma^2((\delta\Delta F + \frac{1}{2}\gamma(\Delta F)^2) = \delta^2\sigma^2(\Delta F) + (\frac{1}{2}\gamma)^2\sigma^2(\Delta F^2)$$

And,

$$\sigma^2(\Delta F^2) = (\sigma^2(\Delta F))^2 + 2m^2\sigma^2(\Delta F)$$

Because,

$$Variance(X.Y) = Variance(X).Variance(Y) + E(X)^2.Variance(Y) + E(Y)^2.Variance(X)$$

m=0, which means:

$$\sigma^2(\Delta F^2) = (\sigma^2(\Delta F))^2$$

Consequently,

$$\sigma^2((\delta\Delta F + \frac{1}{2}\gamma(\Delta F)^2) = \delta^2\sigma^2(\Delta F) + (\frac{1}{2}\gamma)^2(\sigma^2(\Delta F))^2$$

The standard deviation is,

$$\sigma((\delta\Delta F + \frac{1}{2}\gamma(\Delta F)^2) = \sqrt{\sigma^2((\delta\Delta F + \frac{1}{2}\gamma(\Delta F)^2)} = \sqrt{\delta^2\sigma^2(\Delta F) + (\frac{1}{2}\gamma)^2(\sigma^2(\Delta F))^2}$$

If we get back to the VaR, we have:

$$VaR(\Delta M) = N^{-1}(0;1).\sqrt{\delta^2\sigma^2(\Delta F) + (\frac{1}{2}\gamma)^2.(\sigma^2(\Delta F))^2}$$

Furthermore, we can simplify the following term,

$$\sigma^2(\Delta F) = E(\Delta F^2) - \underset{\substack{\uparrow \\ = \mathrm{m} = 0}}{(E(\Delta F))^2} = E(\Delta F^2)$$

This means that, (with n = number of counterparties)

$$(\sigma^2(\Delta F))^2 = E(\Delta F^2)^2 = \left[\frac{1}{n}\sum_{i=1}^{n}\Delta F_i^2\right]^2$$

We assume now the theory of large numbers which is used as an assumption in the Basel frameworks. Consequently, the portfolio is assumed to be constituted of a large number of counterparties which means that the contribution of a counterparty to the CVA of the portfolio is infinitesimal and therefore ΔF = constant.

So,

$$\frac{1}{n}\sum_{i=1}^{n}\Delta F_i^2 = \Delta F^2 < \sum_{i=1}^{n}\Delta F_i = n\Delta F$$

4.3. Determination of the VaR CVA under Basel III – Standardized approach

We consider the following expression which is conservative if we consider retained hypothesis:

$$\frac{1}{n}\sum_{i=1}^{n}\Delta F_i^2 = \Delta F^2 \text{ To be replaced by } \sum_{i=1}^{n}\Delta F_i$$

We obtain:

$$VaR(\Delta M) = N^{-1}(0;1).\sqrt{\delta^2 \sum_{i=1}^{n} \Delta F_i^2 + (\frac{1}{2}\gamma)^2.\left[\sum_{i=1}^{n} \Delta F_i\right]^2}$$

CVA risk depends on 3 factors. First, the longer the maturity is, the higher the risk of downgrade is. Second, the higher credit spread is, the higher the risk of default (the distance to default is shorter). Third, the higher the exposure is, the higher the losses are.
Consequently, we deduce that the risk factors for the CVA are the three components listed above:

$$\Delta F_i = W_i.M_i.EAD_i$$

That gives:

$$VaR(\Delta M) = N^{-1}(0;1).\sqrt{\delta^2 \sum_{i=1}^{n} W_i^2.M^2{}_i EAD^2{}_i + \left[\sum_{i=1}^{n} \frac{1}{2}\gamma.W_i.M_i EAD_i\right]^2}$$

As we said before, we consider a 99% confidence level, and we write VaR(99%):

$$VaR(99\%) = N^{-1}(99\%;0;1).\sqrt{\delta^2 \sum_{i=1}^{n} W_i^2.M^2{}_i EAD_i^2 + \left[\sum_{i=1}^{n} \frac{1}{2}\gamma.W_i.M_i EAD_i\right]^2}$$

$$VaR(99\%) = 2{,}33.\sqrt{\delta^2 \sum_{i=1}^{n} W_i^2.M^2{}_i EAD^2{}_i + \left[\sum_{i=1}^{n} \frac{1}{2}\gamma.W_i.M_i EAD_i\right]^2}$$

Finally,

$$CVA = 2{,}33.\sqrt{\delta^2 \sum_{i=1}^{n} W_i^2.M^2{}_i EAD^2{}_i + \left[\sum_{i=1}^{n} \frac{1}{2}\gamma.W_i.M_i EAD_i\right]^2}$$

If we consider that there are no hedge instruments, we obtain the same formula as proposed by the regulator.

We conclude that the CVA standardized approach is Value at Risk model with a 99% confidence level over a 1 year horizon.

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