Collateral risk for fair valuation and CCR capital*

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The exchange of regulatory initial margin for uncleared derivatives (BCBS 261 due on the 1st of September 2016) implies a massive consumption of collateral. This paper propose a new model to account for collateral and its quality for both fair valuation and CCR capital frameworks defined under IFRS 13 and CRD IV. The industry is expected to enhance and implement more accurate methods for reflecting the "exact" value of collateral, preserving competitiveness and optimising capital consumption.

Introduction

The netting and collateralisation continue to spread through the global banking system as one of the main drivers (CCPs, trade compression, etc..) to reduce systemic risks in the industry and the global economy as detailed in ISDA study [17]. The new regulation on margin requirements for non-centrally cleared derivatives [9] emphasises the view of politicians to move forward in this direction. The amount of collateral directly exchanged between counterparties, through clearing houses or posted to third party grows constantly. Typical credit and funding risk models assume collateral received or posted to be cash. The non-cash collateral (e.g. bonds, equity, gold) may become an important proportion of exchanged margins.

The funding desk provides cash or assets for margin calls following a cheapest to deliver strategy[1] without always assessing its impact on global portfolio valuation or counterparty credit risk capital (CCR[2]). The optionality to post or receive diverse assets or currencies embedded in the unilateral or bilateral contracts may engage an additional cost or benefit to the bank when exchanging a specific type of collateral. The decision process to fund margin calls with a given asset type or currency must at least incorporate two parameters: a cost to deliver an asset or currency, its impact on valuation adjustment and on capital. A parallel could be drawn with collateral optionality adjustment on the risk neutral price of a derivative[3] where the new model could be perceived as a correction to fair valuation adjustments and capital requirements.

This article presents a general model to describe a relation between collateral, P&L and capital through a utility function. Currently, the industry is attentive to the funding adjustments for non-perfect CSA and non-collateralised transactions, this paper also demonstrates the importance of funding adjustments even for perfect CSA. The paper is organised as follows: the section 1 provides a description of general class of collateral models by introducing a collateral multiplier; the section 2 presents a toy model to establish numerical results and exhibits the impacts on P&L; the section 3 debates about a deterministic model; finally the conclusion is presented.

*Disclamer: This paper represents the view of the authors alone, and not the view of their employers

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[1] In terms of cost (repo rate)

[2] The referenced capital includes CCR RWA deduced from EAD and CVA Charge calculated with Advanced Method

[3] The risk neutral pricing theory of valuing derivatives under a Credit Support Amount remains out of scope, please refer to [16], [19], [10].

1 Collateral model

1.1 Mathematical framework for credit and funding valuation adjustments

The equation (1) presents a total valuation adjustment[4], denoted as TA expected to be applied to risk-free mark to market.

$$TA = CVA + DVA + FCA + FBA \tag{1}$$

Assuming the complete probability space $\left(\Omega, (\mathcal{F}_t)_{t\in[0,T]}, \mathbb{Q}\right)$ and finite horizon $[0;T]$, where the state space Ω is a universe of possible outcomes of stochastic market between 0 and T, $\mathcal{F}_t$ is the sigma field of distinguishable events at time t, $\mathbb{Q}$ is a risk-neutral probability measure (no-arbitrage), CVA and DVA defines the credit and debit valuation adjustments, FCA designates the funding cost adjustment and FBA labels the funding benefit adjustment:

$$\begin{aligned} CVA = & - & \mathbb{E}_{\mathbb{Q}}\left[(1-R_\tau^{Int})\cdot DF_\tau \cdot E_\tau^+ \cdot b_\tau\right] \\ DVA = & & \mathbb{E}_{\mathbb{Q}}\left[(1-R_\tau^{Cpt})\cdot DF_\tau \cdot E_\tau^- \cdot b_\tau\right] \end{aligned}$$

and

$$\begin{aligned} FCA = & - & \mathbb{E}_{\mathbb{Q}}\left[\int_0^T x_t \cdot DF_t \cdot NCP_t^+ \cdot b_t \cdot dt\right] \\ FBA = & & \mathbb{E}_{\mathbb{Q}}\left[\int_0^T x_t \cdot DF_t \cdot NCP_t^- \cdot b_t \cdot dt\right] \end{aligned}$$

with the following notation

- τ: a default time or portfolio maturity (institution, counterparty or joint),
- b_t: an institution and/or a counterparty survival indicator at time t,
- DF_t: a discount factor at time t,
- E_t^+: a positive exposure at time t,
- E_t^-: a negative exposure at time$t$,
- NCP_t^+: a positive net collateral position at time t,
- NCP_t^-: a negative net collateral position at time t,
- R_t^{Int}: an institution recovery rate at time t,
- R_t^{Cpt}: a counterparty recovery rate at time t,
- x_t: an institution funding rate at time t.

The reader may refer to [5], [6], [12] or [14] in order to adopt one of the described approaches in the litterature and define an appropriate characterisation for each variable. The paper restricts the attention to analyse collateralised exposure transactions and net collateral position calculation instead of debating on the different views on the fair valuation adjustments.

[4]The capital is a separate cost to derivates trading book

1.2 Standard definition for collateralised exposure and net collateral position

In the event of default, a time lag called a close-out period δ is required to recognise a default and proceed to a full liquidation of transaction and collateral. Market participants frequently estimate the δ period to remain within 5 to 10 days for a CSA with daily frequency. However, the regulators[5] prescribe exact rules based on the number of transactions, liquidity of collateral/transactions or number of disputes which may increase the δ. The collateralised exposure of an institution represents for credit valuation adjustment an estimation of loss if a counterparty defaults and for debit valuation adjustment an evaluation of benefit if the institution itself defaults.

It is common to calculate the exposure in a Monte-Carlo simulation framework where the collateralised exposure E is defined as

$$E_{i,t} = V_{i,t} - C_{i,t} + \Delta V_{i,t,t+\delta} \tag{2}$$

with

- $V_{i,t}$: a mark to market (MtM) of portfolio under CSA at time t, valued with simulated market data of scenario i
- $\Delta V_{i,t,t+\delta}$: a mark to market change between t and $t+\delta$ of the portfolio fixed[6] at time t. It accounts for a MtM variation during the period between the default of a counterparty or the institution and the hedging or position replacement,
- $C_{i,t}$: credit support amount in agreement currency at scenario i time t.

The value of credit support amount in agreement currency C is usually estimated considering the CSA parameters such as threshold, minimum trasfer amount, etc.

In this article, a case study is based on the perfect CSA with daily frequency of margin call, zero threshold, minimum transfert amount. Therefore, the credit support amount may be approximated as equal to a net mark to market of CSA transactions. The model may be easily extended to a non perfect CSA with different margin call frequency and to a contractual[7] or regulatory[8] initial margin.

For simplicity reasons, the mark to market V is in agreement currency of CSA, US dollar, and the settlement currency[9] of transactions in the event of counterparty or institution default is also assumed to be in the same currency, US dollar. Then, the collateralised expected positive exposure EPE for any time point t is defined as

$$EPE_t = \frac{1}{I} \cdot \sum_{i \in I} \max\left(E_{i,t}, 0\right)$$

and the collateralised expected negative exposure ENE may be written as

$$ENE_t = \frac{1}{I} \cdot \sum_{i \in I} \min\left(E_{i,t}, 0\right)$$

where I denotes a total number of simulations.

Furthermore, for funding purposes the classical definition of net collateral position NCP describes a difference between portfolio mark to market with counterparty and received/posted collateral from/to counterparty

$$NCP_{i,t} = V_{i,t} - C_{i,t} \tag{3}$$

with

- $V_{i,t}$: a mark to market of portfolio[10] at time t,

[5] The close-out period is denoted as Margin Period of Risk, Article 285 in [18]

[6] The cash-flows between time t and $t + \delta$ may be ignored to avoid spikes in the exposure.

[7] defined in the usual Credit Support Annex

[8] obligatory and formula based under new regulation [9]

[9] The currency of reference in which the transactions are liquidated in the event of default

[10] All transactions with the counterparty must be netted independently if one is legally under CSA, ISDA or No Documentation.

- $C_{i,t}$: credit support amount at scenario i time t in agreement currency expected to be received/posted for transactions under CSA.

Then, an expected collateral cost ECC

$$ECC_t = \frac{1}{I} \cdot \sum_{i \in I} \max\left(NCP_{i,t}, 0\right)$$

and an expected collateral benefit ECB

$$ECB_t = \frac{1}{I} \cdot \sum_{i \in I} \min\left(NCP_{i,t}, 0\right)$$

are defined for FCA and FBA respectively.

1.3 Collateral multiplier for credit valuation adjustments

The collateralised exposure defined in the equation (2) at the entity level does not include a risk of collateral liquidation[11] after a default of counterparty or the necessity to buyback[12] the posted assets. A simplistic assumption is to consider the collateral as a cash amount in the same currency as the portfolio settlement currency. Indeed the collateral is expected to be received or posted at time t, but a full close-out, i.e. an offset from actual collateral value must be considered at time $t + \delta$. It implies naturally to calculate a potential value change of collateral assets or currencies at time $t + \delta$ received or posted at time t. Therefore, a more formulation of collateralised exposure is

$$E_{i,t} = V_{i,t} - CV_{i,t,t+\delta} + \Delta V_{i,t,t+\delta} \tag{4}$$

As mentioned previously, assessing a composition of collateral assets or currencies, received or posted, at time t is mandatory to determine the value of collateral at time $t + \delta$. The collateral multiplier M represents a collateral composition at scenario i time t and a potential variation of each asset or currency amount against the agreement currency. Then, the collateral value at close-out time may be written as a product of credit support amount and collateral multiplier

$$CV_{i,t,t+\delta} = M_{i,t}^{Y} \cdot C_{i,t}$$

where the variable Y stands for CVA or DVA. This notation is used in the following.

The sign of C is positive (conv. negative), if the institution receives (conv. posts) collateral. Then, the collateral multiplier is separated into two cases

$$M_{i,t}^{Y} = \left(M_{i,t}^{R,Y} \cdot \mathbb{I}_{C_{i,t} \geqslant 0} + M_{i,t}^{P,Y} \cdot \mathbb{I}_{C_{i,t} < 0}\right)$$

where the the receiver parameter (conv. payer parameter) is designated with an upper-script R (conv. an upper-script P). The receiver multiplier describes a composition of collateral assets or currencies and its variation during a close-out when the institution is expected to receive the collateral. The payer multiplier denotes a composition and its variation when the institution has to post collateral.

For both multipliers, the weight of each asset or currency, called ω, is introduced. The choice of asset/currency proportion remain the decision of the party obliged to post collateral under the terms of CSA. Therefore, the weight of asset or currency A is distinguished when the institution receives (conv. payes) collateral from (conv. to) a counterparty under CSA L. The contractual haircuts, named h, set in the CSA L must be also considered. Finally, a variation of asset price or currency exchange between time t and $t + \delta$ for each scenario i is defined as Δ.

$$M_{i,t}^{R,Y} = \sum_{A \in \Psi_L} \omega_A^{R,L}(i,t) \cdot \frac{1 + \Delta_{i,t,t+\delta}^{A}}{1 - h_L^A} \tag{5}$$

[11] When a collateral is received from the counterparty

[12] When a collateral is posted to the counterparty

$$M_{i,t}^{P,Y} = \sum_{A\in\Psi_L} \omega_A^{P,L}(i,t) \cdot \frac{1+\Delta_{i,t,t+\delta}^{A}}{1-h_L^A} \tag{6}$$

The receiver and payer weights depend on a market condition Υ, availability of asset/currency A in the total pool Θ, an eligible set[13] of asset/currency Ψ_L for CSA L and the credit support amount[14] C exchanged at every time point between 0 and t. The utility function g describes the cheapest to deliver strategy of the funding desk or simply a choice of asset/currency by counterparty or institution conditionally on the parameters' state

$$\omega_A^{R,L}(i,t) = g^{Cpt}\left(i,t; C_{[0,t]}, \Upsilon_{i,t}, A, \Theta_{i,t}^{Cpt}, \Psi_L\right) \tag{7}$$

$$\omega_A^{P,L}(i,t) = g^{Int}\left(i,t; C_{[0,t]}, \Upsilon_{i,t}, A, \Theta_{i,t}^{Int}, \Psi_L\right) \tag{8}$$

The function g for payer multiplier is determined by the institution strategy, but the function g for receiver multiplier is fully determined by the counterparty.

Moreover, some logical conditions are to be set on the weight values. Mathematically, the first condition is fixed on the sum of weights for each CSA L that must be equal to one as it is a proportion of credit support amount $C_{i,t}$. The weight represents the proportion of the "haircuted" mark to market of cash or asset by $1-h_L^A$ and normalised against a required amount of credit support amount. The condition is described in the equation

$$\forall(i,t), \forall L: \qquad \sum_{A\in\Psi_L} \omega_A^{R,L}(i,t) = 1 \qquad \text{and} \qquad \sum_{A\in\Psi_L} \omega_A^{P,L}(i,t) = 1.$$

The second condition is the positivity of weights defined by the equation

$$\forall(i,t), \forall A: \qquad \omega_A^{R,L}(i,t) \geq 0 \qquad \text{and} \qquad \omega_A^{P,L}(i,t) \geq 0.$$

It may be seen as a discrete version of probability density.

1.4 Collateral multiplier for funding valuation adjustments

The net collateral position described in the equation (3) does not account for actual value of collateral. If non-cash collateral (e.g. bonds, equity, gold) is received or posted by the institution, the actual value of collateral CV is different from credit support amount C.

$$NCP_{i,t} = V_{i,t} - CV_{i,t} \tag{9}$$

The actual collateral value CV depends on the composition of collateral in the exact way described previously for CVA/DVA. Therefore, the same approach is adopted to define FCA/FBA collateral multiplier to adjust a credit support amount depending on the weight per asset/currency

$$CV_{i,t} = M_{i,t}^{Z} \cdot C_{i,t}$$

where the variable Z replaces FCA or FBA. This notation is used in the following.

As previously seen, the institution may receive or post collateral. The collateral multiplier is split into two variables depending on the sign of collateral at scenarioi level and it may be written as

$$M_{i,t}^{Z} = \left(M_{i,t}^{R,Z} \cdot \mathbb{I}_{C_{i,t}\geqslant 0} + M_{i,t}^{P,Z} \cdot \mathbb{I}_{C_{i,t}<0}\right)$$

with

$$M_{i,t}^{R,Z} = \sum_{A\in\Psi_L} \omega_A^{R,L}(i,t) \cdot \frac{1}{1-h_L^A} \tag{10}$$

[13] The eligible set is assumed to be independent of time and scenario and defined in the Credit Support Annex

[14] A value of collateral in agreement currency after applying the contactual haircut

$$M_{i,t}^{P,Z} = \sum_{A \in \Psi_L} \omega_A^{P,L}(i,t) \cdot \frac{1}{1 - h_L^A} \tag{11}$$

Contrary to CVA/DVA, the variation of collateral during a close-out period is not considered for funding valuation adjustments, only the position of collateral is reflected. For consistency, the weights must be set to the same value between FBA/FCA and CVA/DVA.

1.5 Collateral multiplier for RWA

The collateral multiplier for capital purposes is defined to be the same as for CVA/DVA, refer to (5) and (6). The difference remains only in the required close-out period, i.e. the δ is prescribed by the regulators rules[15] and may be higher than the one used in the calculation of Δ.

1.6 Optimisation

It is essential to take into consideration the new model because the strategy of each counterparty impacts the value of the institution transaction portfolio. Moreover, the latter has to maximise P&L adjustments to portfolio mark to market when selecting the asset/currency proportion for each CSA in case of posting collateral to counterparty. Denoting the cost of capital (RWA[16]) by the scaling parameter of α, the optimisation strategy may be described mathematically as

$$\left(\omega_A^{R,L}, \omega_A^{P,L}\right) = argmax_{\left(\omega_A^{R,L}, \omega_A^{P,L}\right)} \left(TA - \alpha \cdot RWA\right) \tag{12}$$

In conclusion, it is obvious that the classical formulation described in the equations (2) and (3) is the subset of the general collateral model with multiplier where the weights are nil except for the agreement currency. The option of eligible assets and currencies different from agreement currencies are frequently embedded in CSAs. In consequence, the general model with collateral model is required for this type of CSAs and the institution has to adopt the funding strategy as defined in the equation (12).

[15] Please, refer to the article 285 of CRD IV

[16] The approach may be extended to integrate KVA instead of RWA

2 Numerical application

First, we present a toy model for collateral multiplier in order to review the impact on the fair valuation adjustments and exhibit the importance to apply a collateral multiplier model. We then describe the simulation framework, derivative and methods to determine a total valuation adjustment. Finally in the third part, we analyse the results and provide few leads to choose an adequate model to be implemented for an actual portfolio of the institution.

2.1 Constant Asset Weight model (CAW)

The model for utility function described in the equations (7) and (8) may be chosen from large set of statistical or stochastic model. The constant model remains an initial simple choice to study the impact of the model on the fair valuation adjustments. The constant asset weight (CAW) model sets a proportion of asset or currency to a currently exchanged ratio level. In mathematical terms, the relation may be written as

$$\forall A,\, \omega_{i,t}^{Cpt}\left(A\right) = \omega_{0}^{Cpt}\left(A\right) = \omega_{A}^{Cpt} \tag{13}$$

$$\forall A,\, \omega_{i,t}^{Int}\left(A\right) = \omega_{0}^{Int}\left(A\right) = \omega_{A}^{Int} \tag{14}$$

This model assumes that the institution and the counterparty are following the same strategy and are expected to maintain in the future the same proportion of asset/currency till portfolio maturity when obliged to post collateral. This model is simplistic, but very intutive to analyse the effect of collateral multiplier model.

2.2 Framework

The risk factors are simulated with Monte-Carlo method using multi-currency Hull-White framework. The focus of the article remains on the impact of the collateralised exposure model, therefore the simplified formulas for CVA, DVA, FBA and FCA are used to study the impact of the new model. More details may be found in the Appendix. For simplicity reasons, an equity call option is chosen for the case study. The derivative is priced using Black-Scholes-Merton framework. The derivative parameters are summarised in the Table 1a. The reporting and agreement/valuation currency is US dollar. The eligible assets/currencies with corresponding contractual haircut for the CSA are displayed in the Table 1b. The set of eligible collateral is restricted to 10 year US government bond, 10 year German government bond, cash in dollars and euros.

Underlying	Dow Jones
Payoff	Vanilla Call Option
Notional	1 m
Strike (relative)	1.1
Maturity	5 years
Currency	USD

(a) Derivative Parameters

USD Cash	0%
US Bond	3.5%
EUR Cash	0%
EUR Bond	2.75%

(b) Contractual Haircuts

Table 1: Initial Parameters

2.3 Fair valuation adjustments with CAW model

To understand how the constant asset weight model affects the total valuation adjustment, a simple example of call option derivative is considered in the no-overlap framework: the case of buyer (the insitution receives collateral) and seller (the institution posts collateral) using the equations (4) and (9) for collateralised exposure and net collateral exposure. The valuation adjustments are deduced and displayed in the Table 2 for collateral receiver and payer respectively.

Collateral	USD		EUR	
	Cash	Bond	Cash	Bond
TA	-1193	762	-1181	211
CVA	-2386	-1519	-2436	-1740
DVA	1192	1787	1255	1706
FCA	0	0	0	0
FBA	0	494	0	245

(a) Collateral Receiver

Collateral	USD		EUR	
	Cash	Bond	Cash	Bond
TA	-1193	-3309	-1292	-2786
CVA	-2386	-3574	-2510	-3411
DVA	1192	759	1218	870
FCA	0	-494	0	-245
FBA	0	0	0	0

(b) Collateral Payer

Table 2: Global Results

Impact on CVA/DVA

As seen previously, the standard approach is a particular case of the general collateral multiplier model. The results of receiving or posting US dollar cash, i.e. the agreement currency, provide a benchmark for valuation adjustments.

In the case where the euros are exchanged instead of dollars, the total valuation adjustment increases due to higher benefits from DVA than cost in CVA for a collateral receiver. It represents a gain in officially reported P&L of the portfolio. For a collateral payer, the total valuation adjustment decreases, i.e. a loss on P&L is generated. It is due to a higher cost on CVA than benefit on DVA.

Rebalancing a collateral currency affects the credit adjustments, CVA/DVA. The funding desk has to assess those impacts in plus of a cheapest to deliver cost[17] of currency.

Impact on FBA/FCA

The funding adjustments, FCA and FBA, do not appear when cash collateral is received or posted. It is due to the following factors: the assumption of credit support amount equal to mark to market, i.e. fully collateralised, and the zero haircut applied to the non-agreement currency set in the study[18]. The mark to market of non-cash collateral has to be higher in comparison to the credit support amount C due of course to application of a contractual haircut set in CSA.

In the case the USD bonds are exchanged instead of cash in US dollars,a funding benefit [19] is generated for a collateral receiver. On another hand, a collateral payer is exposed to a funding cost. The similar effects are observed for EUR bonds. Therefore, exchanging a non-cash collateral, such as bonds, creates a funding cost or benefit.

Conclusion

The simplified collateral models such as the CAW model provide an overview of collateral risk effects on the valuation adjustments. It also demonstrates the importance of applying the collateral multiplier when calculating valuation adjustments or capital cost. In real market conditions, the cheapest currency to deliver or available collateral may change, therefore keeping a static strategy[20] until portfolio maturity is not realistic. In consequence, the constant asset weight model is only a simplification of real practice, it does not reproduce an actual collateral process.

[17] Repo rate

[18] The industry usually applies a zero contractual haircut cash for G10 currency

[19] It is true only if the asset may be re-hypothecable and its repo rate is ignored

[20] Maintain the same proportion of asset/currency at each margin call

3 Enhanced utility function for asset weights

As established in the previous section, the collateral model with multiplier must be applied in order to correctly adjust the portfolio value. The choice of the utility function g is the main factor allowing to project a potential collateral risk and simulate the strategy of an institution or a counterparty. As seen in the previous section, the static or constant function for g is not a realistic approximation of the collateral and funding management. The equations (7) and (8) do not specify a parametrical form of the function g, only a general definition. Choosing g is a complex statistical problem. Due to computational time, the deterministic functions based on a reduced scope of inputs may be considered, represented mathematically:

$$\omega_A^{R,L}(i,t) : (C_t, \Upsilon_{i,t}, A, \Psi_L) \longrightarrow g^{Cpt}\left(C_t, \Upsilon_{i,t}, A, \Psi_L\right). \tag{15}$$

The parametrical form for the weights transforms an optimisation problem, defined in (12), into a complex multi-dimensional HJB[21] equation of optimal control. A solution may bring an insight on the best strategy to follow for margin calls by the funding desk in order to manage capital requirements and cope with P&L adjustments.

Conclusion

The choice of collateral impacts the capital and P&L of the institution. As well foreseen by [3] back in 2012, the risk weighted asset became the focus of banks. Therefore, the RWA optimisation through adequate collateral models (in the IMM framework) may allow further mitigation of capital increase imposed by regulation rules. The additional accounting measures already enacted or currently in discussion involving valuation adjustments to risk-free P&L may only emphasis the importance of collateral models, described in [15]. It stimulates further research on the collateral optimisation and new methods to be incorporated in valuation adjustments. The aggregation of collateral optimisation and xVA metrics into one big data algorithm seems highly desirable in the future as suggested in [2]. Finally, a global framework is a key component in order to implement and align funding and capital strategies.

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Appendix

The Hull-White/Vasicek framework is defined by

$$\begin{cases} dr_t = (\theta_t - \lambda r_t)\, dt + \vec{\eta} d\vec{W}_t^{\mathbb{Q}} \\ dr_t^{'} = \left(\theta_t^{'} - \lambda^{'} r_t^{'}\right) dt + \vec{\eta}^{'} d\vec{W}_t^{\mathbb{Q}} \\ \frac{dX_t}{X_t} = \left(r_t - r_t^{'}\right) dt + \vec{\sigma}_X d\vec{W}_t^{\mathbb{Q}} \\ \frac{dS_t}{S_t} = r_t dt + \vec{\sigma}_S d\vec{W}_t^{\mathbb{Q}} \end{cases}$$

with

- $\vec{W}_t^{\mathbb{Q}}$: a $\mathbb{R}^4$ domestic brownian motion under $\mathbb{Q}$ domestic risk-neutral probability,
- r: a domestic risk-free interest rate with $r_0 = 0.3$,
- $r^{'}$: a foreign risk-free interest rate with $r_0^{'} = 0.2$,
- $\vec{\eta}$: a constant domestic volatility of interest rate with $\vec{\eta} = (0.005; 0; 0; 0)$,
- $\vec{\eta}^{'}$: a constant foreign volatility of interest rate, $\vec{\eta}^{'} = (0.00175; 0.003031; 0; 0)$,
- θ: a constant domestic mean-reversion level of interest rate, $\theta = 0.026979$,
- $\theta^{'}$: a constant foreign mean-reversion level of interest rate, $\theta^{'} = 0.008863$,
- λ: a constant domestic mean-reversion spead of interest rate with $\lambda = 0.05$,
- $\lambda^{'}$: a constant foreign mean-reversion spead of interest rate with $\lambda^{'} = 0.04$,
- X: an currency exchange rate assuming one unit of domestic currency may be exchanged for X foreign units, i.e. $1d = X \cdot 1f$ with $X_0 = 0.9$,
- $\vec{\sigma}_X$: a constant volatility of the currency exchange rate X with $\vec{\sigma}_X = (-0.02; 0.020785; 0.074619; 0)$,
- S: an equity price in domestic currency with $S_0 = 18000$,
- $\vec{\sigma}_S$: a constant volatility of the equity S with $\vec{\sigma}_S = (-0.03; -0.017320; 0.018226; 0.092021)$.

As stated in [6] and [13], the valuation adjustments in discrete simulations may be written

$$CVA = s^C \cdot \sum_{j=0}^{N-1} EPE_{t_j}^{*} \cdot (t_{j+1} - t_j) \tag{16}$$

$$DVA = s^{Int} \cdot \sum_{j=0}^{N-1} ENE_{t_j}^{*} \cdot (t_{j+1} - t_j) \tag{17}$$

$$FBA = -rf^{Int} \cdot \sum_{j=0}^{N-1} e^{\left(s^{Cpt} + s^{Int}\right) \cdot t_j} \cdot ECC_{t_j}^{*} \cdot (t_{j+1} - t_j) \tag{18}$$

$$FCA = -rf^{Int} \cdot \sum_{j=0}^{N-1} e^{\left(s^{Cpt} + s^{Int}\right) \cdot t_j} \cdot ECB_{t_j}^{*} \cdot (t_{j+1} - t_j) \tag{19}$$

where

- rf^{Int}: a constant institution funding spread, $rf^{Int} = 0.001$,
- s^{Int} : a constant institution credit spread, $s^{Int} = 0.005$,

- s^{Cpt}: a constant counterparty constant credit spread, $s^{Cpt} = 0.006$,
- j: a discrete integer to represent a simualtion time point numbering with $t_j < t_{j'}$ if $j < j^{'}$ and $j \in [0, N]$,
- $EPE^{*}_{t_j}$: a discounted expected positive exposure for time t_j,
- $ENE^{*}_{t_j}$: a discounted expected negative exposure for time t_j,
- $ECC^{*}_{t_j}$: a discounted expected net collateral cost at time t_j,
- $ECB^{*}_{t_j}$: a discounted expected net collateral benefit at time t_j.