ABSTRACT

We suggest that some managers are able to use their unique human capital to innovate, execute and manage abnormally profitable investment opportunities. These skillful entrepreneurial managers may appear to violate rational investor norms when in fact they are behaving quite rationally. Using a real growth options valuation model under highly competitive and risky market conditions, we determine firm value from the perspective of a manager whose human capital is tied to the firm’s prospects. Our model further determines firm valuation when individual behavioral biases, especially overconfidence, are incorporated. Specifically, we derive precise measurements of excessive risk taking, overinvestment, and overvaluation associated with excessive managerial overconfidence.

JEL Classification: D81; G02; G13; G32; J24

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Human Capital, Managerial Overconfidence, and Corporate Valuation

“I believe Apple’s brightest and most innovative days are ahead of it.”
– Steven P. Jobs, Outgoing Apple Inc. CEO, August 21, 2011

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1. Introduction

The question of the true value of a firm has remained a puzzle. Quite often, management of the firm thinks that the company stock is undervalued by the capital markets at the same time that outside investors consider it to be overvalued (see e.g. Graham and Harvey, 2001; Smith, 1986; and Rock, 1986). Empirical research providing strong evidence of the existence of equity pricing discounts by the market have widely shown that whenever managers (CEOs) believe that their company stocks are undervalued, they undertake actions that include stock repurchases, contingent value rights issuance, and cash acquisition decisions (see e.g. Elton and Gruber, 1968; Masulis, 1980; Dann, 1981; Dittmar, 2000; and Chatterjee and Yan, 2008). On the other hand, there has been a large body of evidence suggesting that managers often overestimate their firms’ growth opportunities, hence, the valuation of their firms. For instance, Kurz et al. (2005) report the findings of financial analysts that show top-down valuation of equity composites being generally lower than aggregating the bottom-up forecasts of individual companies. Gao and Shrieves (2002), Burns and Kedia (2006), Bergstresser et al. (2006), Bergstresser and Philippon (2006), and Cornett et al. (2008) document that inflated corporate valuation serves the interest of managers who own stock options. Shleifer and Vishny (2003) suggest that managers favor a high stock price since it gives a firm the currency to make acquisitions while Schrand and Zechman (2012) report a linkage with earnings manipulation and corporate fraud. In fact, recurring episodes of the existence of asset

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1 In a speech announcing his resignation from the position of CEO of Apple Inc. as quoted by the Wall Street Journal (August 27, 2011).
price “bubbles” have given credence to investors’ lingering skepticism about the true corporate valuation.²

Until recently, financial economists have traditionally explained the disagreement between management and investors over the issue of corporate valuation using two main theories. The first is the theory of information asymmetry (Myers and Majluf, 1984), which argues that investors naturally face an information disadvantage. Inside managers always know more about the firm and its prospects than outside investors do and, for fear of competitive forces, prefer to keep corporate information secret. The second theory is that of the principal-agency problem (Jensen and Meckling, 1976), which argues that managers (agents), in their self-interest, tend to act in manners that are at variance with the interests of the shareholders. A plethora of rigorous research conducted over the past three decades has spurred measures including full disclosure legislations, executive compensation designs, and corporate governance reforms that, to some degree, have helped mitigate the information asymmetry and the agency problems (see e.g. Hermalin, 2005; and Shleifer and Vishny, 1997). One prominent example of these measures is the U.S. Sarbanes-Oxley Act of 2002 whose primary goal, among others, is to increase the quality and precision of the information disseminated by the CEOs to investors. However, despite the documented improvements in financial reporting and corporate governance, the persistence of the valuation disparity still remains and the explanation that is increasingly receiving more weight is the behavioral one.

In this paper we present a valuation model of the firm that provides a solution to the puzzling question on why investors and management most often have no consensus on the true value of the

² The collapse of Long-Term Capital Management in 2000, Enron Corporation in 2002, Lehman Brothers in 2008, and several dot.com firms, 2000–2002, close to the peak of their equities’ price levels, are instructive.
firm. In the model, we show how some CEOs may arrive at the wrong assessment that they make about firm value due to biases in the beliefs about the true quality of their human capital. These managers convey to the market or perceive that they have high skill, which they do not have, by exhibiting excessive overconfidence about the prospect of their firms. Under rational capital market conditions investors should obviously view the companies they run as overvalued. However, we suggest that some CEOs who are truly innovative and entrepreneurial, who have a “skin in the game,” provide credible intellectual capital to the firm in deeper ways than outside analysts may be able to quickly comprehend and capture. Typical examples of these CEOs are those with extraordinary skills and who are innovators in the firms that they manage. These CEOs are able to exploit their unique abilities by using their human capital to identify, execute and manage competitively profitable investment opportunities in such a way that may appear to demonstrate overconfidence when in fact they are behaving quite rationally. We particularly examine the behavioral investment decision of the CEOs in a real options framework and show how managerial human capital, or skill, influences corporate investment and valuation. Furthermore, we make the assumption that the competing CEOs (firms) operate in an industry where innovation is important and verifiable.

From early works like Larwood and Whittaker (1977) to contemporary ones like Barber and Odean (2001), Graham and Harvey (2001), Malmendier and Tate (2005, 2008), Sautner and Weber (2009), and Ben-David et al. (2010), an influential body of research on managerial behavior and

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3 Kaplan, Sensoy, and Stromberg (2009) find that the participation and skill contribution of founders of venture capital-financed companies tend to decline over time. In our model, however, the skill contribution of a CEO is assumed to be persistent over the entire model period, consistent with DeBondt and Thaler (1995), Bernado and Welch (2001), Malmendier and Tate (2005, 2008), Goel and Thakor (2008), and Hirshleifer et al. (2012).
corporate policy has added to this debate and has provided support to the market view on the puzzle by arguing that CEOs quite often exhibit excessive overconfidence about the future potential of their businesses leading to overvaluation of the firms. As individuals, the CEOs are driven by emotions which may lead them to believe, for example, that they have better skill than an average CEO, the risks they face are relatively low, and the expected cash flows are higher than average.

The model that we present demonstrates how managerial skill and effort interact to produce competitive advantage which in turn sways the investment decision and valuation by the CEO who continues to believe that her human capital is superior and is tied to the fortune of the firm. While the model reaffirms the traditional view in classical economics of the strong correlation between high skill and high firm value, it also indicates that relative to an identical but rational CEO, an excessively overconfident CEO overvalues the firm, overinvests capital, and underestimates the risk of the venture. These results are consistent with the literature on overconfidence and underestimation of risk (see e.g. Goel and Thakor, 2008; Gervais et al., 2011; and Hirshleifer et al., 2012), overconfidence and overinvestment (see e.g. Malmendier and Tate, 2005; Doukas and Petmezas, 2007; Goel and Thakor, 2008; Billet and Qian, 2008; Ben-David et al., 2010; and Gervais et al., 2011), and overconfidence and overvaluation (see e.g. Barber and Odean, 2001; Graham and Harvey, 2001; and Malmendier and Tate, 2005). Our definition of a CEO who exhibits overconfidence is one that has an upwardly biased belief about her own talent.

Moreover, we are able to precisely measure these effects of overconfidence, an important contribution of the paper. Specifically, under a base case assumption of a firm whose market value consists solely of future growth opportunities and whose volatility is 25 percent, present value of net operating cash flow is $600 million, and capital investment requirement is $250 million, a rational CEO would value the firm at $282.152 million. An identical but otherwise overconfident
manager overvalues the same firm by 5.1 percent (low overconfidence), 10.3 percent (moderate overconfidence), and 19.8 percent (high overconfidence). We also determine that while a rational manager would require a $250 million capital investment to exploit the growth opportunity, an identical but otherwise overconfident manager would overinvest by $19.7 million (low overconfidence), $36.8 million (moderate overconfidence), and $65.2 million (high overconfidence) to generate the same optimal value. Furthermore, our analysis estimates the rational CEO’s assessment of the relative risk of the investment opportunity measured by its capitalization rate at 30.0 percent. For a manager who bears overconfidence biases, the comparable risk is much lower, implied by the capitalization rate of 23.5 percent (low overconfidence), 17.9 percent (moderate overconfidence), and 8.6 percent (high overconfidence).

The study of corporate valuation is of significant importance since valuation is the cornerstone of all financing, investing, operating, and risk management decisions. According to standard finance theory, capital market prices should and are determined by unbiased investors who act rationally using all available information to draw their expectation of the present value of the firm’s future cash flows. On one hand, the persistence of the valuation disparity we have alluded to obviously weakens the usual argument for efficient capital markets. On the other hand, some of the most harmful effects of this disagreement, if it sustains, are the likelihood of increasing correlation among market, credit, and operational risks and the possibility that capital may not be allocated optimally to productive investments in ways that would benefit the firm, investors, the economy, and society. Overall, the findings of a study like ours could help researchers and regulators further strengthen financial reporting and disclosure requirements with the objective to resolve the underinvestment and overinvestment problems and to detect and prevent asset price
“bubbles” as well as fraudulent managers whose reported performances usually rest on the premise that they have high-skill intensity but which they truly do not have.

The rest of the paper is organized as follows. In the next Section we present the setup and motivation for the model and briefly discuss related literature. The model is introduced and discussed in Section 3. The results of the model and the sensitivity analysis are presented in Section 4. Section 5 concludes.

2. The setup

We motivate the development of our model by considering the phenomenal story of the late Steve Jobs, the co-founder of Apple Inc. The company, with Steve Jobs as its CEO, went public on December 12, 1980 at an offer price of $14 per share and the stock closed the first day at a price of $29, garnering a market capitalization of $1.43 billion (see Table 1). On May 31, 1985, Steve Jobs was fired from the CEO post following internal disagreement with the board. On that day the company’s stock closed at $17.37 for a market capitalization of $1.06 billion. Steve Jobs had to leave a company that had ownership of at least three patents that were awarded in his name as the inventor. Interestingly, on his departure he sold only one of the more than four million shares of Apple that he owned, somewhat reflecting the confidence that he had in the firm and the recognition of the value of his human capital still tied to the firm. More than ten years later, on September 16, 1997, Steve Jobs formally returned to the company as CEO after serving for about a year as advisor to then CEO Gil Amelio at a time when the company nearly faced bankruptcy. Apple had just acquired NeXT in 1996, a computer platform company that Steve Jobs had founded. For the next fourteen years as CEO, Steve Jobs managed a company that saw phenomenal transformation and growth, with market capitalization rising from $2.91 billion to $338.49 billion, translating into an annualized average growth rate of 140 percent. When he announced on August
21, 2011 that he was stepping down from the position of CEO of Apple for health reasons, the company had ownership of 313 patents that listed Steve Jobs as the principal inventor or co-inventor. Apple’s stock closed on that day at a price of $376.18 for a market capitalization of $339 billion. Even as he ended his leadership, Steve Jobs still sounded very optimistic about the future potential of the firm as illustrated by the opening quotation attributed to him. According to FactSet, at the time of Steve Jobs’ resignation, Apple had just outperformed Wall Street analysts’ consensus earnings forecasts for thirty five consecutive quarters, implying that the market had consistently undervalued the company throughout that period. By August 20, 2012, one year after Steve Jobs had left the company and barely nine months after his death, Apple stock price had risen by 177 percent to close at total market capitalization of $618 billion consolidating itself as the most valuable company in the world. At the close of July 15, 2015 that value had hit $730 billion. Of the total 454 patents that are listed to date in the name of Steve Jobs as the primary inventor or co-inventor, 141 have been granted since his death.

[Table 1 Here]

Thus, it can be argued that Steve Jobs’ constant expression of optimism in Apple as highlighted above was driven by the belief in the influence that his human capital (“skin-in-the game”) was bearing on the long-term fortune of the company which was constantly backed by verifiable fundamentals, specifically digital innovations and the remarkable growth in real value created. We describe the behavior exhibited by a manager such as that of Steve Jobs as rational confidence (optimism), and not overconfidence.

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4 Apple Inc’s 7-for-1 stock split went into effect on June 9, 2014. The company’s previous stock splits included 2-for-1 on February 28, 2005; 2-for-1 on June 21, 2000; and 2-for-1 on June 16, 1987.
Finance theory and practice both acknowledge that the value of a firm essentially has two identifiable components: the value of assets-in-place and the value of expected future growth opportunities (Tobin, 1969; Myers, 1977). In this paper, we make the assumption that the value of assets-in-place is common knowledge. That is, we argue that there should be no conflict between inside managers and outside investors regarding the assessment of the value of the firm’s existing assets. Rather, the disagreement should center on the estimation of the value of future growth opportunities, the present value of new investments together with the uncertainty regarding the correlation of existing assets with future investment opportunities. We further note that the firm’s growth option value is often driven, among others, by two key risk factors. The first factor is market-priced risk, the long-term product market expectation, which is assumed in this paper to be equally shared by every market participant. In other words, the cash flow forecast or the size of the product market – should the innovation that unleashes that product turn out to be successful – is assumed to be known at the outset by all the competing firms. However, the date of innovation and the identity of the entrepreneur are not known a priori. The second factor is the internal competitive advantage of a firm or the skill required to make the innovation, which is determined by management’s intellectual or entrepreneurial capital. In our model, it is this private risk factor which primarily governs the behavior of a CEO regarding the prospect of the firm, which is either rational optimism or emotional overconfidence.

[Figure 1 Here]

According to behavioral research, individuals who exhibit overconfidence overestimate both their own abilities and the precision of the information that they have (Barber and Odean, 1999; Statman, 1999; Shefrin, 2002); and Barberis and Thaler, 2003). Psychologists have defined overconfidence as the *illusion of knowledge*, which embodies two basic types of overconfidence:
One is *certainty overconfidence* and the other is *prediction overconfidence* (see e.g. Nofsinger, 2008; and Pompian, 2012). *Prediction overconfidence* relates to setting confidence intervals too narrow which leads the individual to underestimate the downside risk while overestimating the upside potential. *Certainty overconfidence*, on the other hand, relates to over-stated probability of success. This psychological bias leads people to overrate their own abilities and underrate an informationally-disadvantaged position. The emotional bias that we introduce to the human capital in this paper strictly focuses on the *certainty overconfidence*. We make the assumption that the product market expectations, or the forecasts of the firm’s future cash flows, are equally shared by all competitors – which would negate the existence of the *prediction overconfidence* in the model. However, we are intrigued by the results of the model that this is not the case since the underestimation of risk emerges inherently in the equilibrium solution when the *certainty overconfidence* is present.

Despite the difficulty usually faced in its measurement, human capital is considered one of the most important determinants of enterprise and economic growth. At the macro level, economists have long utilized the neoclassical Cobb-Douglas growth function to capture human capital, proxied by total factor productivity, as a key driver of the real growth rate in gross domestic product (GDP). At the micro level, however, there is no standard model we are aware of that explicitly incorporates managerial skill or alpha despite the significant role that human capital plays in corporate valuation. A plausible explanation is that individual habit formation, whose absence provides a major shortcoming of the standard finance models, is difficult to model (Rubinstein, 2001). Nevertheless, investors and firms still generally believe that businesses with highly talented employees are competitively superior and relatively more valuable. In this paper, we fill that gap by embedding competitive advantage (or skill) into our real options model.
The model framework and its assumptions allow us to fulfill a key goal of the paper, which is, to explain individual managerial behavior in risk and valuation practices as they relate to and contrast with the standard finance models. In the paper we use the terms “CEO” and “management” interchangeably and make two distinctions of CEO type. The first one is the rationally optimistic high skill type represented by the character of Steve Jobs who expresses strong long-term confidence and whose views are backed by proven inventive contributions that are empirically supported by the company’s positive fundamentals. Such a manager has “skin-in-the game” and believes that the future performance of the company, and especially the value of its growth opportunities, is highly dependent on her human capital. In our model the rational CEO typically bases valuation of the firm on at least eight key factors and the effects of the interactions among them on expected value. First, is the manager’s personal success rate \( g(a) \) in generating valuable information, which defines true skill or human capital that drives the determination of competitive advantage and the optimal time to capture the investment opportunity. Second, is the cost of effort \( c \) reflecting funds incurred each period to finance the manager’s efforts in search of valuable information. We model a CEO’s competitive advantage to be conditional on the skills \( g(b) \) of other competing CEOs, which constitutes competition, our third factor. Fourth, is size of the market or investment opportunity \( V \). This is a measure of the product market expectations, the present value of the forecasted operating cash flow aggregate for the entire industry. While the projected operating cash flows are assumed to be common knowledge, their present value \( V \) which represents the underlying asset of the real investment growth options, is not since the specific schedules of the cash flows are not known and \( V \) must be determined competitively by a firm’s optimal timing.
The fifth factor is *market risk* ($\Delta$), which captures both the volatility of future operating cash flows as well as the volatility of the potential capital investments. It represents the volatility of the growth option in the model. Sixth, is *size of that capital* ($X$) required to finance the production and introduction to market of the new innovation in the event that a decision to invest is made. In the model, this variable is considered the strike price of the real growth option. Our seventh factor is the *correlation* ($\rho_{x\alpha}$) of the rate of return on the market opportunity with the rate of growth of capital investment, which directly affects the measure of volatility. If forecasted capital expenditures are reasonably certain, then correlation is basically zero. However, under some business cycle conditions this may not be so. For instance, if $X$ is acquired debt capital whose credit quality is dependent on the expected operating cash flows from the investment opportunities, then the correlation between capital expenditure and growth opportunities may be high and positive. The final factor in the model is *time horizon* ($\tau$) within which the investment opportunity can be exploited, which also represents the growth option’s time to expiration. Given these inputs and the quality of the human capital represented by Steve Jobs, the capital markets should have rendered a fair valuation of Apple Inc. At worst, they would have incorrectly undervalued it.

We frame the second CEO type as the excessively overconfident agent whose behavior is also governed by the eight factors listed above but whose personal beliefs about her probability of success are biased. In other words, in the model we allow violation of the rational assumption underlying only the first factor, which is skill, by introducing a measure of overconfidence. Under condition of constant competition, an overconfident manager wrongly believes that her skill is higher than it truly is which leads her to underestimate risk, overinvest, and overvalue the investment. We thus utilize two scenarios to simulate and discuss our model results:

- **Case 1 – Rational Confidence (Base Case):**
Competing managers are all assumed to be rational. High-skill manager has a higher conditional probability of success and attains competitive-advantage. Low-skill managers have lower probability of success but they do acknowledge the competitive-advantage level of the high-skill manager once the winner is declared. Holding constant the ability of the low-skill managers, we also examine the effects of varying the rational self-confidence (competitive advantage) of the high-skill manager.

- **Case 2 – Overconfidence:**

  The high-skill manager is irrational and perceives that her conditional probability of success is higher than it truly is. Competing (low-skill) managers are rational and have low probability of success as measured under Case 1 but they acknowledge and believe that the ability of the high-skill manager has not changed from the level depicted under Case 1.

  Our paper documents another category of very interesting outcomes. Under the base case scenario, the real growth option model is able to explain the investment behavior of rationally confident managers, exemplified in Steve Jobs, and find results that are consistent with the implications of overconfidence, loss aversion, and disposition effect, theories that have solely been associated with psychological biases. It is important to emphasize here that these results do not necessarily mean the CEOs are behaviorally biased but that they are likely using a valuation model like ours that is more flexible and fully descriptive of their qualities than the traditional valuation models in standard practice. For instance, similar to the findings in Benabou and Tirole (2002) and Goel and Thakor (2008), our model expressly shows that if the manager has a high competitive advantage, the manager will find it valuable to undertake higher risk. This parallels the tendency of excessively overconfident investors who carry more risks in their “behavioral portfolios” than would be prescribed by traditional portfolio theory (Shefrin and Statman, 2000). We also document
similar results attributed to the positive roles of overconfidence on valuation (see e.g. Goel and Thakor, 2008; Graham et al., 2009; Gervais et al., 2011; and Hirshleifer et al., 2012).

When we characterize the relative size of our product market by its moneyness we also document various business conditions under which the manager makes altering investment decisions. Specifically, whenever the size of the market opportunity (forecasted operating cash flows) is small, it is generally more valuable for the rational manager to terminate a growth investment earlier than later. However, when the risk is larger, but the market size is small, the manager has a higher propensity to postpone terminating the investment. This outcome is akin to the disposition effect in the behavioral finance literature in which it is less emotionally distressing for an investor to sell a losing investment later rather than earlier (see e.g. Shefrin and Statman, 1985; and Genosove and Mayer, 2001). The model also shows that the magnitude of the value sensitivity-to-risk is larger whenever the market is deep out-of-the money than when it is deep in-the-money. In addition, whenever the market opportunity is small (deep out-of-the-money), value sensitivity to risk declines and it remains systematically lower for a rational manager than for an otherwise identical but overconfident manager. However, the reverse is true whenever the market potential is deep in-the-money. This twin result is consistent with the concept of loss aversion which stipulates that individuals tend to assess a higher value to a loss than to a gain of equal magnitude (see Khaneman and Tversky, 1979; and Odean, 1998) and to the claim made by prospect theory that individuals tend to be risk-averse in the domains of gains and risk-seeking over the domains of losses (Khaneman and Tversky, 1979). We therefore conclude that, for managers who are rationally optimistic with truly superior competitive advantage, the market may be incorrectly undervaluing their stocks as it was with Apple Inc.
It turns out from the results we present that our model also helps shed light on a problem arising from the counterintuitive consequences of an overconfident manager that has been highlighted in both theoretical and empirical literature confirming both adverse and beneficial effects of overconfidence (see e.g. Hirshleifer et al., 2012). The findings that we document when we structure overconfidence into the model are largely consistent with the adverse effects of the trait. However, as discussed, we also find outcomes that are similar to the reported beneficial effects of overconfidence especially when we restrict the model to its rational form. Specifically, the positive results are found to be driven by verifiably higher competitive advantage of the manager, which implies rational self-confidence (optimism), and not overconfidence. The reason we are able to obtain these distinctive results is because we treat overconfidence clearly and unambiguously. That is, since overconfidence is a behavioral bias, a manager exhibiting that trait must be acting irrationally, the position we take throughout this paper. It is possible that some of the opposing effects reported especially in the empirical literature may be due to the differences in

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5 Additional counterintuitive effects of managerial overconfidence have been reported in the literature including: On the bright side, improving upon managerial promotional prospects (Goel and Thakor, 2008); presence and survival of entrepreneurs (Bernardo and Welch (2001)); mitigating the debt-overhang problem (Hack Barth, 2008); and achieving greater innovation but only in innovation industries (Hirshleifer et al., 2012). On the dark side, destruction of firm value (Goel and Thakor, 2008; and Gervais et al., 2011); value-destroying mergers and acquisitions (Roll, 1986; Malmendier and Tate, 2005, 2008; Doukas and Petmezas, 2007; and Billet and Qian, 2008)); underinvestment (Goel and Thakor, 2008); and corporate fraud (Schrand and Zechman, 2012).

6 We also note other papers that have provided solutions to the conflicting effects of overconfidence. For instance, Goel and Thakor (2008) and Gervais et al. (2011) show that mild overconfidence increases firm value but extreme overconfidence harms it. Hirshleifer et al. (2012) report positive and significant roles of overconfidence in influencing innovation but only among firms in innovative industries.
defining overconfidence. For instance, in Malmendier and Tate (2005, 2008), a CEO is classified as overconfident if she persistently fails to reduce her personal exposure to the firm’s idiosyncratic risk by voluntarily postponing exercising vested stock options that are at least 67 percent in-the-money. Hirshleifer et al. (2012) use a similar measure but add another proxy for overconfidence based on media coverage. In their approach a CEO is said to be overconfident if the number of press articles linked to the CEO that contain the word “confidence” or a reference to an “optimistic” mood is greater than the number of articles containing the word “cautious” or any “pessimistic” reference. Based on our model framework, it is conceivable that some of the CEOs sampled as overconfident in these studies are just rationally highly confident.

Apart from the studies on behavioral biases on corporate policy that we have cited, our work is also similar to the literature on real growth options (see e.g. Myers, 1977; Kester, 1984; Trigeorgis, 1995; and Ottoo, 1998), and R&D investments (see e.g. Loury, 1979; Dasgupta and Stiglitz, 1980; and Kamien and Schwartz, 1980). However, our paper is most closely related to the theoretical works of Benabou and Tirole (2002), Goel and Thakor (2008), and Gervais et al. (2011). Benabou and Tirole (2002) show how self-confidence improves upon one’s motivation to undertake risky and challenging projects and how learning of bad news can self-handicap the individual. In their model, an agent selects an action that elicits information structure at the end of that first period at which point a decision is made to expend effort and undertake a project whose payoff, if it succeeds, is generated at the end of the second period. The agent’s belief over the probability of project success defines her ability and her self-confidence, a treatment similar to ours. Goel and Thakor (2008) model overconfidence and CEO selection. Here, several managers of a firm who must rely on the strategy of an incumbent CEO choose the riskiness of their own projects. The abilities of the managers are not known a priori even to the managers themselves and
are only inferred from the project payoffs observed at the end of the first period when the highest-ability manager is chosen to succeed the CEO. The contrast we make with their paper is that in our model a manager only knows her own ability but not her relative ability or the ability of her competitors. Gervais et al. (2011) analyze the links among overconfidence, compensation contracts and investment decisions and demonstrate both the risk and benefit of overconfidence. In their model, a manager is assumed to have access to a private signal defining the probability of capturing end-of-period payoff, which denotes the manager’s true skill. In an approach similar to ours, they introduce overconfidence when the manager overestimates own skill.

Our paper however differs from these three related studies in several other ways. First, competition among CEOs for a chance to exploit the exogenous growth opportunities is important in our model as the payoff is not given. A manager’s probability of success is evaluated as conditional on the success rate of the competitors. In Gervais et al. (2011), competition is introduced but it is treated differently in the sense that two different firms compete for the same manager’s labor services. And in Goel and Thakor (2008), competition is intra-firm where several managers compete to replace the incumbent CEO but their abilities to succeed are dependent on the ability of the CEO. Second, in our model the timing of the payoff date is not pre-specified but is determined optimally through competition. Third, despite the distribution of the future cash flow payoffs being common knowledge, the magnitude of the present value of the expected cash flows is not known a priori since it is driven by the optimal time of innovation which is endogenously determined by the conditional success rates of the managers. Fourth, in our model, all the competing managers choose the same market-priced risk regardless of their skills. Thus, managerial ability and project risk are handled separately and produce distinguishable effects. Fifth, the underestimation of risk which is associated with the notion of overconfidence emerges
automatically from the manager’s optimization problem in our model and is not explicitly specified as in Goel and Thakor (2008). Finally, we make a clear distinction between the cost of effort which represents the premium and capital investment which represents the strike price of the real growth option. Unlike, for instance, in Benabou and Tirole (2002), the size of the capital investment determines the capital budgeting decision in our model and it is the variable that is relevant to the overinvestment problem and not the cost of effort.

The central premise of this paper is to attempt to link the formal prescriptive models of standard finance with the intuitive concepts of behavioral finance. The paper acknowledges the potential contribution of behavioral studies in guiding financial economists in fine-tuning rational models and identifying any shifts in the basic paradigm of corporate valuation. Of course, we acknowledge that there are several aspects of individual behavior that may be impossible to capture in a quantitative model. One important interpretation of our model’s output is the possibility that, for some managers, the observed behavioral anomalies could, in fact, be due to the actions of rational agents who might instead be using different valuation models while driven to utilize their unique skills to exploit profitable investment opportunities in search of higher active returns. The real options model we present is testable, relatively flexible and it incorporates competition, skill, risk, timing, finance, and market potential, some of the factors which have not been explicitly captured by the standard corporate valuation models.

3. The model

In a market setting characterized by uncertainty and competition, we offer a model that defines a manager (CEO) who, acting as an entrepreneur, is seeking to innovate and implement a profitable investment for the firm. We assume that the manager has competitors in the industry who are all identical. To simplify the analysis, we present a two-firm (two-manager) structure without loss of
generalities. We posit that the expected value of the investment opportunity, from the perspective of an individual manager, is a function of risk, competitive advantage, time, capital, and the size of the potential investment. In a real call option setting, we assume that the gross present value of the investment, which represents the underlying asset and is denoted as $V$, is identifiable. While the size of the investment payoff is known to every competing firm, the ability to acquire it is not and requires effort. Each firm-manager understands that gaining the right to capture $V$ requires sufficient skill or knowledge that can only be acquired by engaging in “knowledge creation,” or research and development investment, in order to win the competitive race. The model does not rule out the possibility that two or more managers with identical skills can win the right to acquire $V$ simultaneously. However, in this paper, we assume a single winner for ease of tractability.

The investment opportunity in question is presumed to be a new venture not associated with any existing assets. In other words, we assume that the assets-in-place and the future growth opportunities are not correlated. This consideration is important for the purpose of distinctively evaluating the impact of the key value drivers. The manager initiates the financing of own knowledge acquisition, at a cost of $c$ per period at date $t = 0$ and believes that the competitor is doing the same. In the model we assume that the manager who ultimately wins the race to be the first to innovate will also be the sole firm granted the right by a regulatory governmental agency to acquire and manage the investment. The first manager to win is assumed to do so on date $t = \tau$ when the research becomes successful (optimal skill is acquired). That decisive event is publicly announced through a “patent award” secured by the manager. On that date, the simultaneous decision to incur the capital expenditure in order to exploit the investment is made. The acquired investment begins generating cash flows from date $t = \tau + l$ which continues in subsequent periods,
forever, and the gross present value of these cash flows at the time of exercise decision, is equal to \( V(\tau) \). Figure 2 illustrates the sequence of events for the investment.

[Figure 2 Here]

In every period while pursuing the innovative outcome, the manager faces essentially two components of risk which underlie this investment opportunity. One is private risk and the other is market-priced risk as mentioned earlier and illustrated also in Figure 1. From date \( t = 0 \) the primary concern of each of the competing CEO is whether they will develop the right technical skill and eventually capture the investment opportunity. In the model, acquiring skill or technical success and making the innovative discovery are considered to occur simultaneously at \( t = \tau \). Once the innovation is established (private risk is mitigated) and the decision turns to capital investment funding, the dominant worry shifts to market-priced risk, which is comprised of the size and volatility of both the required capital investment and the operating cash flows generated from the investment. Suppose that the manager in question is successful, the investment decision would then be governed by the unique market condition, or scenario, at play. The manager would particularly care about the level of risk, the size of the market, and the size of capital required at the time that the investment must be launched. An optimal mix of these three elements, given a successful innovation, would therefore favor an exercise decision.

3.1. Investing in effort: competitive advantage

In this Section we present the analysis highlighting the first stage of the model (see Figure 2). Individual skill or human capital, which is endogenous, is associated with driving competitive advantage and is considered a measure of private risk. There are two components of private risk: the risk that the manager’s efforts will not culminate into a skill set that is innovatively valuable enough to create a new profitable venture; and the uncertainty about whether the innovation will
be made in time to give the firm a first-mover advantage. To incorporate the effect of competition, we assume that the manager who wins automatically captures the sole right to acquire $V$, and the loser gets nothing. We incorporate the optimal timing model with the real growth option framework and show that if the manager is to win and purchase the right, but not the obligation, to capture $V$ then the manager must make the innovative discovery before the competitor does, specifically at date $t = \tau$. Derivation of the expected value of $\tau$ is a critical element of the model, which we accomplish below.

3.1.1. Rational confidence

In a simplified information structure we consider that the two CEOs are of two skill types: Manager $H$ is assumed to be high-skill CEO in the sense that she is capable of producing superior skill productivity. Manager $L$, on the other hand, is assumed to be low-skill type. We represent the rates of investment spending in effort for manager $H$ and manager $L$ as $a$ and $b$, respectively, where $a \approx \tilde{a}(p_H, c_H)$ and $b \approx \tilde{b}(p_L, c_L)$. Here, $p$ denotes the personal skill of the manager, which we equate with the probability of successfully generating valuable information for the project and is derived as $p_H = \int_{s=0}^{\infty} \tilde{p}(s)ds$ where $s$ represents the state of nature in a given period. On the other hand, $c$ stands for the units of funds incurred on efforts or the R&D expenditure. In discreet time we would derive $a$ as the product of $p$ and $c$. We further assume that during the first stage (Figure 2), $p$ is only known internally to the firm (the manager) and not to the rival manager or the outside market. That is, each of the two managers does not know her relative skill or the skill identity of the competitor. However, in every period, the cost of effort $c$ for each manager is common knowledge by way of the firm’s required financial reporting but this public information by itself is not sufficient to reveal the true value of $a$ or to signal manager type. The two competitors will
continue investing in effort until one of them succeeds and is declared a winner. To further clarify on the estimation of \( p \), suppose that we break down what is often a continuous innovation thought process into discreet units of time and examine the performance of the two managers in, say, a period that has only three states and three outcomes: \( s_1(\text{new information and valuable}) \), \( s_2(\text{new information but not valuable}) \), and \( s_3(\text{no new information}) \). Assume that the states have equal probability of occurrence: 1/3, 1/3, 1/3. Then if we assign and assume the information value of the outcomes in the three states as 1.000; 0.125; and 0.000, respectively, and if \( s_1 \) occurs for manager \( H \) but \( s_3 \) occurs for manager \( L \), nothing else, then as illustrated in Table 2, we estimate \( p_H = 0.3333 \) and \( p_L = 0.0417 \) for the period as the sum of the product of the probability of occurrence and the unit value of outcome.

[Table 2 Here]

Information acquisition is a continuous process. In every period, the objective of each of the competing managers is to maximize the net information value added of effort spent in order to enhance the chance of making the breakthrough innovation. That is, the more valuable the information generated relative to the cost of acquiring that information the higher the probability of eventually being successful. However, we assume that at any given period before the optimal time of discovery a manager is not rewarded (penalized) for positive (negative) net information value added which she strives to maximize. For manager \( H \) the information value added \( \Pi \) generated from investing in effort in a one-period decision plan is therefore expressed as:

\[
\Pi_t = (p_a)(\bar{V}(t, a)) - a,
\]

(1)

where \( \bar{V}(t, a) \) denotes the estimated expected payoff due to the effort committed. We assume that \( p_H > p_L \) and \( \frac{d\bar{V}}{da} > 0 \), implying that increased investing in skill generation is valuable; and \( \frac{d^2\bar{V}}{da^2} < 0 \).
implying that the incremental value of that effort declines with increased investments, other factors held constant.

We also make the assumption that each competing manager’s discovery time $t$ follows an exponential distribution. The benefit of the application of the exponential distribution lies in its memory-less property, which implies that the probability of the arrival of new valuable information does not depend on the arrival of past valuable information. In other words, the distribution of variable $a$ after time $t$ is the same as the original distribution if no success is observed by date $t$. The scope of the technical uncertainty a manager faces before she succeeds ($0 < t \leq \tau$) is defined by two important sources. First is the fact that none of the competing managers has any knowledge of the exact date any one of the managers will succeed in making the innovation. By investing in effort, managers $H$ and $L$ are essentially paying for random variables $t_H(a)$ and $t_L(b)$, respectively, representing uncertain dates at which the investments in effort would be successful. Maximizing the value of the information value added translates to minimizing the variance of the distributions of $t_H$ and $t_L$. The other concern is the uncertainty surrounding the possible date the rival might succeed. From manager $H$’s perspective, we denote this uncertainty by a random variable $t_H(b)$ which also represents the uncertainty that manager $L$ faces about when she will succeed. Thus, under rational expectations assumption, the uncertainty manager $H$ faces about when manager $L$ might succeed is equivalent to the uncertainty manager $L$ has about her own date of innovation. We represented this relationship as:

$$t_H(b) = t_L(b)$$ (2)

The probability of success for each manager, the hazard rate, is a function of the scale of the information cost of the investment effort. Suppose, as we have already noted, that $a$ and $b$ denote the rate of resource utilization for managers $H$ and $L$, respectively, in the skill generation process.
Then, \( g(a) \) and \( g(b) \) represent their respective conditional probabilities of success. We define \( g(a) \) and \( g(b) \), which are key drivers in our model, as skill intensity. If \( g(a) \) is greater than \( g(b) \) then the manager is said to have a higher skill intensity which significantly leads to its competitive advantage in innovation and in capturing the investment opportunity. We utilize the usual assumptions often made in optimal time models, that the function \( g \) is twice continuously differentiable and that it satisfies the following boundary conditions (increasing with diminishing returns to \( a \)):
\[
g(0) = 0; \quad g'(a) = 0 \text{ as } a \to \infty; \quad \text{and } g''(a) < 0.
\]

Suppose that \( T \) denotes the maximum time allowed for the innovation to take place beyond which all profitable investment opportunities would vanish. For manager \( H \) to succeed before manager \( L \) does, the condition that \( t_H(a) < \min[t_L(b), T] \) must be satisfied. For ease of tractability, throughout the paper, we will examine the competition from the perspective of manager \( H \). If manager \( H \) were a monopolist, her probability of making the innovation at date \( \tau \) would be determined as:
\[
prob[t_H(a) \leq \tau] = 1 - e^{-\frac{g(a)}{\tau}}
\]
(3)

Now, introducing manager \( L \) into the competition and utilizing Equations (2) and (3), it follows that manager’s \( H \)'s assessment of the probability of success of the rival is expressed as:
\[
prob \left[ t_H(b) \leq \tau \right] = 1 - prob \left[ t_L(b) > \tau \right] = 1 - e^{-\frac{g(b)}{\tau}}
\]
(4)

Thus, the probability of manager \( H \) winning is conditional on the probability of manager \( L \) not being the first to innovate and is computed as:
\[
\left[ \frac{g(a)}{g(a) + g(b)} \right] \left( 1 - e^{-\frac{g(a)}{\tau} + \frac{g(b)}{\tau}} \right)
\]
(5)

Manager \( H \)'s optimal innovation date \( \tau \) (the conditional expectation of \( t_H(a) \)) is therefore estimated as:
\[ \tau = E[t_H(a)|t_H(a) \leq (t_L(b), T)] = \frac{g(a)}{(g(a)+g(b))^2} \]  

(6)

The term \( \tau \) in Equation (6) is the measure of manager \( H \)'s competitive advantage under rational conditions. The smaller \( \tau \) is, the larger the competitive advantage manager \( H \) commands since she is able to innovate faster than manager \( L \). By implication, we also claim that \( \tau \) measures the degree of rational confidence of the manager. That is, the shorter the optimal discovery time the more confident (optimistic) the manager becomes.

We conclude this Section by framing the decision to invest in effort. It should be noted that both managers will continue to invest in effort as long as there is no winner or until one of them has succeeded in making the discovery. From the perspective of manager \( H \), the flow of investment will occur with probability density \([g(a)+g(b)]e^{-\tau [r_{H(a)+r_{L(b)}}]}\). Hence, her expected value cost of information generation is expressed as:

\[
\int_{\tau=0}^{\infty} \left[ g(a)+g(b) \right] e^{-\tau [r_{H(a)+r_{L(b)}}]} \left( \int_{t=0}^{\tau} ae^{-r_{L(b)} dt} \right) d\tau = \frac{a}{[r + g(a)+g(b)]} \]  

(7)

That is, the expected value of the cost of information for manager \( H \) is the investment in effort discounted by the sum of the risk free rate \( r \) and the relative conditional probabilities of success.

The optimal choice for the manager should be the level of investment effort that maximizes her information value added. Utilizing Equation (5), we can show that the present value of information that manager \( H \) will receive on successfully completing the investment in effort and recording a win at date \( \tau \) before manager \( L \) does is given as:

\[
\frac{[g(a)][V(\tau)]}{[r + g(a)+g(b)]} \]  

(8)

Thus, her objective function is to maximize the net present value of information generated:

\[
\text{Maximize} \left( \frac{[g(a)][V(\tau)]-a}{[r + g(a)+g(b)]} \right) \]  

(9)
3.1.2. Overconfidence

We now introduce overconfidence in the decision framework and assume the action of manager \( H \) to be irrational while manager \( L \) continues to behave rationally. Since we restrict our incorporation of overconfidence in this paper to the individual’s overstated probability of success (certainty overconfidence), we make the assumption that manager \( H \) believes that her personal skill level now is higher at \( p_H + \lambda \), where \( \lambda \) is a measure of overconfidence similar to the specification in Gervais et al. (2011). By implication, since \( p_H + \lambda > p_H \) and the cost of effort level \( c \) for the manager is known without any ambiguity and has not changed, then \( a^* = (p_H + \lambda)(c) > a \) and the manager’s perceived (inflated) probability of success \( (g(a^*)) \) is much higher than the true (rational) value \( (g(a)) \). At the same time, manager \( H \) believes that the probability of success for manager \( L \) is \( g(\hat{b}) \), which is consistent with the assumed manager \( H \)’s assessment that her uncertainty regarding a date her competitors will win before she does is equal to the uncertainty that the competitors have about their own chance of winning. That is, \( \text{prob}_H(t_H(\hat{b})) = \text{prob}_L(t_L(\hat{b})) \).

Thus, the derivation of the conditional expectation of the optimal innovation date \( \tau^* \) for manager \( H \), given her overconfidence, takes the same steps as in Section 3.1.1 above and is expressed as:

\[
\tau^* = E[t_H(a^*)|t_H(a^*) \leq t_L(\hat{b}), T] = \frac{g(a^*)}{[g(a^*) + g(\hat{b})]^2}
\]  

(10)

However, according to rational manager \( L \) (and outside investors), manager \( H \)’s true competitive advantage has not changed and so, from Equations (6) and (10), the condition \( \tau^* = \tau \) must hold. Therefore, if \( \tau^* = \tau \), then \( \frac{g(a^*)}{[g(a^*) + g(\hat{b})]^2} = \frac{g(a)}{[g(a) + g(\hat{b})]^2} \), and since \( g(a) > 0 \) and \( a^* > a \), we can easily show that \( g(\hat{b}) < g(b) \). It turns out that \( a^* \) and \( \hat{b} \) provide an equilibrium solution from the perspective of manager \( H \). Thus, in her overconfident mindset, the manager falsely is driven to
believe that her competitive advantage is far larger than what it actually is because in her own view she is more skillful than is reported, \( g(a^*) > g(a) \), and the competitor has even weaker skill than she had first feared, \( g(b) < g(b) \). We later demonstrate numerically how the manager’s overconfidence has direct impact on her valuation and investment decisions.

3.2. Market opportunity and capital investment

Once the problem of private risk is resolved by successfully making a discovery at time \( \tau \), the manager must next focus on the nature and magnitude of the uncertainty in the product market. Technical success is not a guarantee for generating cash flow benefits in the future. We suppose that expected periodic operating cash flows become available to the manager on the date immediately following the innovation. At date \( t = \tau \) the sum of the present values of these periodic net operating cash flows is assessed and denoted as \( V(\tau) \). However, the product market is constantly evolving adding to the uncertainty about the value of market potential. The manager’s success in patenting the innovation grants her the right, but not the obligation, to acquire \( V(\tau) \), or the present value of a contingent stream of operating cash flows whose purchase decision requires an immediate capital investment \( X \) at time \( \tau \). This is a typical real growth option exercise decision, where \( V(\tau) \) represents the value of the underlying asset and \( X \) is the strike (exercise) price of the real option.

3.2.1. Market opportunity

Achieving the optimal innovation date is a major victory for the successful manager. However, the decision to invest is not automatic and must be exercised so that the firm can benefit from its future performances. The exercise decision is largely a function of the relative magnitudes of \( V \)
and $X$. It pays to exercise the real option only if the option is at least in-the-money ($V \geq X$). If the manager exercises the option, the investment payoff would amount to $Max[V - X, 0]$. Following Black and Scholes (1973) and Merton (1973), we assume that the expected value of the net operating cash flows $V$ evolves according to the following diffusion process:

$$\frac{dV(t)}{V(t)} = \alpha dt + \sigma d\pi$$

(11)

where:

$\alpha = \text{the instantaneous expected return on the new investment;}$

$\sigma^2 = \text{the instantaneous variance of its return;}$ and

$d\pi = \text{the Gauss-Wiener process.}^{8}$

Additionally, we assume that $V(t)$ is spanned by the cash flows of traded securities whose instantaneous equilibrium rate of return equals $\alpha$.

3.2.2. Capital investment

The sheer size of the capital expenditure required in producing and marketing the new product can drive many firms out of contention despite the success in winning the competitive race. Managers are, therefore, concerned about the level of $X$ in determining the exercise decision. Just like the market potential $V$, the distribution of $X$ also changes over time. Our model argues that pre-setting the strike price to a fixed level may be unrealistic because of the role of competition and the uncertainty in defining the innovation race but we adopt it nevertheless in illustrating the

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7 We also leave open the possibility that the volatilities of $V$ and $X$ and their correlation might influence the exercise decision in spite of the magnitude of the relative sizes of $V$ and $X$. We verify this opportunity with the sensitivity analysis of the model in Section IV.

8 The Gauss-Wiener process defines the uncertainty underlying the process generating $V(t)$. 

27
base case scenario. In practice, a corporation that initiates investment in effort (R&D) at $t = t_0$ usually estimates the required capital expenditure, were it to succeed, as $X(0)$, which is known. However, this estimate may change subsequently and could be very different at the time of making the required capital outlays. The nature of this uncertainty affects individual managers in our model as well.

To account for randomness in expected capital expenditures we follow Fischer (1978) and Margrabe (1978) and model the strike price as a stochastic process. The uniqueness of our model is that it considers the optimal timing and the speed of skill generation. Thus, the payoff from exercising the option at $t = \tau$ critically depends on the volatility in both $V(t)$ and $X(t)$. We make the usual assumption that the process of $X(t)$ takes the following dynamics:

$$\frac{dX(t)}{X(t)} = \mu dt + \nu d\omega \tag{12}$$

where:

$\mu =$ is the instantaneous expected rate of change of the strike price;

$\nu^2 =$ is the instantaneous variance of its return;

$d\omega =$ is the Gauss-Wiener process.

While it is more straightforward to derive the option value by assuming a constant strike price, Fischer (1978) suggests that we can resolve the problem of the volatility of the exercise price by making a simple assumption that one could purchase a hedge security to provide insurance against unanticipated changes in $X(t)$. In that case the expected rate of return ($\gamma$) on the hedge security would be equivalent to the economy’s risk-free rate of interest ($r$) plus the risk premium on the hedge asset. Under perfect capital market assumptions, the condition $r = \gamma - \mu$ must hold. A constant strike price implies that the risk premium on the hedge asset is zero. While the hedge
security is important for the no-arbitrage principle to hold, we acknowledge that, in practice, managers may sometimes assess the impact of the risk premium on their investment decisions. We relax this condition for the most part of our analysis but consider it when evaluating the impact of correlation between returns on investment opportunity and capital cost.

3.3. Risk and value of investment opportunity

It is clear from the discussions above that the processes of \( V(t) \) and \( X(t) \) are correlated. When making a decision, the manager whose capital stock is dominated by human capital will consider not only the volatility in \( V(t) \) but also the covariance with \( X(t) \). We denote the total market risk as \( \Delta \) which captures two sources of uncertainty – the unpredictability in the changes in streams of both the operating cash flows and the capital expenditures. Thus, we obtain the following relationship:

\[
\Delta^2 = \sigma^2 + \nu^2 - 2\sigma\nu \rho_{\sigma \nu}
\]

where:

\( \sigma^2 = \) the variance of the returns in operating cash flows;

\( \nu^2 = \) the variance of the rate of change in capital investment;

\( \rho_{\sigma \nu} = \) correlation coefficient between \( d\pi \) and \( d\omega \).

We assume that both \( \Delta^2 \) and \( \rho_{\sigma \nu} \) are constant. This is consistent with the assumptions underlying log-normal distributions. From the perspective of the winning firm, the real call option value (\( Q \)) of the new investment opportunity as at time zero, following a successful exercise of the investment option, is therefore computed as follows:

\[
Q = \left[ g(a) \right] \left[ V\phi(Z) - Xe^{-r\tau} \phi(Z - \Delta\sqrt{\tau}) \right]
\]

where:
\[ Q = \text{market value of the future growth opportunities}; \]
\[ V = \text{present value of net operating cash flows of the investment on the date of innovation (and,} \]
\[ \text{also on the date of receipt of patent);} \]
\[ X = \text{strike price, also the capital expenditure (investment cost);} \]
\[ r = \text{risk-free rate of interest;} \]
\[ \Delta^2 = \text{the conditional variance of the underlying cash flows and investment costs, a measure of} \]
\[ \text{total market risk;} \]
\[ \phi = \text{cumulative standard normal distribution function;} \]
\[ \tau = \frac{g(a)}{[g(a) + g(b)]^2}, \text{optimal innovation time, measuring competitive advantage;} \text{and} \]
\[ Z = \left[ \ln \left( \frac{V}{X} \right) + \left( \frac{\Delta^2}{2} \right) \right] \left( \frac{\tau}{\Delta \sqrt{\tau}} \right). \]

From Equation (14) we can see that the underlying discount rate, the implied capitalization rate for the investment opportunity, is represented by the first term on the right hand side, and is computed by \( \frac{g(a)}{r + g(a) + g(b)} \). This discount rate can further be rearranged and expressed as \( \frac{r + g(b)}{g(a)} \).

3.4. The overinvestment problem

To introduce the condition of excessive overconfidence, we maintain our assumption that the rival manager is rational but that manager \( H \) is now overconfident. However, as we have shown under Section 3.1, if manager \( H \) bears overconfidence then conditions \( g(a^*) > g(a) \) and \( \tau^* = \tau \) should hold. To determine whether there is overinvestment or not, we derive the value of \( X^* \) that the overconfident manager would require to be able to generate the market value of the firm \( Q^* \)
that would be obtained by a comparable but rational manager. We know that under excessive overconfidence, the value of the firm is expressed as

\[ Q^* = \left[ \frac{g(a^*)}{r + g(a) + g(b)} \right] V \phi(Z) - X^* e^{-\tau r} \phi(Z - \Delta \sqrt{r}) \]  

(15)

where \( Z = \left[ \ln \left( \frac{V}{X^*} \right) + \left( r + \frac{\Delta^2}{2} \right) \frac{\tau}{\Delta \sqrt{r}} \right] . \)

If \( Q^* = Q, \tau^* = \tau \) and \( r, V, g(b) \) are unchanged, and since \( g(a^*) > g(a) \), then we can easily verify that \( X^* > X \). Thus, an overconfident manager is shown to invest more than an identical but rational manager in order to generate the same level of real option value.

4. Valuation and risk: analysis and discussion

Our analysis focuses on the high-skill winning manager (manager H) since our interest is to examine the impact of human capital on the behavior of the manager under conditions of rationality and also when the manager is acting irrationally. In this section, we apply Equation 14 to estimate the value of the real growth options \( (Q) \), which measures the market value of the investment opportunity from the perspective of the manager who is rated as the first to innovate because of her higher competitive advantage. As noted earlier, a key feature of the model is the requirement that the success of a manager is conditional on the performance of the competitors. To assess the impact of the decision habit of the high-skill manager on investment and value we apply the base case and changing values of the model parameters. Specifically, we frame the competitive advantage of the manager in two ways and present the parameter values in Table 2. In the base case where both manager H and manager L are assumed to be rational, the conditional probability of success (true skill) of manager H is assumed to be 50% and that of manager L equals 10%, resulting in a competitive advantage factor of 1.389 for manager H. We examine the impact of increasing rational confidence of manager H on firm value, assuming that manager L holds her
probability of success at 10% while acknowledging the higher success rate of manager $H$. We present these relationships in Figure 3. The results indicate what is widely reported in the standard literature, that high managerial skill leads to increased firm value.

Under case 2, manager $H$ acts irrationally and exhibits overconfidence in her ability thus inflating the quality of her own human capital, which puts the manager’s probability of success at 55% (low overconfidence), 60% (moderate overconfidence), and 70% (high overconfidence) in comparison with her probability of success of 50% under rationality. Manager $L$ on the other hand believes that her probability of success and that of manager $H$ have not changed from 10% and 50%, respectively, implying that the competitive advantage of manager $H$ remains at 1.389. Consistent with manager $L$’s belief but biasedly believing that her probability of success is higher than it truly is (as alluded to above), manager $H$ assesses the corresponding success rate of manager $L$ at 7.929% (low overconfidence), 5.726% (moderate overconfidence), and 0.99% (high overconfidence).

In the base case scenario, the strike price $X$ of the real option is assumed to remain constant at a market value of $250 million. Thus, the correlation coefficient between $V$ and $X$ is zero (that is, $\rho_{ov} = 0$). Other base case parameter assumptions are as follows: the market value of the operating cash flows of the real growth option ($V$) equals $600 million with volatility measured by a standard deviation ($\sigma$) of 25%. The risk-free rate of interest ($r$) is taken to be 5%. It is important to note that any investment under uncertainty typically faces several scenarios with the most optimal valuation for the manager generated whenever the competitive advantage is superior, capital investment requirement is low, market potential is high, and investment risk is high. Table 3 reports the base case parameter values and the computed market value of the real growth opportunities ($Q$) amounting to $282.152 million from the perspective of a rational manager $H$. By using the base
case parameter values and by changing the market value of the underlying investment ($V$), we obtain corresponding values of $Q$ under different levels of competitive advantage. The interesting finding depicted in Figure 3 (data not shown) while using base case assumptions is that for a range of values, for example, when the investment is out-of-the money ($V < X$) $Q$ remains positive and non-zero and the manager still finds it worthwhile to continue undertaking the investment. By holding the investment cost $X$ constant at $250 million, $Q$ eventually drops to zero only when $V$ has declined to $65 million. In the traditional discounted cash flow analysis without considering optionality, $Q$ would be expected to amount to zero when $V = X = $250 million. This result also provides very strong support to the widely documented attractiveness in using real options for valuing growth opportunities. We can see that the strict use of the traditional DCF model clearly undervalues investments since market opportunities with negative NPV ($V \leq $250 million) would be rejected using our assumed parameter values.

We also derive firm values under case 2 when manager $H$ is considered excessively overconfident. Our results show that while a rational manager values the firm at $282.152$ million, an overconfident manager would put the valuation at $296.984$ million (low overconfidence), $311.171$ million (moderate overconfidence), and $337.885$ million (high overconfidence), implying valuation premiums of 5.057 percent, 10.285 percent, and 19.753 percent, respectively. Figure 4 plots the graph of excess firm value at different levels of overconfidence as market opportunity changes. Additionally, we determine that the underlying capitalization rate (proxy for the cost of capital, see Equation 14) for the firm is 30.00 percent for a rational manager. For the overconfident manager, the comparable rates are 23.50 percent (low overconfidence), 17.88 percent (moderate overconfidence), and 8.56 percent (high overconfidence), implying that a manager who exhibits excessive overconfidence underestimates the risk of the investment. These
results are also reported in Table 4. We also generate values pertaining to the overinvestment problem captured by the model (see Table 3). Given a volatility of 25 percent for the investment opportunity, we document that an overconfident manager would invest $269.678 million (low overconfidence), $286.780 million (moderate overconfidence), and $315.222 million (high overconfidence) when an identical but otherwise rational manager would incur only $250 million in capital expenditures to capture the same firm value of $282.152 million. The extent of overinvestment rises with increasing degree of overconfidence. However, when we increase the correlation coefficient between cash flow and capital investment returns, the magnitude of overinvestment somewhat declines but the decrease is more pronounced when the manager is highly overconfident.

We also derive the value of the investment opportunity under different levels of risk and competitive advantage, specifically investigating firm valuation from the perspective of manager $H$ when the size of the market opportunity ($V$) is either significantly low or significantly high. The relationships of the results appear in Figure 5 for deep in-the-money growth option (Panel A) and for deep out-of-the-money option (Panel B). Both the volatility of the investment cash flows and the degree of competitive advantage are found to have a positive effect on value. However, assuming rationality, at every level of volatility the value of the investment opportunity for manager $H$ is higher when the manager’s human capital is at higher skill ($g(a) = 60$ percent) than when it is at lower skill level ($g(a) = 50$ percent) whenever the investment is deep in-the-money (Panel A). However, when the investment is deep out-of-the-money (Panel B), the value of the investment opportunity is larger for the lower skill than it is for the higher skill level. The intuition of this finding is that at some specific points when the market opportunity is considered low a
manager who perceives own human capital as being at a lower level than usual has a tendency to hold on to the investment much longer at a time when the same manager, but at a higher skill level, would have made a decision to exit the investment sooner. This result is consistent with the disposition effect of behavioral finance.

We further examine the value sensitivity of the manager at different levels of investment volatility ($\sigma$) and competitive advantage $g(a)$ in a down market ($V=$50m) and a boom market ($V=$600m). In our calculation, price sensitivity is computed by the percentage change in real growth investment value ($Q$) divided by the percentage change in the investment volatility ($\sigma$) and it denotes the degree of aversion to risk. In interpreting the results, we adopt the standard portfolio theory definition of risk aversion – a marginal reward for accepting additional risk – and state that a high measure of value sensitivity implies that the investor has a low tolerance for risk. When the value sensitivity is infinite the manager is considered to have abandoned the investment (or the decision to invest) and dropped out of the market. In the field of investment management, this is akin to the investor selling the asset initially owned. Our results point out the following: whenever the investment is deep out-of-the-money (down market), the manager at a lower skill level feels a much smaller value sensitivity than when she is at higher skill level. This implies that at lower skill level the manager has a stronger tendency to continue riding the investment (market) than when the human capital is at higher skill state. However, when the investment is deep in-the-money (rising market), our result is reversed: the lower skill manager has a stronger tendency, or higher value sensitivity, than the higher skill manager to exit the investment earlier. These findings are consistent with the arguments of loss aversion and disposition effect in behavioral finance.
Figure 6, which plots value sensitivity-to-risk over a range of investment volatility also reveals additional information. Whenever the real investment opportunity is low (deep out-of-the-money), value sensitivity declines. However, as investment volatility increases, the lower skill level manager almost always requires a lower marginal reward to accept additional risk (has a lower value sensitivity) than the higher skill manager does. But the opposite occurs when the market is up (deep in-the-money): the lower skill manager almost always exhibits higher value sensitivity to risk than the higher skill manager. The switching state of aversion-to-risk for either manager under the two different market conditions parallels the evidence claimed by the concept of Prospect Theory. That is, individuals tend to be risk-averse over the domains of gains ($V > X$) and risk-seeking over the domains of losses ($V < X$).

[Figure 6 Here]

5. Conclusion

This paper presents a real options valuation model that describes the investment decision-making habit of a typical manager who, faced with competition and a risky investment opportunity, exploits her unique abilities to capture growth opportunities. The paper responds to the observed shortcomings of standard finance theories in recognizing the psychological biases that often inhibit investors’ and managers’ abilities to make good investment decisions. Behavioral finance has argued that individual managers and investors often act irrationally, making, for instance, inflated assessment of the value of their own stock due to their excessive overconfidence in the firm. In particular, it has shown that individuals typically do not use the standard finance tools in their valuation and investment decision-making.

The paper introduces a real growth options model that may help explain investors’ behavioral tendencies under uncertainty that have been solely attributed to psychological biases. It particularly
argues that some managers are able to use their unique human capital to identify abnormal profitable opportunities. These managers may appear to violate rational investor norms when in fact they are behaving quite rationally as they seek to apply their unique skills in much the same way as entrepreneurs do. Thus, faced with a growth investment opportunity under uncertainty, they base their valuation decisions on several key factors: timing, competitive advantage, riskiness, financing, size of the market opportunities, and the correlation of the investment and market returns. The real growth options valuation model that we propose has the ability to incorporate these factors and the potential to capture valuation and investment decision-making habit of these managers. Overall, we conclude that the model can explain behavioral valuation decisions of individual managers that have often solely been interpreted in terms of psychological biases. At the same time, we highlight a framework that is capable of analyzing an excessively overconfident manager, for instance, one who engages in manipulative financial reporting practices in efforts to make outside investors believe in the skills which the manager does not truly have and in the valuation of the firm that is both inflated and unsustainable. The model helps in the understanding of the often cited disagreement between inside managers and outside investors regarding the true value of the firm. Our main results include the following:

- Under rational expectations, high managerial skill (and competitive advantage) produces high firm value. Volatility of market growth opportunities (expected operating cash flows) has a strong positive effect on firm value.
- Excessive overconfidence leads to underestimation of risk of an investment. However, if a manager is rational and has higher competitive advantage, the manager will find it valuable to undertake a higher risk than a rational manager with relatively lower skill.
• An overconfident manager overinvests in a project and the extent of overinvestment increases with the degree of overconfidence. However, the extent of overinvestment tends to be mitigated somewhat by a positive correlation between the investment cost and the operating cash flows. The effect of correlation is stronger whenever managerial overconfidence is very high.

• An overconfident manager overvalues the firm. The valuation premium becomes larger the higher the level of excessive overconfidence.

• Under rational expectations, value sensitivity to risk (risk aversion) is found to be larger in absolute terms when the investment opportunity is deep out-of-the-money (small) than when it is deep in-the-money (large). Whenever the real investment is deep out-of-the-money, value sensitivity to risk declines and its level is systematically lower for a low-skill than for a high-skill manager. However, the opposite result is established when the investment is deep in-the-money. Thus, we document findings that parallel the claim made by prospect theory, that is, individuals tend to be risk-averse over the domains of gains and risk-seeking over the domains of losses.

We are cautious to point out that, as with all models, our model rests on the assumptions that the parameter estimates are correct. Overall, the model and the results do provide sound implications for risk management, corporate valuation, investment decisions, financial reporting, regulatory policy, and ethical standards.
References


Table 1
Summary of events and patenting activities of Steve Jobs at Apple Inc.

<table>
<thead>
<tr>
<th>Date</th>
<th>Event Related to Steve Jobs</th>
<th>Apple Inc. Market Capitalization</th>
<th>Cumulative Number of Patents in Steve Jobs’ Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1976</td>
<td>Co-founded Apple Inc. with Steve Wozniak</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>December 12, 1980</td>
<td>Founding CEO at the IPO</td>
<td>$1.43 billion</td>
<td>2</td>
</tr>
<tr>
<td>May 31, 1985</td>
<td>Fired, left the company</td>
<td>$1.06 billion</td>
<td>3</td>
</tr>
<tr>
<td>September 17, 1997</td>
<td>Returned to company, CEO</td>
<td>$2.09 billion</td>
<td></td>
</tr>
<tr>
<td>August 21, 2011</td>
<td>Resigned as CEO for health</td>
<td>$339 billion</td>
<td>313</td>
</tr>
<tr>
<td>October 5, 2011</td>
<td>Died of pancreatic cancer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>August 20, 2012</td>
<td>1 year after his resignation and 10 months since his death</td>
<td>$618 billion</td>
<td></td>
</tr>
<tr>
<td>February 23, 2015</td>
<td>Apple Inc. achieves peak valuation</td>
<td>$775 billion</td>
<td></td>
</tr>
<tr>
<td>July 15, 2015</td>
<td></td>
<td>$730 billion</td>
<td>454</td>
</tr>
</tbody>
</table>

This table presents a summary of Steve Jobs’ human capital impact on Apple Inc. A total of 141 patents have been awarded in Steve Jobs’ name as primary inventor or co-inventor since his death on October 5, 2011. Of the 454 patents awarded, 346 are U.S. patents. Information on events come from Apple, Inc. annual and company reports. Data on patents are gathered from the U.S. Patents and Trademark Office (USPTO) database, and financial data are from Bloomberg and Yahoo Finance.
Fig. 1. Components of firm value and key risk drivers of the value of growth opportunities.

Value of the Firm = Value of Assets-in-Place + Value of Growth Opportunities

- Product Market Expectations
  - Market-Priced Risk
    - Cash Flow Distribution: Known

- Competitive Advantage Drive
  - Private Risk
    - Unknown
Breakthrough innovation is announced; high-skill CEO wins.

High-skill CEO makes the decision whether to exercise the real option to undertake the capital investment or not.

Capital expenditure $X$ is incurred if investment is accepted.

Investment payoff $V(t)$ is captured only if outlay $X$ is incurred.

Each of the competing CEOs initiates spending on effort (R&D).

Distribution of investment payoff $\bar{V}(t)$ is known but the value of the optimal discovery time $t = \tau$ is not.

Growth opportunities expire if no winner

**Fig. 2.** Sequence of events in the model
**Table 2**
Hypothetical discreet illustration of the evaluation of the productivity of investment in effort: A first-stage sample period.

<table>
<thead>
<tr>
<th>State ($j$)</th>
<th>Probability of Occurrence</th>
<th>Possible Outcome</th>
<th>Unit Value if Outcome is True</th>
<th>Is Outcome True?</th>
<th>Probability Contribution ($p_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>0.3333</td>
<td>New information and valuable to project</td>
<td>1.0000</td>
<td>Yes</td>
<td>0.3333</td>
</tr>
<tr>
<td>$s_2$</td>
<td>0.3333</td>
<td>New information but not valuable to project</td>
<td>0.1250</td>
<td>No</td>
<td>0.0000</td>
</tr>
<tr>
<td>$s_3$</td>
<td>0.3333</td>
<td>No new information</td>
<td>0.0000</td>
<td>No</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Probability of success ($p_H$) 0.3333

<table>
<thead>
<tr>
<th>State ($j$)</th>
<th>Probability of Occurrence</th>
<th>Possible Outcome</th>
<th>Unit Value if Outcome is True</th>
<th>Is Outcome True?</th>
<th>Probability Contribution ($p_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>0.3333</td>
<td>New information and valuable to project</td>
<td>1.0000</td>
<td>No</td>
<td>0.0000</td>
</tr>
<tr>
<td>$s_2$</td>
<td>0.3333</td>
<td>New information but not valuable to project</td>
<td>0.1250</td>
<td>Yes</td>
<td>0.0417</td>
</tr>
<tr>
<td>$s_3$</td>
<td>0.3333</td>
<td>No new information</td>
<td>0.0000</td>
<td>No</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Probability of success ($p_L$) 0.0417
Table 3  
Firm valuation under both rational base case and overconfidence conditions.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>Rational Base Case</th>
<th>Low Overconfidence</th>
<th>Moderate Overconfidence</th>
<th>High Overconfidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present value (market size) of operating cash flows</td>
<td>$V$</td>
<td>$600.000m$</td>
<td>$600.000m$</td>
<td>$600.000m$</td>
<td>$600.000m$</td>
</tr>
<tr>
<td>Capital investment (strike price)</td>
<td>$X$</td>
<td>$250.000m$</td>
<td>$250.000m$</td>
<td>$250.000m$</td>
<td>$250.000m$</td>
</tr>
<tr>
<td>Risk (standard deviation) of operating cash flow</td>
<td>$\sigma$</td>
<td>25.000%</td>
<td>25.000%</td>
<td>25.000%</td>
<td>25.000%</td>
</tr>
<tr>
<td>Correlation coefficient of $V$ and $X$</td>
<td>$\rho_{ov}$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Conditional standard deviation of investment opportunity</td>
<td>$\Delta$</td>
<td>25.495%</td>
<td>25.495%</td>
<td>25.495%</td>
<td>25.495%</td>
</tr>
<tr>
<td>Probability of success of the high-skill manager ($H$)</td>
<td>$g(a)$</td>
<td>50.000%</td>
<td>55.000%</td>
<td>60.000%</td>
<td>70.000%</td>
</tr>
<tr>
<td>Probability of success of the low-skill manager ($L$)</td>
<td>$g(b)$</td>
<td>10.000%</td>
<td>7.929%</td>
<td>5.726%</td>
<td>0.990%</td>
</tr>
<tr>
<td>Competitive advantage factor</td>
<td>$\tau$</td>
<td>1.389</td>
<td>1.389</td>
<td>1.389</td>
<td>1.389</td>
</tr>
<tr>
<td>Underlying discount rate</td>
<td>$(1/DF) - 1$</td>
<td>30.000%</td>
<td>23.503%</td>
<td>17.887%</td>
<td>8.557%</td>
</tr>
<tr>
<td>Firm value (value of the investment opportunity)</td>
<td>$Q$</td>
<td>$282.152m$</td>
<td>$296.984m$</td>
<td>$311.171m$</td>
<td>$337.885m$</td>
</tr>
<tr>
<td>Excess (inflated) firm value due to overconfidence</td>
<td>$\Delta Q$</td>
<td>---</td>
<td>$14.269m$</td>
<td>$29.019m$</td>
<td>$55.733m$</td>
</tr>
<tr>
<td>Valuation premium due to overconfidence</td>
<td>---</td>
<td>5.057%</td>
<td>10.285%</td>
<td>19.753%</td>
<td></td>
</tr>
</tbody>
</table>

This table reports the parameter values as well as the expected value of the firm (real option value of the investment opportunity). We assume that the size of the value of the underlying market opportunity ($V$), presented as the market value of the operating cash flow, is $600 million; the investment requirement ($X$) is $250 million; the standard deviation of the operating cash flow ($\sigma$) is 25 percent; the rate of return ($\mu$) and the standard deviation of the investment requirement ($\nu$) are both at 5 percent. The probability of success (the competitive advantage) of the high-skill, winning, manager, $g(a)$ is 50 percent, and that of the low-skill competing managers $g(b)$ is 10 percent under base case rational expectations. We also assume that risk-free rate of return ($r$) is 5 percent and the return on a hedge security ($\gamma$) 10 percent. The correlation coefficient between the underlying asset and investment cost ($\rho_{ov}$) is assumed to be zero. Thus, the conditional standard deviation of the investment opportunity ($\Delta$) is derived to be equal to 25.495 percent and is computed by $\Delta^2 = \sigma^2 + \nu^2 - 2\sigma\nu\rho_{ov}$. The firm value, which is the real growth option value of the investment opportunity ($Q$), is determined to be equal to $282.152 million and is computed by $Q = (DF)\{V \phi(Z) - X e^{-rt} \phi(Z - \Delta \sqrt{r})\}$, where $DF = \frac{g(a)}{r + g(a) + g(b)}$; $Z = \left[ \ln \left( \frac{V}{X} \right) + \left( \frac{\Delta}{2} \right) \right] \left( \frac{\Delta}{\Delta \sqrt{r}} \right)$; $\tau = \frac{g(a)}{[g(a) + g(b)]^2}$ and $\phi$ denotes the cumulative standard normal distribution function. Under managerial overconfidence, $g(a)$ is increased to 55, 60, and 70 percent for low, moderate, and high overconfidence. All other parameter values remain the same.
Fig. 3. Value of the firm ($Q$) under rational conditions and varying values of the underlying market opportunity ($V$) and manager probability of success [$g(a)$]. The figure plots the relationship between the firm value (in millions of U.S. dollars) and the value of the underlying market opportunity (in millions of U.S. dollars), assuming rational expectations, when the probability of success of the high-skill manager $g(a)$ is 15 percent; 50 percent; and 95 percent. We assume that the standard deviation ($\sigma$) of the underlying operating cash flows ($V$) is 25 percent, the standard deviation ($\nu$) of the investment requirement ($X$) is 5 percent, and the correlation coefficient between the investment value and investment cost ($\rho_{\nu v}$) is zero – giving the conditional standard deviation of the investment opportunity ($\Delta$) to be equal to 25.5 percent, which is computed by $\Delta^2 = \sigma^2 + \nu^2 - 2\sigma\nu\rho_{\nu v}$, and is assumed to remain constant. We also assume that the investment requirement ($X$) is $250$ million; the probability of success for the low-skill competitor managers $g(b)$ is 10 percent; and the risk-free rate of return ($r$) is 5 percent.
Fig. 4. Overconfidence and overvaluation of the firm. The figure graphs the excess value of the firm at each level of the present value of the market opportunity (operating cash flow) under low, moderate and high overconfidence. Cash flow volatility is held constant at 25 percent and the correlation between cash flow and capital investment is assumed to be zero. The strike price of the real growth option remains constant at $250 million and the probability of success of the low-skill manager (manager L) is 10 percent while the overconfident manager (manager H) believes that it is 7.929% (low overconfidence), 5.726% (moderate overconfidence), and 0.99% (high overconfidence).
Table 4

Estimation of overinvestment due to overconfidence.

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Amount of Capital Invested ($ in millions) in Excess of $250 million Under</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low Overconfidence [g(a) = 55%]</td>
</tr>
<tr>
<td>σ = 25%</td>
<td>$19.678</td>
</tr>
<tr>
<td>σ = 45%</td>
<td>$21.451</td>
</tr>
<tr>
<td>σ = 75%</td>
<td>$29.282</td>
</tr>
</tbody>
</table>

The table presents the levels overinvestment made when the high skill manager is overconfident in her abilities at different levels of volatility of the operating cash flow. When the manager is rational, and has true skill \( g(a) \) measure of 50%, the manager requires $250 million to generate firm value of $282.152 million. However, to achieve the same value under overconfidence, the manager tends to overinvest by the amounts shown. We assume that the size of the value of the underlying market opportunity (\( V \)), presented as the market value of the operating cash flow, is $600 million; the standard deviation of the operating cash flow (\( \sigma \)) is 25, 45 and 75 percent; the rate of return (\( \mu \)) and the standard deviation of the investment requirement (\( \nu \)) are both at 5 percent. We also assume that risk-free rate of return (\( r \)) is 5 percent and the return on a hedge security (\( \gamma \)) 10 percent. The correlation coefficient between the underlying asset and investment cost (\( \rho_{\sigma\nu} \)) is assumed to be zero. The probability of success of the low-skill competing managers \( g(b) \) is held constant at 10 percent and that of the high skill manager \( g(a) \), who now acts irrationally 55 percent (low overconfidence), 60 percent (moderate overconfidence), and 70 percent (high overconfidence).
In order to maintain overconfident manager that the competitive advantage factor whose probability of success is 60 percent while the probability of success is 10 percent such that the competitive advantage factor $\tau$ is equal to 1.389. The solid line denotes firm valuation of a rationally confident manager $H$ whose probability of success is 60% and believes that the probability of success of manager $L$ is 10 percent, such that the competitive advantage factor $\tau$ is enhanced which equals 1.225. The barbed line presents valuation of an overconfident manager $H$ whose probability of success $g(a)$ is 60 percent but who is truly overestimating her ability. In order to maintain manager $H$’s competitive advantage factor $\tau$ at 1.389, the probability of success $g(b)$ of manager $L$ drops to 5.726 percent.

**Fig. 5.** Firm value at base case and at rational confidence (optimism) level of the high-skill manager. The first graph (i) plots firm value against volatility of the underlying investment opportunity when underlying market is deep in-the-money ($V=750m$). The second graph (ii) denotes the case for a deep out-of-the-money market ($V=50m$). The broken line presents firm valuation of the rational high-skill manager (manager $H$, base case) whose probability of success $g(a)$ is 50 percent while the probability of success $g(b)$ of manager $L$ is 10 percent such that the competitive advantage factor $\tau$ is equal to 1.389. The solid line denotes firm valuation of a rationally confident manager $H$ whose probability of success is 60% and believes that the probability of success of manager $L$ is 10 percent, such that the competitive advantage factor $\tau$ is enhanced which equals 1.225. The barbed line presents valuation of an overconfident manager $H$ whose probability of success $g(a)$ is 60 percent but who is truly overestimating her ability.
Fig. 6. Value sensitivity-to-risk at various levels of volatility. The figures plot the sensitivity of firm value-to-risk of the investment opportunity (y-axis) at various levels of investment risk (x-axis) as derived in Table 4 assuming that the probability of success of the high-skill manager \( g(a) \) is 50 percent and the probability of success of the competing low-skill managers is 10 percent. Graph (i) shows the magnitude of and how firm value sensitivity-to-risk changes with volatility when the underlying market opportunity is deep in-the-money \( (V = \$750 \text{ million}) \) and graph (ii) depicts the case for a deep out-of-the-money investment opportunity \( (V = \$50 \text{ million}) \). The correlation coefficient between the investment cost and the investment value is assumed to equal 0; the investment cost \( (X) \) is assumed to remain constant at \$250 million, and its standard deviation \( (\sigma) \) is 5 percent.