Banks' credit portfolio choice and risk-based capital regulation☆

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Abstract

In order to address banks' risk-taking during the recent financial crisis, we develop a model of credit portfolio optimization and study the impacts of risk-based capital regulation (Basel Accords) on banks' asset allocations. The model shows that, when a bank's capital is constrained by regulation, regulatory cost (risk weightings in Basel Accords) alters the risk and value calculations for the bank's assets. The model predicts that the effect of a tightening of the capital requirements – for banks for which these requirements are (will become) binding – will be to skew the risky portfolio towards high-risk high-earning assets (low-risk low-earning assets), provided that the asset valuation, i.e. reward-to-regulatory-cost ratio, of the high-risk asset is higher than that of the low-risk asset. Empirical examination of U.S. banks supports the predictions applicable in the dataset. In addition, our tests show the characteristics of banks with different risk-taking. In particular, the core banks that use the internal ratings-based approach under Basel II invest more in high-risk assets.

Keywords: Banks; Asset risk; Credit risk; Portfolio choice; Risk-based capital regulation

JEL classification: G11;G21;G28

1. Introduction

The recent financial crisis has put a sharp spotlight on banks' risk-taking. Banks are blamed for shrugging off risk concerns while pursuing higher earnings, such as on loans with high credit risk.

To investigate banks' risk-taking on credit risk, we look into banks' total assets with

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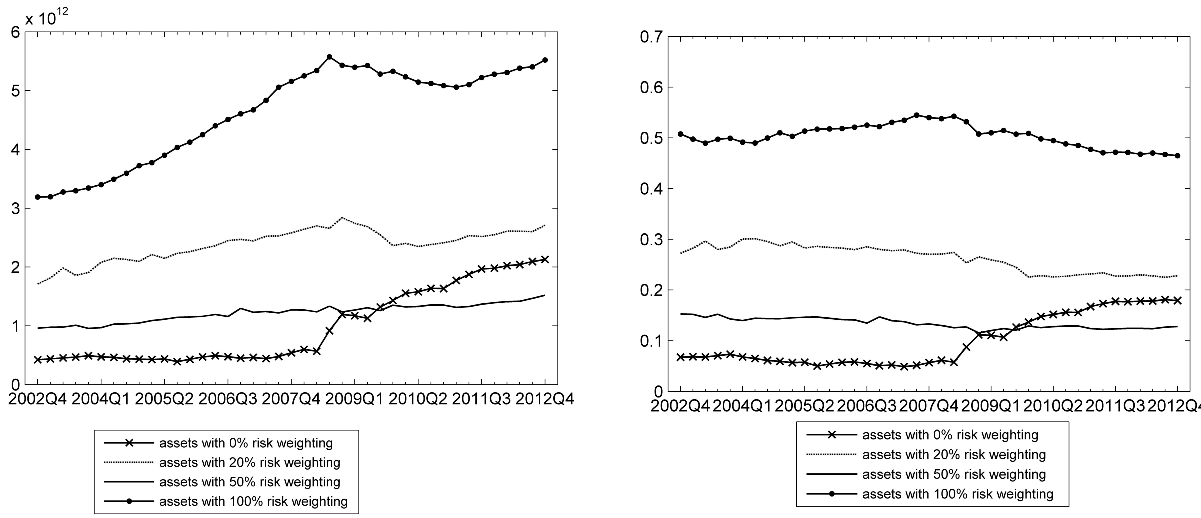
different levels of credit risk, defined by the risk weightings under the Basel Accords[1]. To value the overall risk of a bank's assets, the Basel Accords adopt the total risk-weighted assets, where a higher weight is assigned to assets with higher credit risk.[2]

Figure 1: Banks' allocation in assets with different risk weightings

(a) Allocation in different assets in dollars   **(b)** Allocation in different assets in proportion



Figures 1 and 2 display the sum of the assets with a certain risk weighting for all U.S. banks that are insured by the Federal Deposit Insurance Corporation (FDIC) from 2002 to 2012. According to Figure 1, there is a distinct increase in the amount of assets with the highest credit risk, namely 100% risk weighting, although the trend of its proportion is not obvious. Notice that, from the second quarter in 2008, there is an apparent increase in the assets with the lowest credit risk, namely 0% risk weighting, measured in dollars or in proportion, which verifies the phenomenon of "flying to safety" since the crisis. Figure 2 shows banks' total allocation among risky assets whose risk weightings are non-zero. The proportion of assets with 100% risk weighting, i.e. high-risk assets, increases through the years, compared to the sum of assets with 20% and 50% risk weightings, i.e. low-risk assets. The maximum of the difference in their allocations is 17.9% of risky assets, that is 1.62 trillions of dollars. This could be due to "flying to earnings" targeting the assets with 100% risk weighting.

[1]The Basel Accords are the supervision Accords for banks promulgated by the Basel Committee on Bank Supervision. This study limits its focus on the aspects of the Accords that address issues related to the capital adequacy requirement, which is the centre of the Accords.

[2]Under the standard approach of Basel I and II, there are four broad categories of risk, i.e. 0%, 20%, 50%, and 100% risk weightings. To determine capital adequacy, the Basel Accords use a risk-based capital ratio, the ratio of total regulatory capital to the total risk-weighted assets.

Figure 2: Banks' allocation within risky assets with non-zero risk weightings

One question that arises from the above observations is how banks allocate resources (deposits and capital) in assets with different credit risks conditional on the information on assets' payoffs and default probabilities. Our paper addresses this question. Moreover, the risk-based capital adequacy requirements under the Basel Accords[3] pose additional costs to riskier assets, since banks have to reserve more capital for assets with a higher credit risk. However, how banks react to this regulation is ambiguous. Banks also have incentives to take more risk in order to gain higher earnings and compensate for the higher costs of their capital reserves. Thus, we are not certain that a tightening capital requirement would have the desired effect.

We regard a bank as its assets' manager and develop a model of portfolio allocation with a minimum regulatory capital requirement as a possible binding condition. Then we examine how the bank reshuffles the portfolio basket when the conditional information, i.e. assets' payoffs, default probabilities, default correlation, or regulation, changes. This allows us to examine explicitly the banks' credit risk-taking from the point of view of asset portfolio management and to study specifically the impacts of the risk-based capital requirements on banks asset choice.

Our model shows that, when a bank's capital is not constrained by regulation, its asset allocation decision depends on the risk measure of assets, namely the cash flow volatility around the expected loss due to default risk, and on the key measure of assets' valuation, the

[3]Under Basel I (1998) and II (2004), a bank has to reserve total capital at least as 8% of the value of the bank's total risk-weighted assets; under Basel III (2010), a bank has to hold additional conservation and counter-cyclical buffers.

Sharpe ratio (Sharpe, 1966), modified according to our settings. However, when the bank's capital is constrained by regulation, regulatory cost (risk weighting in risk-based capital regulation) steps in and weights the cash flow volatility and even replaces the volatility in the measure of the assets' valuation (reward-to-regulatory-cost ratio instead of reward-to-variability ratio (the Sharpe ratio)). If the regulator imposes a new and more stringent regulation, i.e. a higher risk-based capital requirement, the bank whose capital is already constrained will skew the risky portfolio to high-risk high-earning asset, while the bank whose capital will become constrained by the new regulation will do the opposite, i.e. investing less in high-risk assets, provided that the valuation (reward-to-regulatory-cost ratio) of a high-risk asset is higher than that of a low-risk asset. The latter effect is similar to the immediate risk-reducing effect of capital regulation demonstrated in the literature.

Moreover, we test the model with bank-level data on assets with different credit risk categories for all commercial banks insured by the FDIC. The empirical examination verifies the model's predictions of how banks' choices between high-risk high-earning assets and low-risk low-earning assets react to the updated information of assets' earnings and default probabilities and of the impacts of a higher capital requirement. In addition, our tests show the characteristics of banks with different risk-taking. In particular, the core banks that use the internal ratings-based approach under Basel II invest more in high-risk assets.

Although there are models evaluating portfolio credit risk, there are only a few articles on credit portfolio optimization (see e.g. Altman and Saunders, 1998; Kealhofer and Bohn, 2001; Mencía, 2012). Regarding the impacts of capital regulation on banks' asset risk, theoretical literature yields mixed predictions, with a few studies from the point of view of portfolio management (see e.g. Koehn and Santomero, 1980; Kim and Santomero, 1988; Rochet, 1992; Furfine, 2001; Milne, 2002). In order to understand banks' risk-taking on credit risk and the impacts of a tightening capital requirement on banks' asset risk, we derive an analytical and tractable solution for credit portfolio optimization, similar to Koehn and Santomero (1980) and Kim and Santomero (1988). Our paper contributes to studies of banks' risk-taking by investigating banks' asset allocation explicitly with respect to credit risk and by disentangling the impacts of risk-based capital regulation on banks' asset risk.

The remainder of the paper is organized as follows. Section 2 reviews the literature. Section 3 presents the model. In Section 4, we examine the model empirically using a panel data set, including details on the estimation of conditional default probabilities, default correlation, and payoffs. Section 5 concludes.

2. Literature review

Over the past two decades, we have seen important advances in modelling correlated defaults and evaluating portfolio credit risk, such as Moody's KMV *PortfolioManager*, JPMorgan's *CreditMetrics*, Credit Suisse's *CreditRisk*$^{+}$, McKinsey's *CreditPortfolioView*, correlated default-intensity models, and copula-based modelling.[4]

Yet, there are only a few articles on credit portfolio optimization. Altman and Saunders (1998) measure the risk of a portfolio by its unexpected loss, determined by the standard deviation around the expected loss, which is estimated historically over time using a bond rating equivalence, i.e. the Z"-Score (Altman, 1993). Similarly, Kealhofer and Bohn (2001) measure unexpected loss by the standard deviation of loss only due to default in a default-only model, where there are two states, default and no default. Mencía (2012) models homogeneous loan classes, each comprising conditional independent loans whose conditional default probability is a probit function of a Gaussian state variable. Thus, the return distribution of the loan portfolio is presented by the mean and variance of the state variable. Then, all three articles adapt the mean-variance framework, introduced by Markowitz (1952), to analyse the risk and returns on credit portfolios.

Due to the difficulty of presenting the distribution of portfolio credit losses with sufficient accuracy, various simulation techniques have been developed to approximate the distribution.[5] Iscoe et al. (2012) adapt factor models for conditional default probabilities and compare different techniques to minimizing the variance, Value-at-Risk, and expected shortfall of portfolio credit losses. Yet, they find that a Normal approximation to the conditional loss distribution performs best. Normal distribution is also widely used to calculate Value-at-Risk and expected shortfall by practitioners.

In order to serve the purpose of understanding banks' risk-taking on credit risk, we need an analytical and tractable solution for credit portfolio optimization, which could not be provided by simulation techniques. Hence, we develop a default-only model of one-period conditional credit portfolio management for a bank manager, and the risk of a type of assets is measured by the volatility around the expected loss of assets' cash profits. This risk measure is similar to those from Altman and Saunders (1998) and Kealhofer and Bohn (2001). Different from Mencía (2012), capital regulation, i.e. the minimum capital requirement, is explicitly presented in our analytical solution of portfolio allocation. Although we do not model the drivers for the conditional default probabilities, default correlation, and payoffs

[4]See detailed descriptions of the models by Gordy (2000), Crouhy et al. (2000), and Duffie and Singleton (2003), among others.

[5]See Iscoe et al. (1999), Mausser and Rosen (2001), and Zagst et al. (2003), among others.

of assets, which are given before the bank manager makes decision, they are determined by the market data in the empirical examination in Section 4. Moreover, the losses of risky assets in our study are Bernoulli distributed with a positive skewness, and the skewness and kurtosis of the distribution are functions of the first two moments.

Regarding the impacts of capital regulation on banks' asset risk, theoretical literature yields mixed predictions, although there are general agreements about the immediate effects of capital requirements on bank lending and the longer-term impacts on capital ratio.[6] As yet, there are just a few studies of banks' asset risk from the point of view of portfolio management. Koehn and Santomero (1980) and Kim and Santomero (1988) consider a mean-variance portfolio-selection model, showing that a higher uniform regulatory capital ratio constrains the efficient asset investment frontier and might actually result in a higher asset risk and an increase in the insolvency risk of banks, i.e. the opposite of the intended effect. Nevertheless, Kim and Santomero (1988) model the optimal weights for the risk-based capital requirement, and predict that, with higher weights for riskier assets, banks would hold more liquid safe assets and less risky assets. Rochet (1992) argues that if banks behave as portfolio managers (maximizing utility instead of the market value of their future profits, such as in Furlong and Keeley (1989) among others), capital regulation can be effective, but only if the risk weights are proportional to the systematic risks of the assets (their betas). Furfine (2001) develops a dynamic value-maximizing model and calibrates it to U.S. data. He finds that Basel I was involved in the credit crunch experienced in the 1990s and predicts that, under Basel II, banks would increase loans relative to securities and safer loans relative to risky ones. Milne (2002) interprets capital regulation as a system of sanctions for *ex post* violation instead of *ex ante* enforcement, and his value-maximizing model suggests that there is relatively less need to match risk weightings accurately to portfolio risk.

To study the impacts of risk-based capital regulation explicitly, we set up a portfolio-selection model similar to Koehn and Santomero (1980) and Kim and Santomero (1988). Differently, in our model, the bank manager maximizes a utility of one-period net value of assets, instead of a utility of equity returns as in their model, which is questioned for modelling returns of a bank's capital regardless of its actual asset risk. The advantage of using a utility of assets' net value is to focus on the bank's asset risk and that the model is close to practice and could be enriched with additional features, such as capital structure.

The most remote difference from the aforementioned banking literature is that this paper focuses on credit risk, where there are only two conditional states, default or no default. Thus, we can adapt similar approaches from the credit portfolio optimization literature.

[6]See VanHoose (2007), among others.

Moreover, we study whether banks restructure the portfolio from low-risk low-earning assets to high-risk high-earning assets in order to compensate for additional costs imposed by capital requirements, as suggested. Therefore, our model contributes not only to rationalizing banks' asset allocation with respect to credit risk, but also to the gap in the literature studying the impacts of capital regulation on the credit risk of banks' assets.

3. Model

We model a commercial bank as its assets' manager who makes one-period decisions on allocating resources (deposits and capital) in the assets with different levels of credit risk and its capital might be constrained by risk-based capital regulation. The model predicts how the bank manager restructures the portfolio of different assets when their conditional default probabilities, default correlation, or payoffs change, or when the regulator imposes a tightening risk-based capital requirement.

3.1. Model set-up

The bank aims to maximize a single-period expected quadratic utility of the random cash profit of its assets, and the expected utility is an increasing function of the expected cash flow and a decreasing function of the cash flow variance.[7] The bank chooses among three types of assets: a high-risk high-earning asset, a low-risk low-earning asset, and a risk-free asset.[8] For simplicity, only the relative sizes of assets are assumed to be under the control of bank management.[9] In addition, the regulator decides that the bank has to hold capital as minimum k times of the total risk-weighted assets, where risky assets are assigned higher weights. It is also assumed that the holding period perfectly matches the maturity of the assets.

The random cash flow, $\tilde{CF}$, of a risky asset is $(1 + C)(1 - \tilde{Z})$, where C is the payoff of the asset if not defaulted and $\tilde{Z}$ is a variable for a default event with Bernoulli distribution,

[7]Bawa and Lindenberg (1977) show that, as long as the expected utility can be written as an increasing function of the expected return and a decreasing function of the variance of the portfolio only, without any assumption of probability distributions of assets' returns, the optimal portfolio lies on the efficient frontier in the mean-variance framework of Markowitz (1952), and when there is a risk-free asset, the two-fund theorem is valid. Here, by applying a quadratic utility function, we could sidestep most of the problems with solving a general utility-based portfolio choice and obtain an analytical solution.

[8]The choice of three types of assets is also consistent with the empirical examination in Section 4. In addition, the model with four types of assets, which corresponds to the four risk categories of assets in the Basel Accords, is qualitatively the same.

[9]This simplification is to serve the purpose of this study. While we could enrich the model with additional features, such as variations on the bank's liabilities and capital, how the bank allocates among assets with different credit risks in a *ceteris paribus* environment is not altered.

which takes value 1 with probability p if default happens[10] and 0 otherwise; i.e. there are two credit states, default or no default, and

$$\tilde{Z} = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability 1-}p. \end{cases} \tag{1}$$

The third and fourth moments of Bernoulli distribution are functions of its mean and variance. Therefore, we can use the first two moments to represent the distribution. Naturally, the measure of risk is the volatility around the expected loss (mean) and an approximation of unexpected loss, which is a usual term in credit portfolio literature.

The expected value and variance of its cash flow are

$$\begin{aligned} &E[\tilde{CF}] = (1+C) - E[\tilde{Z}](1+C) = (1-p)(1+C) \quad \text{and} \\ &Var[\tilde{CF}] = Var[\tilde{Z}](1+C)^2 = p(1-p)(1+C)^2, \quad \text{respectively,} \end{aligned} \tag{2}$$

where E stands for *Expectation* and Var for *Variance*. To ensure that the expected utility is decreasing as default probability increases, we only consider $p < 0.5$, which is consistent with the estimated probabilities of default (far less than 0.5) in the empirical examination (Section 4). Then, the loss due to default risk is captured by a positively skewed Bernoulli distribution.

For the two types of risky assets in the model, high-risk high-earning asset with type h (*high*) and low-risk low-earning asset with type l (*low*), their cash flow covariance is

$$Cov[\tilde{Z}_h, \tilde{Z}_l] = p_B - p_h p_l = \rho\sqrt{p_h(1-p_h)p_l(1-p_l)}, \tag{3}$$

where p_B and ρ are the pair-wise probability of default and the default correlation between the two types.

For the bank, its utility function is

$$u(\tilde{\pi}) = 2a\tilde{\pi} - \tilde{\pi}^2, \tag{4}$$

where $\tilde{\pi}$ is the random cash profit at the end of the decision period, which is

$$\tilde{\pi} = A_h\tilde{CF}_h + A_l\tilde{CF}_l + G(1+r_f) - Dr_D \tag{5}$$

where $\tilde{CF}_h$ and $\tilde{CF}_l$ are the random cash flows of high-risk high-earning and low-risk

[10]For simplification, it is assumed that recovery rate is zero.

low-earning assets respectively, A_h and A_l are their respective amounts in dollars, G is the amount of the risk-free asset with return r_f, and D is for deposits with rate r_D.

Consequently, the decision problem for the bank manager is:

$$\arg\max_{A_h,A_l,G}\{E[u(\tilde{\pi})]\} = \arg\max_{A_h,A_l,G} \quad \{2aE[\tilde{\pi}] - (E[\tilde{\pi}]^2 + Var[\tilde{\pi}])\} \tag{6}$$

Subject to:

$$A_h + A_l + G = D + K \tag{7a}$$

$$A_h \geqslant 0,\ A_l \geqslant 0,\ G \geqslant 0 \tag{7b}$$

$$\frac{K}{W_h A_h + W_l A_l} \geqslant k \text{ and } 1 \geqslant W_h > W_l > 0 \tag{7c}$$

The objective function is a function of $a, A_h, A_l, G, C_h, C_l, p_h, p_l, \rho, r_f, D, \text{and} r_D$. a is positive, and $a \geq (1 + C_h)(D + K) - Dr_D > 0$ which ensures that the marginal utility of cash profit is positive; C_h and C_l are payoffs of type h and type l assets respectively, $1 \geq C_h > C_l > r_f > 0$, p_h and p_l are their respective probabilities of default, and $0.5 > p_h > p_l > 0$; K stands for capital and the first restriction (equation 7a) states the balance-sheet constraint; W_h and W_l are the risk weightings for risky assets used in the calculation of the total risk-weighted assets, they are constant and determined by the regulator, and by definition, $1 \geq W_h > W_l > 0$; k denotes the minimum risk-based capital ratio determined by the regulator, which is 8% under Basel I and II, and the third restriction (equation 7c) expresses the regulatory capital constraint. In practice, actual defaults are positively but not perfectly positively correlated.[11] Hence, default correlation ρ here belongs to interval $(0, 1)$.

In addition, the following subsections are based on solutions to the above maximization problem when the risky assets generate positive excess cash flows over risk-free asset, i.e. $X_h \equiv (1 - p_h)(1 + C_h) - (1 + r_f) > X_l \equiv (1 - p_l)(1 + C_l) - (1 + r_f) > 0$.

3.2. Optimal portfolio allocation when the capital requirement is not binding

When the capital requirement is not binding, we get the following inner solution to the maximization problem (equation (6)):

$$A_h^* = \frac{(a - B)(SR_h - \rho SR_l)}{(SR_h^2 + SR_l^2 - 2\rho SR_h SR_l + 1 - \rho^2)\sqrt{V_h}} \tag{8}$$

[11] See Kealhofer and Bohn (2001).

$$A_l^* = \frac{(a-B)(SR_l - \rho SR_h)}{(SR_h^2 + SR_l^2 - 2\rho SR_h SR_l + 1 - \rho^2)\sqrt{V_l}} \tag{9}$$

$$G^* = D + K - \frac{(a-B)[SR_h(\sqrt{V_l} - \rho\sqrt{V_h}) + SR_l(\sqrt{V_h} - \rho\sqrt{V_l})]}{(SR_h^2 + SR_l^2 - 2\rho SR_h SR_l + 1 - \rho^2)\sqrt{V_h V_l}} \tag{10}$$

where $B \equiv (D+K)(1+r_f) - Dr_D < a$[12], V stands for cash flow variance, and SR for Sharpe ratio[13] which is the ratio of excess cash flow over risk-free asset (X) to cash flow volatility ($\sqrt{V}$).

The optimal allocations to the risky assets are determined by their Sharpe ratios, cash flow variances, default correlation, and the bank's risk aversion. Finally, the balance-sheet constraint controls the investment in risk-free assets.

From now on, we consider a change of the risk or payoff of one type of the risky assets or default correlation, and derive how the optimal allocation adjusts. This is to reveal a more dynamic picture of how the bank restructures the portfolio of different assets when the conditional information on assets changes. Here, we use the two-fund separation theorem of Markowitz (1952). The two funds refer to the risky fund, which is comprised of high-risk high-earning and low-risk low-earning assets, and the risk-free fund, which essentially is risk-free asset. Then, the risky fund is the tangency portfolio on the capital market line, which is the ray from the risk-free cash flow with a tangency to the mean-variance efficient frontier of risky assets. Within the risky fund, the portfolio weights of type h and type l assets are defined as $\omega_h^* = \dfrac{A_h^*}{A_h^* + A_l^*}$ and $\omega_l^* = \dfrac{A_l^*}{A_h^* + A_l^*}$ respectively. For the portfolio composed of the risky and risk-free fund, the amounts of the allocations $A_h^* + A_l^*$ and G^* represent their relative portfolio weights, since the bank's size does not change.

3.2.1. The risky fund

The following proposition is derived from the first derivative of the optimal weight of high-risk high-earning asset (ω_h^*) with respect to the payoff or default probability of any risky asset, or default correlation. Obviously, the weight of low-risk low-earning asset (ω_l^*) would consequently change in an opposite direction.

Proposition 1. *Within the risky fund, the bank invests proportionally more (less) in high-*

[12] Note that, because we assume $a \geq (1 + C_h)(D + K) - Dr_D > 0$, $B < a$.

[13] This ratio is a reward-to-variability ratio introduced by Sharpe (1966), which is modified according to the settings in our model.

risk asset, ceteris paribus, if
(a) its payoff C_h increases (decreases);
or (b) its probability of default p_h decreases (increases);
or (c) the payoff of low-risk asset C_l decreases (increases);
or (d) the default probability of low-risk asset p_l increases (decreases);
or (e) default correlation ρ increases (decreases), given that $SR_h > SR_l$.

The proof is provided in Appendix A.1. As expected, the bank invests more in high-risk high-earning assets, when the asset generates higher payoff or its obligor has a lower probability of defaulting, or the other risky asset generates lower payoff or its obligor has a higher probability of defaulting, *ceteris paribus*. In short, a high-risk high-earning asset acts as a substitute for a low-risk low-yield asset, and provides a natural hedge against losses stemming from low-risk assets.

When default correlation increases, the bank allocates more to high-risk high-earning assets if their Sharpe ratio is larger than that of low-risk low-yield assets. That is, when the two types of assets are more likely to default at the same time, the best strategy is to compare their Sharpe ratios and go for the asset type with a higher Sharpe ratio.

3.2.2. The risk-free fund

For the risky fund as a single asset, there is no measure of its overall payoff or probability of default. Therefore, as in Section 3.2.1, we consider any change of the payoff or default probability of any risky asset or of the default correlation, which measures how the earning or risk of the whole fund varies. The following proposition is derived from the first derivative of the optimal investment in risk-free asset (G^* (equation (10)) with respect to each of these measures. Obviously, the allocation to the risky fund ($A_h^*+A_l^*$) would consequently change in an opposite direction. Recall that the amounts of the allocations in different funds represent their relative portfolio weights since the bank's size does not change.

Proposition 2. *The bank invests more (less) in the risk-free fund, ceteris paribus, if*
(a) the risk-free rate r_f increases (decreases);
or (b) the payoff of high-risk asset C_h increases (decreases), given that $\rho\sqrt{V_h} \geq \sqrt{V_l}$;
or (c) the default probability of high-risk asset p_h decreases (increases), given that $\dfrac{\rho\sqrt{V_h}}{\sqrt{V_l}} \geq \dfrac{2X_h}{X_h+X_l} > 1$;
or (d) the payoff of low-risk asset C_l decreases (increases), given that $\dfrac{\rho\sqrt{V_h}}{\sqrt{V_l}} \geq \dfrac{X_h+1+r_f}{X_l+1+r_f}$, $\rho^2 X_h \geq X_l$, $\rho(1+r_f) > X_l$ and $\dfrac{X_h^2}{X_l^2} \geq \dfrac{V_h-\rho\sqrt{V_h V_l}}{V_l-\rho^2 V_l}$;

or (e) the default probability of low-risk asset p_l increases(decreases), given that $\rho\sqrt{V_h} \geq \sqrt{V_l}$ and $SR_l^2 \leq 1 - \rho^2$;
or (f) default correlation ρ increases (decreases), assuming $\rho\sqrt{V_h} \leq \sqrt{V_l}$ and $SR_h \geq SR_l$.

The proof is in Appendix A.2. As expected, the bank buys more securities when the risk-free rate is higher.

This proposition shows that when a high-risk asset is far riskier than a low-risk asset in terms of cash flow variance, i.e. $\rho\sqrt{V_h} > \sqrt{V_l}$, the risk-free fund is a complement to the high-risk asset (in statement (b) and (c)) and a substitute for the low-risk asset (in statement (d) and (e)), given possible additional conditions, such as the relative difference in variance is bigger than that in excess cash flow, and some measures of excess cash flows are limited.

When a high-risk asset generates less payoff or is more likely to default, the bank invests less in high-risk assets and more in low-risk assets, as Proposition 1 tells us. However, when a high-risk asset is far riskier than a low-risk asset in terms of cash flow variance, the increase of the investment in the low-risk asset is much more than the decrease of that in high-risk asset, so that the investment in the risk-free asset actually decreases. Yet, a risk-free asset is always a substitute for a low-risk asset, since the low-risk asset is always less risky than a high-risk asset.

When default correlation increases, the bank invests more in the risk-free fund, given that the cash flow variances of the risky assets are relatively close and a high-risk asset earns a higher Sharpe ratio than a low-risk asset. Here, the risk-free fund mitigates the risk stemming from that both types of the risky assets default at the same time.

3.3. Impacts of risk-based capital regulation

This section disentangles the impacts of risk-based capital regulation on the bank's asset risk through analysing how the bank changes its asset allocation when the regulator imposes a new and more stringent capital requirement in the situation that the bank's capital is already constrained by the current regulation or will become constrained by the new regulation.

3.3.1. Optimal portfolio allocation when the capital constraint is currently binding

For the bank that hits the minimum capital requirement and whose capital is constrained, the allocations to the risky assets are restricted by the risk-based capital ratio, which is the ratio of capital to the total risk-weighted assets. Under this condition, we derive the following inner solution to the maximization problem (equation (6)):

$$A^*_{bh} = \frac{(a-B)\varphi_l(\varphi_l SR_h - \varphi_h SR_l) - \dfrac{K}{k}\{SR_l(\varphi_l SR_h - \varphi_h SR_l) + (\rho\varphi_l - \varphi_h)\}}{(\varphi_h^2 - 2\rho\varphi_h\varphi_l + \varphi_l^2)\sqrt{V_h} + (\varphi_l SR_h - \varphi_h SR_l)^2\sqrt{V_h}} \tag{11}$$

$$A^*_{bl} = \frac{(a-B)\varphi_h(\varphi_h SR_l - \varphi_l SR_h) - \dfrac{K}{k}\{SR_h(\varphi_h SR_l - \varphi_l SR_h) + (\rho\varphi_h - \varphi_l)\}}{(\varphi_h^2 - 2\rho\varphi_h\varphi_l + \varphi_l^2)\sqrt{V_l} + (\varphi_l SR_h - \varphi_h SR_l)^2\sqrt{V_l}} \tag{12}$$

$$\begin{aligned} G^*_b =& D + K - \frac{K}{kW_l} \\ &+ \frac{W_h - W_l}{W_l}\frac{(a-B)\varphi_l(\varphi_l SR_h - \varphi_h SR_l) - \dfrac{K}{k}\{SR_l(\varphi_l SR_h - \varphi_h SR_l) + (\rho\varphi_l - \varphi_h)\}}{(\varphi_h^2 - 2\rho\varphi_h\varphi_l + \varphi_l^2)\sqrt{V_h} + (\varphi_l SR_h - \varphi_h SR_l)^2\sqrt{V_h}} \end{aligned} \tag{13}$$

where b stands for *binding*, W_h and W_l are the risk weightings for risky assets used in the calculation of the total risk-weighted assets, and $\varphi_h \equiv \frac{W_h}{\sqrt{V_h}}$ and $\varphi_l \equiv \frac{W_l}{\sqrt{V_l}}$.

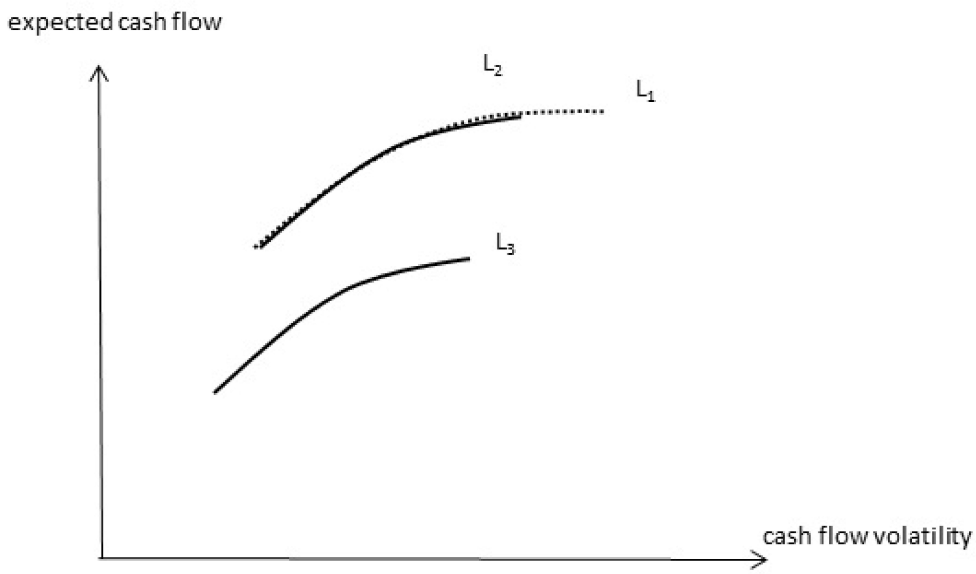
We interpret φ_h and φ_l as the regulatory cost per asset risk for high-risk and low-risk assets respectively. It is obvious that these costs play an imperative role in determining the optimal allocation.

3.3.2. Impacts of a tightening capital requirement

We then study how the bank reshuffles the portfolio due to new and more stringent capital regulation, such as an increase in the risk-based capital requirement k as in Basel Accord III. This would provide a prediction on the impacts of Basel III on banks' behaviour.

One important effect of a tightening capital requirement on the bank's portfolio is to change its efficient asset investment frontier. For a bank whose capital is not constrained by the current regulation, since it still faces risk-based capital regulation: $W_h A_h + W_l A_l \leq \frac{K}{k}$ (equation (7c)), there are upper limits for the portfolio weights in the risky funds and hence also for their expected values and variances of cash flows. When the regulator imposes a new regulation and requires the bank to have a higher capital ratio, these upper limits for the risky funds are smaller. Subsequently, those risky funds, whose expected values and variances of cash flows are too high and that locate far upward and right on the efficient frontier, are out of reach for the bank now. Therefore, the available efficient frontier shrinks from upward and right, which is illustrated by a move from line L_1 to line L_2 in Figure 3, and

Figure 3: Banks' efficient asset investment frontier



This figure shows banks' efficient asset investment frontier of risky assets. Each curve represents the best possible expected cash flow of a bank's portfolio of assets for its level of risk (cash flow volatility) under a certain capital regulation rule.

the bank's capital might become constrained by the new regulation. See proof in Appendix B.2.

For a bank whose capital is already constrained by regulation, all the risky funds on the efficient frontier reach the upper limits of their expected values and variances of cash flows. Thus, when the regulator imposes a new and more stringent regulation, i.e. a higher k, the bank's efficient frontier falls downward and to the left since any risky fund's expected value and variance of cash flows decrease, which is illustrated by a move from line L_2 to line L_3 in Figure 3. See proof in Appendix B.2.

How will the bank whose capital is already constrained by the current regulation reshuffle its optimal portfolio due to the new regulation? The following proposition is derived from the first derivatives of the optimal portfolio weight of high-risk asset in the risky fund ($\omega_{bh}^* \equiv \dfrac{A_{bh}^*}{A_{bh}^* + A_{bl}^*}$) and of the optimal investment in the risk-free fund (G_b^*) with respect to k.

It turns out that there are two key parameters determining the bank's choice: $\vartheta_h \equiv [(1-p_h)(1+C_h)-(1+r_f)]/W_h$ and $\vartheta_l \equiv [(1-p_l)(1+C_l)-(1+r_f)]/W_l$. ϑ_h and ϑ_l measure expected excess cash flows per capital cost due to the regulation for high-risk and low-risk assets respectively. We call ϑ_h and ϑ_l reward-to-regulatory-cost ratios.

Proposition 3. *When the bank's capital is constrained by regulation and the regulator imposes a new and more stringent regulation with a higher capital requirement k,*

(a) within the risky fund, the bank invests proportionally more in high-risk asset, ceteris paribus, given that $\vartheta_h > \vartheta_l$;
and (b) the bank invests more in the risk-free fund, ceteris paribus, given that $\vartheta_h \geq \vartheta_l$ and $\rho\varphi_l \geq \varphi_h$.

The proof is in Appendix B.1. Statement (a) tells us that if the bank whose capital is already constrained by the regulation is required to have a higher capital ratio, it would reshuffle the risky fund and invest proportionally more in the asset with higher reward-to-regulatory-cost ratio. Different from the Sharpe ratio, which is a reward-to-variability ratio, the denominator of reward-to-regulatory-cost ratio is the risk weighting assigned to that type of risky asset by the regulator. Therefore, if the risk weightings are not consistent with the assets' cash flow variances, which are measures of risk in this model, we could predict that there would be opportunities for regulatory arbitrage.

The statement (b) shows that when the bank is required to have a higher capital ratio, it invests more in the risk-free fund, when the regulatory cost per asset risk for low-risk asset is much higher than that for high-risk asset whose valuation (reward-to-regulatory-cost ratio) is not lower. That is, the risk-free fund mitigates the risk of a higher regulatory cost, when low-risk assets are too costly to provide such a mitigation.

How will the portfolio change for the bank whose capital is not constrained by the current regulation but will become constrained by the new regulation? We derive the following proposition by comparing the optimal portfolio weight of high-risk asset in the risky fund for a bank whose capital is not constrained by regulation (ω_h^*), and that for a bank whose capital is constrained (ω_{bh}^*).

Proposition 4. *When the regulator imposes a higher risk-based capital requirement k and the bank's capital will become constrained by the new regulation, within the risky fund, the bank invests proportionally less in high-risk assets, if $\vartheta_h > \vartheta_l$.*

The proof is in Appendix B.3. Proposition 4 states that, within the risky fund, the bank invests proportionally more in the asset with a lower reward-to-regulatory-cost ratio, when its capital becomes constrained due to a higher capital requirement. That is, a more stringent regulation poses an additional cost, so that the bank cannot invest in the asset with a higher reward-to-regulatory-cost ratio as it could otherwise. Thus, the regulation does restrict the bank from taking more risk. However, this effect does not exist for the bank whose capital is already constrained by regulation before a more stringent capital requirement is imposed, as stated in Proposition 3.

4. Empirical examination

This section tests the model using bank-level data for all U.S. commercial banks insured by the FDIC from 2002 to 2012. Due to concerns regarding domestic and international competitiveness, the implementation of capital regulation in the U.S. closely follows the Basel Accords.[14]

Ideally, an empirical test of the model should be conducted using detailed micro data reflecting each bank's individual assets and therefore the credit risk and payoff of each asset. Unfortunately, due to business confidentiality, it is not possible to obtain such detailed data. Yet, we have data on the total holding of assets within each risk category characterized by a risk weighting under the Basel Accords, i.e. the total value of assets with 0%, 20%, 50%, or 100% risk. Thus, we presume each bank does business with all the corporations with senior unsecured debentures issued in the market. Thereafter, the credit risk and payoff of each bank's assets within one risk category are valued by those bonds' issuers with external ratings matching this risk category according to Basel II. Therefore, the empirical tests aim to investigate whether banks absorb the market-wide macro information on credit risk in their decision making on asset allocations as predicted by the model. We also test the model's prediction on the impacts of a tightening capital regulation on banks' asset allocations.

4.1. Data

We obtain the bank financial data in the Uniform Bank Performance Report (UBPR), provided by the Federal Financial Institutions Examination Council. The source of the data in the UBPR is the Report of Condition and Report of Income (Call Reports), filed quarterly by each bank. In order to capture banks' variations in risk appetites and their decision making on asset allocations, we use yearly data for the empirical tests. We exclude banks with assets of less than one billion dollars and with capital ratio or risk-based capital ratio equal to or larger than 25%, in order to ensure the relevance of the tests.

We use Standard & Poor's long-term credit ratings of the issuers of all senior unsecured corporate debentures in the market and actual defaults among these issuers to estimate default probability of corporations with certain ratings and the default correlation between

[14] General risk-based capital rules based on Basel I have been implemented since 1989; the standardized approach for general banking organizations and the advanced internal ratings-based approach for core banks, based on Basel II, have been implemented since 2008. Core banks are those with consolidated total assets of $250 billion or more or with a consolidated total on-balance sheet foreign exposure of $10 billion or more. See 'Risk-based capital standards: Advanced capital adequacy framework – Basel II' (2007) and 'Risk-based capital guidelines; Capital adequacy guidelines: Standardized framework' (2008).

these corporations and other corporations holding other ratings. To value the payoff of assets within one risk category, we use average yield[15] on the bonds issued by the corporations holding the ratings corresponding to the risk category according to Basel II. These data are obtained from Standard & Poor's Capital IQ.

4.2. Asset categories

Under the standardized approach in Basel Accord II, assets are classified into different risk categories according to their external ratings when the ratings are applicable. Based on Basel Accord II and the requirements for Call Reports, the ratings corresponding to assets with 20%, 50%, and 100% risk are AAA to AA, A, and BBB to BB,[16] respectively.

Yet, there are so few observations of defaults for corporations holding ratings AAA to AA, or A, that it results in a zero default rate in much of the sample period. Therefore, we combine assets with 20% and 50% risk and assign them an average risk level of 35%. Consequently, there are three types of assets in the sample: risk-free assets, low-risk low-earning assets, and high-risk high-earning assets, with 0%, 35%, and 100% risk respectively, consistent with the model in Section 3. Hence, the credit qualities of low-risk assets and high-risk assets are estimated by the market-wide bond issuers holding ratings AAA to A and BBB to BB, respectively.

4.3. Estimating default probability, default correlation, and payoff

We estimate the probabilities of default by empirical average cumulative default rates for a historical time period, which are commonly used by the major rating agencies. These historical default rates, based on issuer, give equal weights to all issuers in the calculation, regardless of differences in the nominal sizes of the bonds issued by each issuer.[17] This approach is also cohort based, which tracks the default rates of firms with a certain rating on a given calendar date, and this pool of issuers is a *cohort*. We adopt the method of calculating average cumulative default rates with adjustment for rating withdrawals used by Moody's, as

[15]Since the yield on a bond already counts for the risk associated with the bond, using yield would underestimate the payoff of a type of assets. Nevertheless, the average yield in the market provides a macro level (actually a macro low bound) of the average payoff of the type of assets, which is comparable across time.

[16]According to the instructions for Call Reports, only the ratings above B are eligible for the ratings-based approach. Although in accordance with the Dodd-Frank Wall Street Reform and Consumer Protection Act the U.S. rules do not reference external credit ratings from 2010, in practice U.S. banks often use external ratings, see "Regulatory Consistency Assessment Programme (RCAP), Assessment of Bsel III regulations-United States of America" available at www.bis.org/bcbs/publ/d301.pdf.

[17]We use average default information on issuers instead of issues to fetch the low bound of macro level of default, since the ratings of issues are generally not higher than that of their issuer.

demonstrated by Cantor and Hamilton (2007). We then modify their methodology accordingly in order to estimate the default correlations. The methods are illustrated in Appendix C.

Since there are relatively more default observations on a quarterly basis than on a monthly basis, especially for investment-grade corporations, we employ quarterly cohort spacing, which also produces more accurate estimates of default correlations. For the same reason, we also choose a longer investment horizon of four years.[18] Then, it is assumed that, when banks' managers make portfolio choices on assets, they hold expectations on default probabilities and default correlations based on the historical information during the previous four years.

To value the credit risk and payoffs of the assets with certain risk type and to preserve the creditworthiness of issuers, we only employ the senior unsecured straight bonds, which are fixed-rate U.S. dollar bonds without any asset-backed or credit-enhancement feature, e.g. callable, puttable, sinking fund, or convertible. We then estimate the payoff of each type of assets at a date by an average of four-year-to-maturity yields[19] at that date on all available straight bonds whose issuers hold certain ratings.

4.4. Results

Table 1 summarizes statistics of the data on the variables used in the empirical tests. *proportion of high risk assets* and *proportion of low risk assets* are the banks' actual shares of high risk (100% risk) and low risk (20% and 50% risk) assets in the risky (20%, 50%, and 100% risk) assets respectively. So they sum up to one for each bank. Consistent with Figure 2, on average banks allocate resources more to high risk assets. The *payoff* and *default probability* of each type of risky assets and their *default correlation* are the average macro credit information from our estimation. Consistent with the assumptions in the model, high risk assets have higher probability of default and payoff compared to low risk assets. These variables are used to test whether banks do absorb macro credit information in the direction as the model predicts.

Basel II is a dummy variable for the years since Basel II was implemented. We use *Basel II* as a proxy for a tightening capital requirement. This approximation is applicable since assets' risk is valued by the types of their obligors under Basel I instead of by the actual risk of the obligors. For example, assets involving banks in OECD countries are classified as

[18]We perform robustness checks on two-year and three-year default rates and correlations and the results are qualitatively similar. The maximum possible length of the estimation window is four years because the data on actual defaults date back to 1998.

[19]The yield that represents one issuer is an average of yields on all available straight bonds issued by that corporation.

Table 1: Summary statistics of the data

VARIABLES	Mean	Std. Dev.	Min.	Max.
proportion of high risk assets	61.83	15.10	0.08	99.31
proportion of low risk assets	38.17	15.10	0.69	99.92
payoff of high risk assets	5.80	1.69	2.49	8.46
payoff of low risk assets	3.62	1.41	1.19	5.58
default prob. of high risk assets	159.9	126.6	20.1	432.0
default prob. of low risk assets	14.75	12.97	0.00	43.52
default correlation	1.55	1.77	0.00	5.83
Basel II	0.48	0.50	0	1
capital becomes constrained	0.01	0.10	0	1
risk-based capital ratio	12.98	2.74	1.97	24.96
capital ratio	10.33	3.03	1.00	24.95
non-earning assets	9.45	3.72	0.34	35.66
loan and lease allowance	1.70	1.71	0.00	100
size	0.02	0.11	0.00	1.90
core banks	0.01	0.08	0	1

proportion of high-risk assets and *proportion of low-risk assets* are the proportions of banks' allocations within risky assets. Therefore, for each bank, they sum up to one. *payoff*, *default probability*, and *default correlation* are the average macro credit information on the risky assets from our estimation. *Basel II* is a time dummy taking one from 2008 when Basel II was implemented. *capital becomes constrained* is a dummy variable which takes one for a bank whose ratio of capital to the total risk-weighted assets is more than 8% at time $t-1$ but not at time t. The bank level controls are *risk-based capital ratio* (the ratio of total capital to total risk-weighted assets), *capital ratio* (the ratio of total capital to total assets), *non-earning assets* (average non-earning assets divided by average total assets), *loan and lease allowance* (loan and lease allowance to total loans and lease), *size* (total assets), and *core banks* (a dummy variable for banks using the advanced internal ratings-based approach under Basel II). The estimated payoffs, default probabilities, and default correlation, which are used to calibrate the model, are valued at the beginning of the last quarter. The variables are valued in percent, except *default probability* and *default correlation* in basis points, *size* in trillions of dollars, and dummy variables.

20% risk category under Basel I, but among which the ones whose obligors have high credit risk will fall to 50 % or 100 % risk category under Basel II. *capital becomes constrained* is a dummy which takes one for a bank who fulfils the capital requirement at $t-1$ but not at t. We use these variables to test the model's predictions on the impacts of a tightening capital requirement proxied by Basel II on banks' asset risk. The rest of the variables are bank level controls.

For the panel data, since estimated default probabilities, default correlation, and payoffs are macro information, we use bank fixed-effect regressions to capture the evolution of banks' choices of assets across time. Moreover, we adopt Hoechle's (2007) approach with Driscoll and Kraay (1998) standard errors to produce standard error estimates that are heteroskedasticity, autocorrelation, and cross-sectionally consistent.

Table 2 presents the results. The dependent variable is each bank's actual share of high risk assets within the risky assets. The tests are conducted for the years when the excess cash flows over risk-free asset of high risk assets and that of low risk assets are positive (the condition for the propositions in the model[20]). We only test the most part of Proposition 1 and a proxy to Proposition 4 in the model, since the conditions in Proposition 1(e), 2, and 3[21] are not met due to the limited information on assets' actual payoffs and credit risk besides at the average macro level. Specification (1) and (2) test the statements (a) to (d) of Proposition 1 for banks whose capitals are not constrained by regulation. It is verified that banks do increase allocation of resources in high-risk assets when, in general, they have a higher average payoff or on average less likely to default, and low-risk assets are substitutes to high-risk assets. We cannot test the exact version of Proposition 4, since the condition for assets' valuations ($\vartheta_h > \vartheta_l$) based on the macro information is not met in 2008 when Basel II was implemented. However, there are banks whose capitals became constrained in 2008. Thus, we test the concept of an immediate risk reduction due to a tightening capital requirement, a proxy to Proposition 4. Specification (3) tells that the banks whose capitals became constrained due to the tightening capital requirement do invest less in high-risk assets, although its explaining power is weaker when we add bank-level controls (specification (4)). After counting for banks' actual capital levels (*risk-based capital ratio* and *capital ratio*) and their valuations of asset risk (*non-earning assets* and *loan and lease allowance*), on average, banks still skew the risky portfolio to high-risk assets since 2008 (*Basel II*).

[20]The condition is not fulfilled in year 2003 or 2004.

[21]The condition $\vartheta_h > \vartheta_l$ in Proposition 3 is only met in 2011 and there is no bank whose capital is constrained in both 2007 and 2008 when there was a shift of regulation.

Table 2: Bank fixed-effect panel regressions on banks' allocations in high-risk assets

	(1)	(2)	(3)	(4)
VARIABLES	Prop. 1	Prop. 1	Prop. 4'	Prop. 4'
payoff of high-risk assets	0.18*	0.18		
	(0.08)	(0.17)		
payoff of low-risk assets	-0.44***	-1.48***		
	(0.10)	(0.16)		
default prob. of high-risk assets	-0.03***	-0.03***		
	(0.00)	(0.00)		
default prob. of low-risk assets	0.49***	0.46***		
	(0.01)	(0.03)		
default correlation	-3.15***	-2.82***		
	(0.11)	(0.27)		
Basel II			1.03	4.27***
			(1.25)	(0.65)
d2008			2.50**	-0.42
			(0.82)	(0.28)
capital becomes constrained			4.91***	-0.10
			(0.40)	(0.77)
(d2008)*(capital becomes constrained)			-5.31***	-1.68*
			(0.61)	(0.84)
capital ratio		1.58***		1.72***
		(0.24)		(0.18)
loan and lease allowance		0.14*		0.16**
		(0.06)		(0.07)
risk-based capital ratio		-2.01***		-2.17***
		(0.18)		(0.16)
non-earning assets		-0.48***		-0.50***
		(0.09)		(0.08)
size		-11.14***		-9.25***
		(1.50)		(1.56)
core banks		4.25***		3.82***
		(0.87)		(1.11)
Constant	65.54***	82.92***	61.70***	75.27***
	(0.45)	(1.17)	(1.01)	(1.35)
Observations	4,068	4,064	4,121	4,117
Number of groups	803	801	804	802
F	3445	4129	93.48	168.6
Prob>F	0	0	9.46e-07	3.43e-08
within R-squared	0.0964	0.292	0.0331	0.278

payoff, *default probability*, and *default correlation* are the average macro credit information on the risky assets from our estimation. *Basel II* is a time dummy taking one from 2008 when Basel II was implemented. *d*2008 is a dummy for year 2008. *capital becomes constrained* is a dummy variable which takes one for a bank whose risk-based capital ratio is more than 8% at time $t-1$ but not at time t. The bank level controls are *risk-based capital ratio* (the ratio of total capital to total risk-weighted assets), *capital ratio* (the ratio of total capital to total assets), *non-earning assets* (average non-earning assets divided by average total assets), *loan and lease allowance* (loan and lease allowance to total loans and lease), *size* (total assets), and *core banks* (a dummy variable for banks using the advanced internal ratings-based approach under Basel II). The variables are valued in percent, except *default probability* and *default correlation* in basis points, *size* in trillions of dollars, and dummy variables. In parentheses are heteroskedasticity, autocorrelation, and corss-sectionally consistent Driscoll and Kraay (1998) standard errors. $*$, $**$, and $***$ denote significance at 10%, 5%, and 1% confidence level respectively.

In addition, bank-level controls reveal that the banks that are rich in capital in terms of capital ratio or reserve more loan and lease allowances (which cover expected losses of assets) invest more in high-risk assets in order to pursue earnings. Naturally, a higher risk-based capital ratio is associated with a higher portion of high-risk assets. Yet, the banks with bigger sizes or more non-earning assets are more risk averse in that they invest proportionally less in high-risk assets. However, the core banks[22] invest proportionally more in high-risk assets compared to other banks. This suggests an inconsistency between the internal ratings-based approach used by the core banks and the standardized approach used by other banks.

5. Conclusion

This paper explicitly investigates the credit risk of banks' assets and addresses banks' portfolio allocations under risk-based capital regulation. Adopting a methodology from credit portfolio optimization literature allows us to disentangle the impacts of risk-based capital regulation on the credit risk of banks' assets, which have hitherto not been explored by the previous banking literature.

Our model of portfolio allocation shows that, when risk-based capital regulation is binding, the risk weightings assigned by the regulator affect the original measures of risk and valuation of assets, namely volatility around expected loss due to default risk and Sharpe ratio respectively. This raises concerns that if the risk weightings are not consistent with the true risk measures of assets, there could be opportunities for regulatory arbitrage that banks invest more in the assets with a high level of true risk but a low regulatory risk weighting. If the regulator imposes a new and more stringent regulation, i.e. a higher risk-based capital requirement, the bank whose capital is already constrained will skew the risky portfolio to high-risk high-earning asset, while the bank whose capital will become constrained by the new regulation will do the opposite, i.e. investing less in high risk asset, provided that the valuation (reward-to-regulatory-cost ratio) of high risk asset is higher than that of low risk asset. The immediate risk reduction of the latter is consistent with the literature.

The empirical tests support the model's predictions applicable in the dataset. Due to business confidentiality, detailed data on each bank's each asset are not available. Yet, the average macro information on payoffs and credit risk of assets in each risk category that we estimate is very informative in explaining banks' actual asset choices. The tests support the prediction of flying to earnings and avoiding default risk (Proposition 1) and the

[22]By core banks, we mean banks with total assets of at least $250 billion at the end of a year since 2008, identified as those using the advanced internal ratings-based approach under Basel II.

immediate risk reduction for banks whose capitals become constrained due to a tightening capital regulation (Proposition 4'). In addition, our results also reveal the characteristics of banks with different risk-taking. In particular, the core banks, which use the internal ratings-based approach under Basel II, take more asset risks compared to other banks, which suggests an inconsistency between the internal ratings-based approach and the standard approach that the regulation is softer to the core banks.

Our study contributes to the literature and ongoing debates on banks' risk-taking and capital regulation from the angle of credit risk, which hopefully paves a way for future research on banks' asset risk. For example, our analysis could be extended by using detailed data on assets at individual bank level.

Appendix

A. Optimal portfolio allocation when the capital requirement is not binding

A.1. Proof of Proposition 1

(a) Within the risky fund, the bank invests proportionally more (less) in high-risk asset, ceteris paribus, if its payoff C_h increases (decreases).
Proof:

$$\begin{aligned}\frac{\partial \omega_h^*}{\partial C_h} &= \frac{\partial \dfrac{A_h^*}{A_h^* + A_l^*}}{\partial C_h} \\ &= \frac{(A_h^*)^2\sqrt{p_h(1-p_h)}}{(A_h^* + A_l^*)^2(E_h\sqrt{V_l} - \rho E_l\sqrt{V_h})^2\sqrt{V_l}}\{\rho(E_l\sqrt{V_h} - E_h\sqrt{V_l})^2 \\ &\quad + (1-\rho)E_l\sqrt{V_hV_l}[\rho(1-p_h)(1+C_h) + 2(1+r_f) - (1-p_h)(1+C_h)]\} \\ &> 0\end{aligned} \tag{A.1}$$

since $1 > \rho > 0$ and $2(1+r_f) \geq 2 > (1-p_h)(1+C_h)$. Then $\dfrac{\partial \omega_h^*}{\partial C_h}$ is positive, i.e. the bank invests proportionally more (less) in high-risk asset, *ceteris paribus*, if its yield C_h increases (decreases).

(b) Within the risky fund, the bank invests proportionally more (less) in high-risk asset, ceteris paribus, if its probability of default p_h decreases (increases).
Proof:

$$
\begin{aligned}
\frac{\partial \omega_h^*}{\partial p_h} &= \frac{\partial \dfrac{A_h^*}{A_h^* + A_l^*}}{\partial p_h} \\
&= -\frac{(A_h^*)^2}{(A_h^* + A_l^*)^2 (E_h V_l - \rho E_l \sqrt{V_h V_l})^2} \{(1-\rho^2) E_l V_h V_l (1 + C_h) \\
&+ \frac{V_l}{2}(1 - 2p_h)(1 + C_h)^2 [E_l \sqrt{V_h}(SR_h - \rho SR_l) + E_h \sqrt{V_l}(SR_l - \rho SR_h)]\} \\
&< 0
\end{aligned}
\tag{A.2}
$$

since $p_h < 0.5$, $SR_h - \rho SR_l > 0$ and $SR_l - \rho SR_h > 0$ (as $A_h^* > 0$ and $A_l^* > 0$). Hence, $\frac{\partial \omega_h^*}{\partial p_h} < 0$, i.e. the bank invests proportionally more (less) in high-risk asset, *ceteris paribus*, if its probability of default p_h decreases (increases).

(c) Within the risky fund, the bank invests proportionally more (less) in high-risk asset, ceteris paribus, if the payoff of low-risk asset C_l decreases (increases).

Proof: It can be shown similarly as in the proof of statement *(a)*.

(d) Within the risky fund, the bank invests proportionally more (less) in high-risk asset, ceteris paribus, if the default probability of low-risk asset p_l increases (decreases).

Proof: It can be shown similarly as in the proof of statement *(b)*.

(e) Within the risky fund, the bank invests proportionally more (less) in high-risk asset, ceteris paribus, if default correlation ρ increases (decreases), given that $SR_h > SR_l$.

Proof:

$$
\frac{\partial \omega_h^*}{\partial \rho} = \frac{\partial \dfrac{A_h^*}{A_h^* + A_l^*}}{\partial \rho} = \frac{(A_h^*)^2 V_h \sqrt{V_h V_l}}{(A_h^* + A_l^*)^2 (E_h \sqrt{V_l} - \rho E_l \sqrt{V_h})^2} (SR_h^2 - SR_l^2) > 0 \tag{A.3}
$$

if $SR_h > SR_l$. Then the bank invests proportionally more (less) in high-risk asset, *ceteris paribus*, if default correlation ρ increases (decreases), given that $SR_h > SR_l$.

A.2. Proof of Proposition 2

(a) The bank invests more (less) in the risk-free fund, ceteris paribus, if the risk-free rate r_f increases (decreases).

Proof:

$$
\frac{\partial G^*}{\partial r_f} = (D+K) \frac{\sqrt{V_l}(X_h \sqrt{V_l} - \rho X_l \sqrt{V_h}) + \sqrt{V_h}(X_l \sqrt{V_h} - \rho X_h \sqrt{V_l})}{X_h \sqrt{V_l}(X_h \sqrt{V_l} - \rho X_l \sqrt{V_h}) + X_l \sqrt{V_h}(X_l \sqrt{V_h} - \rho X_h \sqrt{V_l}) + (1-\rho^2) V_h V_l} > 0 \tag{A.4}
$$

since $X_h\sqrt{V_l} > \rho X_l\sqrt{V_h}$ and $X_l\sqrt{V_h} > \rho X_h\sqrt{V_l}$ (as $A_h^* > 0$ and $A_l^* > 0$).

(b) The bank invests more (less) in the risk-free fund, ceteris paribus, if the payoff of high-risk asset C_h increases (decreases), given that $\rho\sqrt{V_h} \geq \sqrt{V_l}$.

Proof:

$$\begin{aligned}\frac{\partial G^*}{\partial C_h} &= \frac{(a-B)\sqrt{V_l}(X_h\sqrt{V_l}-\rho X_l\sqrt{V_h})}{\{X_h\sqrt{V_l}(X_h\sqrt{V_l}-\rho X_l\sqrt{V_h})+X_l\sqrt{V_h}(X_l\sqrt{V_h}-\rho X_h\sqrt{V_l})+(1-\rho^2)V_hV_l\}(1+C_h)} \\ &+ \frac{(a-B)\sqrt{V_l}(1+r_f)(X_h\sqrt{V_l}-\rho X_l\sqrt{V_h})\{X_l\sqrt{V_h}(\sqrt{V_h}-\rho\sqrt{V_l})+\sqrt{V_l}(X_h\sqrt{V_l}-\rho X_l\sqrt{V_h})\}}{\{X_h^2V_l+X_l^2V_h-2\rho X_hX_l\sqrt{V_hV_l}+(1-\rho^2)V_hV_l\}^2(1+C_h)} \\ &\frac{(a-B)\sqrt{V_l}(1+r_f)\{\sqrt{V_hV_l}(X_l\sqrt{V_h}-\rho X_h\sqrt{V_l})(X_h-X_l)+(1-\rho^2)V_hV_l(\rho\sqrt{V_h}-\sqrt{V_l})\}}{\{X_h^2V_l+X_l^2V_h-2\rho X_hX_l\sqrt{V_hV_l}+(1-\rho^2)V_hV_l\}^2(1+C_h)}\end{aligned} \tag{A.5}$$

Since $a > B$, $X_h\sqrt{V_l} > \rho X_l\sqrt{V_h}$, $X_l\sqrt{V_h} > \rho X_h\sqrt{V_l}$, $\sqrt{V_h} > \rho\sqrt{V_l}$ (as $V_h > V_l$ and $\rho < 1$) and $X_h > X_l$, $\dfrac{\partial G^*}{\partial C_h} > 0$ given that $\rho\sqrt{V_h} \geq \sqrt{V_l}$.

(c) The bank invests more (less) in the risk-free fund, ceteris paribus, if the default probability of high-risk asset p_h decreases (increases), given that $\dfrac{\rho\sqrt{V_h}}{\sqrt{V_l}} \geq \dfrac{2X_h}{X_h+X_l} > 1$.

Proof:

$$\begin{aligned}\frac{\partial G^*}{\partial p_h} &= -\frac{(a-B)(1+C_h)\sqrt{V_l}\{\sqrt{V_hV_l}(X_l\sqrt{V_h}-\rho X_h\sqrt{V_l})(X_h-X_l)+(X_h\sqrt{V_l}-\rho X_l\sqrt{V_h})^2\sqrt{V_l}\}}{\{X_h^2V_l+X_l^2V_h-2\rho X_hX_l\sqrt{V_hV_l}+(1-\rho^2)V_hV_l\}^2} \\ &- \frac{(a-B)(1+C_h)\sqrt{V_l}\{X_l\sqrt{V_h}(X_h\sqrt{V_l}-\rho X_l\sqrt{V_h})(\sqrt{V_h}-\rho\sqrt{V_l})+(1-\rho^2)V_hV_l(\rho\sqrt{V_h}-\sqrt{V_l}))\}}{\{X_h^2V_l+X_l^2V_h-2\rho X_hX_l\sqrt{V_hV_l}+(1-\rho^2)V_hV_l\}^2} \\ &- \frac{(a-B)(1-2p_h)\sqrt{V_l}(X_h-X_l)\{X_l\sqrt{V_h}(X_h\sqrt{V_l}-\rho X_l\sqrt{V_h})+X_h\sqrt{V_l}(X_l\sqrt{V_h}-\rho X_h\sqrt{V_l})\}}{2\sqrt{V_h}\{X_h^2V_l+X_l^2V_h-2\rho X_hX_l\sqrt{V_hV_l}+(1-\rho^2)V_hV_l\}^2} \\ &- \frac{(a-B)(1-2p_h)\sqrt{V_l}V_l(1-\rho^2)\{(X_h+X_l)\rho\sqrt{V_h}-2X_h\sqrt{V_l}\}}{2\{X_h^2V_l+X_l^2V_h-2\rho X_hX_l\sqrt{V_hV_l}+(1-\rho^2)V_hV_l\}^2}\end{aligned} \tag{A.6}$$

The first term is negative since $a > B$, $X_l\sqrt{V_h} > \rho X_h\sqrt{V_l}$ and $X_h > X_l$. The second term is negative given $\rho\sqrt{V_h} \geq \sqrt{V_l}$ since $X_h\sqrt{V_l} > \rho X_l\sqrt{V_h}$ and $\sqrt{V_h} > \rho\sqrt{V_l}$. The third term is negative since $p_h < 0.5$. The last term is non-positive given that $\dfrac{\rho\sqrt{V_h}}{\sqrt{V_l}} \geq \dfrac{2X_h}{X_h+X_l} > 1$.

Therefore, $\dfrac{\partial G^*}{\partial p_h} < 0$ given that $\dfrac{\rho\sqrt{V_h}}{\sqrt{V_l}} \geq \dfrac{2X_h}{X_h+X_l} > 1$.

(d) The bank invests more (less) in the risk-free fund, ceteris paribus, if the payoff of low-risk asset C_l decreases (increases), given that $\dfrac{\rho\sqrt{V_h}}{\sqrt{V_l}} \geq \dfrac{X_h+1+r_f}{X_l+1+r_f}$, $\rho^2X_h \geq X_l$, $\rho(1+r_f) > X_l$

and $\dfrac{X_h^2}{X_l^2} \geq \dfrac{V_h - \rho\sqrt{V_h V_l}}{V_l - \rho^2 V_l}$.

Proof:

$$
\begin{aligned}
\frac{\partial G^*}{\partial C_l} &= -\frac{(a-B)\rho X_h^2 \sqrt{V_h} V_l (X_h\sqrt{V_l} - \rho X_l \sqrt{V_h})}{\{X_h^2 V_l + X_l^2 V_h - 2\rho X_h X_l \sqrt{V_h V_l} + (1-\rho^2)V_h V_l\}^2 (1+C_l)} \\
&- \frac{(a-B)(1-\rho^2)V_h V_l \sqrt{V_h}\{\sqrt{V_h}(1+r_f - X_l) - \rho\sqrt{V_l}(1+r_f - X_h)\}}{\{X_h^2 V_l + X_l^2 V_h - 2\rho X_h X_l \sqrt{V_h V_l} + (1-\rho^2)V_h V_l\}^2 (1+C_l)} \\
&- \frac{(a-B)X_h X_l V_h \sqrt{V_l}\{(1+r_f)(\rho\sqrt{V_h} - \sqrt{V_l}) - (X_h\sqrt{V_l} - \rho X_l\sqrt{V_h})\}}{\{X_h^2 V_l + X_l^2 V_h - 2\rho X_h X_l \sqrt{V_h V_l} + (1-\rho^2)V_h V_l\}^2 (1+C_l)} \\
&- \frac{(a-B)(1+r_f)V_h\{(1-\rho^2)X_h^2 V_l - X_l^2\sqrt{V_h}(\sqrt{V_h} - \rho\sqrt{V_l})\}}{\{X_h^2 V_l + X_l^2 V_h - 2\rho X_h X_l \sqrt{V_h V_l} + (1-\rho^2)V_h V_l\}^2 (1+C_l)} \\
&+ \frac{(a-B)\sqrt{V_h}(X_l\sqrt{V_h} - \rho X_h\sqrt{V_l})\{X_l\sqrt{V_h}(X_l\sqrt{V_h} - \rho X_h\sqrt{V_l}) - (1+r_f)X_h\sqrt{V_l}(\rho\sqrt{V_h} - \sqrt{V_l})\}}{\{X_h^2 V_l + X_l^2 V_h - 2\rho X_h X_l \sqrt{V_h V_l} + (1-\rho^2)V_h V_l\}^2 (1+C_l)}
\end{aligned}
\tag{A.7}
$$

The first term is negative since $a > B$ and $X_h\sqrt{V_l} > \rho X_l\sqrt{V_h}$. The second term is negative since $\sqrt{V_h} > \rho\sqrt{V_l}$ and $1 + r_f > X_h > X_l$. The third term is non-positive given that $\dfrac{\rho\sqrt{V_h}}{\sqrt{V_l}} \geq \dfrac{X_h + 1 + r_f}{X_l + 1 + r_f} > 1$. The fourth term is non-positive given that $\dfrac{X_h^2}{X_l^2} \geq \dfrac{V_h - \rho\sqrt{V_h V_l}}{V_l - \rho^2 V_l}$ since $\sqrt{V_h} > \rho\sqrt{V_l}$. In addition, $\dfrac{X_h^2}{X_l^2} \geq \dfrac{V_h - \rho\sqrt{V_h V_l}}{V_l - \rho^2 V_l}$ implies that $X_h\sqrt{V_l} > X_l\sqrt{V_h}$ if $\rho\sqrt{V_h} > \sqrt{V_l}$. Then the last term is negative if $(1 + r_f)(\rho\sqrt{V_h} - \sqrt{V_l}) \geq X_l\sqrt{V_h} - \rho X_h\sqrt{V_l}$, which is true given that $\rho\sqrt{V_h} > \sqrt{V_l}$, $\rho(1 + r_f) > X_l$ and $\rho^2 X_h \geq X_l$.

Therefore, $\dfrac{\partial G^*}{\partial C_l} < 0$ given that $\dfrac{\rho\sqrt{V_h}}{\sqrt{V_l}} \geq \dfrac{X_h + 1 + r_f}{X_l + 1 + r_f} > 1$, $\rho^2 X_h \geq X_l$, $\rho(1 + r_f) > X_l$ and $\dfrac{X_h^2}{X_l^2} \geq \dfrac{V_h - \rho\sqrt{V_h V_l}}{V_l - \rho^2 V_l}$.

(e) The bank invests more (less) in the risk-free fund, ceteris paribus, if the default probability of low-risk asset p_l increases(decreases), given that $\rho\sqrt{V_h} \geq \sqrt{V_l}$ and $SR_l^2 \leq 1 - \rho^2$.

Proof:

$$
\begin{aligned}
\frac{\partial G^*}{\partial p_l} = & \frac{(a-B)(1+C_l)\sqrt{V_h}\{X_h\sqrt{V_l}(\rho\sqrt{V_h}-\sqrt{V_l})(2X_l\sqrt{V_h}-\rho X_h\sqrt{V_l})+(1-\rho^2)X_h^2V_l\sqrt{V_h}\}}{\{X_h^2V_l+X_l^2V_h-2\rho X_hX_l\sqrt{V_hV_l}+(1-\rho^2)V_hV_l\}^2} \\
& + \frac{(a-B)(1+C_l)\sqrt{V_h}V_h(\sqrt{V_h}-\rho\sqrt{V_l})\{(1-\rho^2)V_l-X_l^2\}}{\{X_h^2V_l+X_l^2V_h-2\rho X_hX_l\sqrt{V_hV_l}+(1-\rho^2)V_hV_l\}^2} \\
& + \frac{(a-B)(1-2p_l)\sqrt{V_h}(X_h-X_l)\{X_l\sqrt{V_h}(X_h\sqrt{V_l}-\rho X_l\sqrt{V_h})+X_h\sqrt{V_l}(X_l\sqrt{V_h}-\rho X_h\sqrt{V_l})\}}{2\sqrt{V_l}\{X_h^2V_l+X_l^2V_h-2\rho X_hX_l\sqrt{V_hV_l}+(1-\rho^2)V_hV_l\}^2} \\
& + \frac{(a-B)(1-2p_l)\sqrt{V_h}V_h(1-\rho^2)\{\rho\sqrt{V_l}(X_h-X_l)+2(X_l\sqrt{V_h}-\rho X_h\sqrt{V_l})\}}{2\{X_h^2V_l+X_l^2V_h-2\rho X_hX_l\sqrt{V_hV_l}+(1-\rho^2)V_hV_l\}^2}
\end{aligned}
\tag{A.8}
$$

The first term is positive given $\rho\sqrt{V_h} \geq \sqrt{V_l}$, since $X_l\sqrt{V_h} > \rho X_h\sqrt{V_l}$. The second term is positive given that $X_l^2 \leq (1-\rho^2)V_l$, i.e. $SR_l^2 \leq 1-\rho^2$, since $\sqrt{V_h} > \rho\sqrt{V_l}$. The third term is positive since $p_l < 0.5$, $X_h > X_l$, $X_h\sqrt{V_l} > \rho X_l\sqrt{V_h}$ and $X_l\sqrt{V_h} > \rho X_h\sqrt{V_l}$, which also implies that the last term is positive.

Therefore, $\dfrac{\partial G^*}{\partial p_l} > 0$ given that $\rho\sqrt{V_h} \geq \sqrt{V_l}$ and $SR_l^2 \leq 1-\rho^2$.

(f) The bank invests more (less) in the risk-free fund, ceteris paribus, if default correlation ρ increases (decreases), given that $\rho\sqrt{V_h} \leq \sqrt{V_l}$ and $SR_h \geq SR_l$.

Proof:

$$
\begin{aligned}
\frac{\partial G^*}{\partial \rho} = & \frac{(a-B)V_hV_l\{(\sqrt{V_h}-\rho\sqrt{V_l})(X_h\sqrt{V_l}-\rho X_l\sqrt{V_h})+(\sqrt{V_l}-\rho\sqrt{V_h})(X_l\sqrt{V_h}-\rho X_h\sqrt{V_l})\}}{\{X_h^2V_l+X_l^2V_h-2\rho X_hX_l\sqrt{V_hV_l}+(1-\rho^2)V_hV_l\}^2} \\
& + \frac{(a-B)\sqrt{V_hV_l}(X_h-X_l)(X_h^2V_l-X_l^2V_h)}{\{X_h^2V_l+X_l^2V_h-2\rho X_hX_l\sqrt{V_hV_l}+(1-\rho^2)V_hV_l\}^2}
\end{aligned}
\tag{A.9}
$$

The first term is positive given that $\rho\sqrt{V_h} \leq \sqrt{V_l}$, since $a > B$, $\sqrt{V_h} > \rho\sqrt{V_l}$, $X_h\sqrt{V_l} > \rho X_l\sqrt{V_h}$ and $X_l\sqrt{V_h} > \rho X_h\sqrt{V_l}$. The second term is non-negative given that $X_h^2V_l \geq X_l^2V_h$, i.e. $SR_h \geq SR_l$, since $X_h > X_l$. Therefore, $\dfrac{\partial G^*}{\partial \rho} > 0$ given that $\rho\sqrt{V_h} \leq \sqrt{V_l}$ and $SR_h \geq SR_l$.

B. Impacts of the risk-based capital regulation

B.1. Proof of Proposition 3

(a) When the bank's capital is constrained by regulation and the regulator imposes a new and more stringent regulation with a higher capital requirement k, within the risky fund, the bank invests proportionally more in high-risk assets, ceteris paribus, given that $\vartheta_h > \vartheta_l$.

Proof:

$$\begin{aligned}\frac{\partial \omega_{bh}^*}{\partial k} =& \frac{\partial \dfrac{A_{bh}^*}{A_{bh}^* + A_{bl}^*}}{\partial k} \\ =& \frac{A_{bh}^{*2} K\sqrt{V_h}(a-B)(\varphi_l SR_h - \varphi_h SR_l)[(\varphi_l SR_h - \varphi_h SR_l)^2 + (\varphi_h^2 - 2\rho\varphi_h\varphi_l + \varphi_l^2)]}{(A_{bh}^* + A_{bl}^*)^2\sqrt{V_l}\{k(a-B)\varphi_l(\varphi_l SR_h - \varphi_h SR_l) - K[SR_l(\varphi_l SR_h - \varphi_h SR_l) + (\rho\varphi_l - \varphi_h)]\}^2} \\ &> 0\end{aligned} \tag{B.1}$$

given that $\varphi_l SR_h > \varphi_h SR_l$, i.e. $\vartheta_h > \vartheta_l$, since $a > B$ and $\varphi_h^2 - 2\rho\varphi_h\varphi_l + \varphi_l^2 = (\varphi_h - \varphi_l)^2 + 2(1-\rho)\varphi_h\varphi_l > 0$.

(b) When the bank's capital is constrained by regulation and the regulator imposes a new and more stringent regulation with a higher capital requirement k, the bank invests more in the risk-free fund, ceteris paribus, given that $\vartheta_h \geq \vartheta_l$ and $\rho\varphi_l \geq \varphi_h$.

Proof:

$$\begin{aligned}\frac{\partial G_b^*}{\partial k} =& \frac{K}{k^2 W_l} + \frac{W_h - W_l}{W_l}\frac{\partial A_{bh}^*}{\partial k} \\ =& \frac{K}{k^2 W_l} + \frac{W_h - W_l}{W_l}\frac{K\{SR_l(\varphi_l SR_h - \varphi_h SR_l) + (\rho\varphi_l - \varphi_h)\}}{k^2(\varphi_h^2 - 2\rho\varphi_h\varphi_l + \varphi_l^2)\sqrt{V_h} + k^2(\varphi_l SR_h - \varphi_h SR_l)^2\sqrt{V_h}} > 0\end{aligned} \tag{B.2}$$

given that $\varphi_l SR_h \geq \varphi_h SR_l$, i.e. $\vartheta_h \geq \vartheta_l$, and $\rho\varphi_l \geq \varphi_h$, since $W_h > W_l$.

B.2. How the efficient frontier changes

For a bank whose capital is not constrained by regulation, since it still faces the regulatory capital constraint ($W_h A_h + W_l A_l \leq \dfrac{K}{k}$, i.e. equation (7c)), for each risky fund P, there is a constraint for the portfolio weight of low-risk asset ω_l, which is $\omega_l \leq \dfrac{1}{A_h + A_l}(\dfrac{K}{W_l k} - \dfrac{W_h}{W_l}A_h)$.

Then the expected value and variance of random cash flow for each risky fund P are:

$$\begin{aligned}E[\tilde{CF}_P] &= \omega_h(1-p_h)(1+C_h) + \omega_l(1-p_l)(1+C_l) \\ &\leq \frac{1}{A_h + A_l}[A_h(1-p_h)(1+C_h) + (\frac{K}{kW_l} - \frac{W_h}{W_l}A_h)(1-p_l)(1+C_l)] \\ &= \omega_h(E[\tilde{CF}_h] - \frac{W_h}{W_l}E[\tilde{CF}_l]) + \frac{K}{(A_h + A_l)kW_l}E[\tilde{CF}_l] \equiv E[\tilde{CF}_P]_{bound}\end{aligned} \tag{B.3}$$

and

$$
\begin{aligned}
Var[\tilde{CF_P}] =& \omega_h^2 V_h + \omega_l^2 V_l + 2\omega_h\omega_l\rho\sqrt{V_hV_l} \\
\le & \frac{1}{(A_h+A_l)^2}[A_h^2V_h + (\frac{K}{kW_l} - \frac{W_h}{W_l}A_h)^2V_l + 2A_h(\frac{K}{kW_l} - \frac{W_h}{W_l}A_h)\rho\sqrt{V_hV_l}] \\
=& \omega_h^2(V_h + \frac{W_h^2}{W_l^2}V_l - 2\frac{W_h}{W_l}\rho\sqrt{V_hV_l}) \\
&+ 2\frac{\omega_h K\sqrt{V_l}}{(A_h+A_l)kW_l}(\rho\sqrt{V_h} - \frac{W_h}{W_l}\sqrt{V_l}) + \frac{K^2V_l}{k^2W_l^2(V_h+V_l)^2} \\
\equiv & Var[\tilde{CF_P}]_{bound},
\end{aligned}
\tag{B.4}
$$

where $E[\tilde{CF_P}]_{bound}$ and $Var[\tilde{CF_P}]_{bound}$ are upper limits for $E[\tilde{CF_P}]$ and $Var[\tilde{CF_P}]$ respectively.

Furthermore, as k increases, $E[\tilde{CF_P}]_{bound}$ decreases, and $Var[\tilde{CF_P}]_{bound}$ decreases since $\frac{\partial Var[\tilde{CF_P}]_{bound}}{\partial k} = \frac{2K\sqrt{V_l}}{(A_h+A_l)^2k^2W_l^2}[(A_hW_h - \frac{K}{k})\sqrt{V_l} - A_h\rho W_l\sqrt{V_h}] < 0$ (as $A_hW_h \le \frac{K}{k}$ and $\rho > 0$).

When the regulator imposes a new and more stringent capital requirement, i.e. k increases, for some risky funds, the expected value and variance of their random cash flows are higher than the respective upper limits. Hence, these risky funds are out of reach and the efficient frontier for the bank shrinks from upward and right.

For a bank whose capital is already constrained by regulation, the expected values and variances of cash flows of all the risky funds on its efficient frontier reach their respective upper limits, which decrease as k increases. Therefore, the new regulation forces the efficient frontier to move downward and to the left.

B.3. Proof of Proposition 4

Proposition 4. *When the regulator imposes a higher risk-based capital requirement k and the bank's capital will become constrained by the new regulation, within the risky fund, the bank invests proportionally less in high-risk assets, if $\vartheta_h > \vartheta_l$.*
Proof:

$$
\begin{aligned}
\frac{A_l^*/A_h^*}{A_{bl}^*/A_{bh}^*} =& \frac{SR_l - \rho SR_h}{SR_h - \rho SR_l}\frac{(a-B)\varphi_l(\varphi_l SR_h - \varphi_h SR_l) - \frac{K}{k}[SR_l(\varphi_l SR_h - \varphi_h SR_l) + (\rho\varphi_l - \varphi_h)]}{(a-B)\varphi_h(\varphi_h SR_l - \varphi_l SR_h) - \frac{K}{k}[SR_h(\varphi_h SR_l - \varphi_l SR_h) + (\rho\varphi_h - \varphi_l)]} \\
&< 1, \text{i.e.} \frac{A_l^*}{A_h^*} < \frac{A_{bl}^*}{A_{bh}^*}, \text{given that } \vartheta_h > \vartheta_l,
\end{aligned}
\tag{B.5}
$$

since

$$
\begin{aligned}
&(SR_l - \rho SR_h)\{(a-B)\varphi_l(\varphi_l SR_h - \varphi_h SR_l) - \frac{K}{k}[SR_l(\varphi_l SR_h - \varphi_h SR_l) + (\rho\varphi_l - \varphi_h)]\} \\
&- (SR_h - \rho SR_l)\{(a-B)\varphi_h(\varphi_h SR_l - \varphi_l SR_h) - \frac{K}{k}[SR_h(\varphi_h SR_l - \varphi_l SR_h) + (\rho\varphi_h - \varphi_l)]\} \\
=&(\varphi_l SR_h - \varphi_h SR_l)(A_h^* W_h + A_l^* W_l - \frac{K}{k})(SR_h^2 - 2\rho SR_h SR_l + SR_l^2 + 1 - \rho^2) < 0
\end{aligned}
$$

given that $\varphi_l SR_h > \varphi_h SR_l$, i.e. $\vartheta_h > \vartheta_l$, because $A_h^* W_h + A_l^* W_l < \frac{K}{k}$, $SR_h^2 - 2\rho SR_h SR_l + SR_l^2 > 0$ and $\rho^2 < 1$.

Therefore, $\omega_h^* = \frac{A_h^*}{A_h^* + A_l^*} = \frac{1}{1 + A_l^*/A_h^*} > \omega_{bh}^* = \frac{A_{bh}^*}{A_{bh}^* + A_{bl}^*} = \frac{1}{1 + A_{bl}^*/A_{bh}^*}$ if $\vartheta_h > \vartheta_l$, that is, when its capital becomes constrained by the capital regulation, the bank invests proportionally more in the asset with a lower reward-to-regulatory-cost ratio.

C. Estimating default probability and default correlation

We adopt the method of calculating average cumulative default rates with adjustment for rating withdrawals used by Moody's, as demonstrated by Cantor and Hamilton (2007).

A cumulative default rate for an investment horizon of length T, denoted as $D(T)$ is formulated as:

$$
\begin{aligned}
D_y(T) &= d_y(1) + d_y(2)[1 - d_y(1)] + d_y(3)[(1 - d_y(1))(1 - d_y(2))] + ... \\
&+ d_y(T)(\prod_{t=1}^{T-1}[1 - d_y(t)]) = 1 - \prod_{t=1}^{T}[1 - d_y(t)]
\end{aligned}
\tag{C.1}
$$

where $d_y(t)$ is the marginal default rate in the time interval t[23] for a cohort of issuers formed on date y holding a certain rating and calculated as $d_y(t) = \frac{x_y(t)}{n_y(t)}$, where x is the number of defaults and n is the effective size of the cohort adjusted for rating withdrawals. As displayed, the cumulative default rate is essentially a discrete-time approximation of the nonparametric continuous-time hazard rate approach and a conditional probability.

We adopt average cumulative default rates, where the average is taken over many cohort periods, to estimate default probabilities in our study. The average cumulative default rate for an investment horizon of length T, denoted as $\bar{D}(T)$, is derived from the weighted average marginal default rates, $\bar{d}(t)$, where the average is taken over all the available cohort marginal default rates in the historical data set Y.

[23] For example, in the first period after the formation of a cohort, $t = 1$; in the second period after the formation of a cohort, $t = 2$; etc..

Then $\bar{D}(T) = 1 - \prod_{t=1}^{T}[1 - \bar{d}(t)]$, where $\bar{d}(t) = \dfrac{\sum_{y\in Y} x_y(t)}{\sum_{y\in Y} n_y(t)}$.

As we estimate the default correlations, we modify the above methodology accordingly.

Since the pair-wise default probability for a pair, one corporation with rating 1 and one corporation with rating 2 in the time interval t, is $\dfrac{x_y^1(t)x_y^2(t)}{n_y^1(t)n_y^2(t)}$, where x_y^1 and x_y^2 are the numbers of defaults for cohorts of issuers holding rating 1 and 2 formed on date y respectively, and n_y^1 and n_y^2 are the corresponding effective sizes of the cohorts. Then the average pair-wise default rate in the time interval t over all available cohorts is $\bar{d_{12}}(t) = \dfrac{\sum_{y\in Y} x_y^1(t)x_y^2(t)}{\sum_{y\in Y} n_y^1(t)n_y^2(t)}$.

Hence, we could estimate default correlation for investment horizon of length T by an average over all available marginal default correlations in the data set Y:

$$\bar{\rho}_{12}(T) = \frac{1}{T-1}\sum_{t=1}^{T-1}\rho_{12}(t) = \frac{1}{T-1}\sum_{t=1}^{T-1}\frac{\bar{d_{12}}(t) - \bar{d_1}(t)\bar{d_2}(t)}{\sqrt{\bar{d_1}(t)[1-\bar{d_1}(t)]\bar{d_2}(t)[1-\bar{d_2}(t)]}} \tag{C.2}$$

where $\rho_{12}(t)$ is the marginal default correlation in the time interval t.[24]

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[24] $\rho_{12}(T)$ is excluded in the calculation of $\bar{\rho}_{12}(T)$ because there is only one observation at T resulting from one cohort left with a certain rating, which results in zero correlation.

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