Analyzing Hedging Strategies for Fixed Income Portfolios: A Bayesian Approach for Model Selection

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Current Version: April 15, 2014

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Abstract

During the recent European sovereign debt crisis, returns on EMU government bond portfolios experienced substantial volatility clustering, leptokurtosis and skewed returns, as well as correlation spikes. Asset managers invested in European government bonds had to derive new hedging strategies to deal with the changing return properties and the higher level of uncertainty. In this market environment, conditional time series approaches such as GARCH models might be better suited to achieve a superior hedging performance relative to unconditional hedging approaches such as OLS. The aim of this study is to develop and investigate improved hedging strategies for EMU bond portfolios for non-crises and crises periods. The empirical analysis includes OLS, constant conditional correlation (CCC), and dynamic conditional correlation (DCC) multivariate GARCH models. In addition, we introduce a Bayesian composite hedging strategy, attempting to combine the strengths of OLS and GARCH models, thereby endogenizing the dilemma of selecting the best estimation model. During the recent sovereign debt crisis yield spreads among EMU member countries widened and the well established hedging instruments such as the Bund futures suddenly were inapt to minimize the risk exposure of European government bond portfolios. As a consequence, Eurex introduced new future contracts on Italian government securities (BTP). Therefore, in this study we analyze single and composite hedging strategies with the German Bund and the Italian BTP futures contracts empirically and evaluate the hedging effectiveness in an out-of-sample setting. Thus, the pivotal research question is whether it is more important to introduce new and better suited futures contracts, or to employ more sophisticated statistical models to determine optimal hedge ratios. Our empirical results demonstrate that the Bayesian composite hedging strategy was particularly beneficial during the recent sovereign debt crisis period.

Current Version: April 15, 2014

Keywords: Sovereign Debt Crisis, Bond Portfolio Management, Interest Rate Futures, MGARCH, Bayesian composite hedging, Hedge Ratio and Hedging Effectiveness.

JEL Classification C52, G11, G19,

I. Introduction

Subsequent to the introduction of the Euro in 1999, government bond yields of countries within the European monetary union (EMU) converged. Consequently, EMU government bond yields basically co-moved, resulting in relatively stable yield spreads (Figure 1), revealing little differences in sovereign risk at least before the beginning of the financial crisis in 2008. Consequently, there was a high level of substitutability in a sense that futures contracts on bonds of one country could be used to efficiently hedge government bonds of other EMU countries. Thus, the theoretically ideal situation occurred that trading was concentrated in one or two future contracts based on German government bonds which were traded at Eurex, offering market participants market depth and liquidity. Turnover on futures contracts based on French, Italian and Spanish bonds decreased to historical low levels and, as a consequence, were finally closed and removed from the market (Blanco, 2001).

With the emerging of the financial crisis in mid 2008, sovereign risk became increasingly important for EMU bond portfolios, and government bond returns became highly dependent on political events. Figure 2 presents important political events that occurred during the sovereign debt crisis and illustrates its influence on EMU bond-portfolio and German Bund and Italian BTP futures returns. During the sovereign debt crisis the time series of EMU government bond returns exhibits positive excess kurtosis and GARCH-effects (Sibbertsen, Wegener, and Basse, 2014). Asset managers invested in European government bonds had to derive new hedging strategies to deal with the increased uncertainty and changing return dynamics. In addition, as sovereign risk levels of EMU member countries diverged, the established hedging instruments on German government bonds were inapt for hedging sovereign risk of lower rated EMU countries. Thus, futures contracts on Italian government securities (BTP) were (re-) introduced.

The objective of this study is to develop and investigate improved hedging strategies for EMU bond portfolios for non-crises and crises periods. Given the heterogeneity of sovereign risk levels during the recent sovereign debt crisis, we analyze the improvement in hedging performance by extending the hedging framework from one instrument hedges (single hedges) to two instrument-hedges (composite hedges), employing traditional (Bund-futures) and newly (re-)introduced futures contracts on Italian government bonds (BTP-futures). From a modeling perspective, traditional unconditional hedging approaches such as the minimum variance OLS hedge (Houthakker, 1959; Johnson, 1960; Stein, 1961) might not yield efficient hedging results during the sovereign debt crisis due to the assumption of a constant return covariance matrix. In contrast, hedging strategies based on multivariate GARCH (MGARCH) models (Baillie and Myers, 1991; Cecchetti, Cumby and Finglewski, 1988)[1] might provide a superior hedging efficiency. Therefore, in this study we employ OLS, constant conditional correlation (CCC) (Bollerslev, 1990), and dynamic conditional correlation (DCC) (Engle, 2002) multivariate GARCH models for hedging European government bond portfolios before and during the sovereign debt crisis and evaluate and compare their out-of-sample hedging effectiveness. Finally, we employ a Bayesian hedging strategy (Poomimars, Cadle and Theobald, 2003), attempting to control for estimation errors in GARCH models and to reduce futures turnover.

Our research contributes to the literature in several dimensions. First, we provide empirical evidence on the hedging effectiveness of simple OLS and sophisticated time series hedging (MGARCH) strategies for bond portfolios during the pre-crisis and the sovereign debt crisis period. While, there are numerous studies on the hedging effectiveness of OLS- compared to MGARCH hedging strategies for commodities, currency, and equity portfolios (Baillie and Myers, 1991; Myers, 1991; Kroner and Sultan, 1993; Tong 1996; Lien, Tse and Tsui,

[1] Related approaches include cointegration methods (e.g. Ghosh, 1993; Kroner and Sultan, 1993).

2002; Alexander and Barbosa, 2006), there is still little evidence on the performance of these models for bond portfolios and during financial crisis periods (Cecchetti, Cumby and Finglewski, 1988; Koutmos and Pericil 1999). The success of a hedging strategy is particularly important during crisis periods in which asset returns are highly volatile and the risk of severe losses is increased. Conditional time series hedging approaches (MGARCH) might be particularly beneficial compared to unconditional approaches (OLS) during crisis periods. Second, we analyze single and composite hedges for EMU government bond portfolios, employing German and the (re)introduced Italian futures contracts during crisis and non-crisis periods for conditional (MGARCH) and unconditional (OLS) hedging methods. Chen and Sutcliffe (2012), Bookstaber and Jacob (1986), Ramaswami, (1991), Piepta, (1990), Morgan (2008) and Leschhorn (2001), Bessler and Wolff (2012) provide evidence that combining multiple futures contracts improves the hedging effectiveness. However, so far these potential benefits were not investigated for conditional estimation methods such as MGARCH for bond portfolios. Moreover, most studies neglect the disadvantages associated with an increase in transaction costs resulting from employing additional hedging instruments. Therefore, we explicitly take transaction costs into account and analyze the relative importance of employing additional hedging instruments (single vs. composite hedges) compared to employing more sophisticated hedge ratio estimation methods (MGARCH vs. OLS hedges). Third, we extend Poomimars, Cadle and Theobald's (2003) Bayesian hedging strategy on composite hedges in order to control for estimation errors in MGARCH models.

Overall, our empirical results indicate, in hindsight, that EMU bond portfolio managers should have employed composite hedges with the Bund and BTP-futures relying on OLS or Bayesian hedging techniques. We find evidence that Bayesian hedging techniques dominated MGARCH hedges, resulting in a superior hedging effectiveness and reduced futures turnover relative to pure MGARCH hedges. Compared to OLS, the Bayesian composite hedging strategy turns out to be particularly beneficial during the sovereign debt crisis period, while in the pre-crisis period the difference in hedging effectiveness is only marginal.

The remainder of the paper is organized as follows. Chapter 2 reviews the relevant literature on hedging, hedge ratios and hedging effectiveness measures. The employed dataset is discussed in chapter 3. In Chapter 4 we discuss OLS as well as CCC- and DCC-MGARCH hedging methodologies. Chapter 5 presents descriptive statistics, the GARCH model selection process and the empirical hedging results for OLS and GARCH hedging strategies. Chapter 6 presents the methodology as well as the empirical results for the Bayesian hedging technique. Chapter 7 concludes.

II. Literature Review

Two crucial aspects in determining the optimal hedging strategy are the selection of adequate hedging instruments and the computation of the optimal hedge ratios. In the simplest case of a direct hedge, derivatives on the spot position are used as hedging instruments. However, if derivatives on the spot position are not available (as it is the case for EMU bond portfolios), other hedging instruments which may be selected based on the magnitude of correlation between the asset and the future returns have to be employed (cross hedge). Usually the hedging instrument(s) having the highest return correlation(s) with the spot position should be selected (Ederington, 1979). In a large number of cross hedges a hedging strategy with more than one hedging instrument (composite hedge) might be more effective compared to hedges with only one instrument.

1. Single versus Composite Hedging

Several studies provide evidence for the benefits of composite hedges for hedging bond portfolios. Bookstaber and Jacob (1986) and Ramaswami (1991) hedge high-yield corporate bonds using US Treasury bond futures and futures on the corresponding company's equity, finding that composite hedges achieve superior hedging results compared to single hedges. Grieves (1986) and Marcus and Ors (1996) provide similar results when hedging US investment-grade corporate bonds with S&P500 and Treasury bond futures.

Leschhorn (2001), Pieptea (1990) and Morgan (2008) report that hedges with futures on long- and short-term bonds, thus using information along the yield curve, are superior to single futures hedges when hedging US or German government bonds. In contrast, Koutmos and Pericili (2000) employ multiple futures contracts on Treasury Notes with different maturities (2, 5 and 10 years) to hedge mortgage-backed securities (MBS). They conclude that composite hedges - employed in an out-of-sample setting, - are inferior compared to single hedges with the 10-year Treasury-Note futures only. However, this may be more due to the fact that the characteristics of MBS are quite different from T-Notes rather than due to using different contracts along the yield curve.

Overall, the academic literature provides some empirical evidence for the benefits of composite hedges with equity and fixed income futures in the presence of default risk, but mixed results for hedging with various fixed income instruments that differ only in the maturity of the underlying. However, a drawback of most studies is that they neglect the disadvantages associated with an increase in transaction costs due to employing additional hedging instruments.

2. Determination of Optimal Hedge Ratios

While the selection of the optimal hedging instruments is not trivial, the determination of the optimal hedge ratios might be even more challenging. Various approaches for computing optimal hedge ratios were proposed in the literature. The optimal hedge ratio depends on the particular objective function which may either focus only on minimizing risk of the hedge portfolio (one-dimensional) or may include return characteristics as well (two-dimensional).

Moreover, various risk measures can be employed (e.g. return variance, semi-variance, value-at-risk etc.), resulting in different hedge ratios. Chen, Lee, and Shrestha (2003) and Lien and Tse (2011) provide an extensive review of different theoretical approaches to derive the optimal hedge ratio.

Unconditional Minimum Variance Hedging Approach

The probably most commonly used hedging approach in the academic literature and amongst practitioners is the one-dimensional minimum variance approach due to its simplicity and its validity under 'reasonable' assumptions. Moreover, if the expected returns of the hedging instruments are zero, as it might be the case for instance for fixed income futures contracts if interest rate changes are not anticipated, the one and two dimensional approaches result in identical optimal hedge ratios. Houthakker (1959), Johnson (1960) and Stein (1961) propose unconditional minimum variance hedges based on sample variances and covariances of the spot and futures returns. Ederington (1979) shows that the unconditional minimum variance hedge ratio is equivalent to the OLS regression coefficient when regressing spot on futures returns. Implicitly in OLS hedging strategies it is assumed that variances and covariances of futures and spot returns are constant over time. Moreover, all observations during the sample period obtain equal weights. Thus, two shortcomings of this approach are that first most recent developments might not be considered adequately and second fluctuations of return variances and covariances are ignored.

Conditional Hedging Approaches

If the return variances and covariances are time varying and follow certain regularities, hedgers might benefit from including information on the contemporaneous market condition Θ when estimating the optimal hedge ratio (Bell and Krasker, 1986). Following this argument, conditional hedging approaches estimate variances and covariances conditional on the available information set Θ. Several empirical studies (e.g. Mandelbrot, 1963) document the phenomenon of volatility clustering in financial return time series. Engel (1982) and Bollerslev (1986) develop autoregressive conditional heteroskedasticity models (ARCH) and generalized autoregressive conditional heteroskedasticity models (GARCH) for modeling and estimating time varying volatilities. The extension of these models from univariate to multivariate cases was proposed by Engel, Granger and Kraft (1984) for ARCH and by Bollerslev, Engel and Wooldridge (1988) for GARCH models. The multivariate GARCH (MGARCH) models transfer the notion of volatility clustering to a dynamic modeling of covariances in general and to covariance clustering specifically. A few studies apply the MGARCH framework for estimating minimum variance single hedge ratios for commodity and equity markets (e.g. Cecchetti, Cumby and Flingwelisky, 1988; Baillie and Myers, 1991; Meyers, 1991; Sephton, 1993; Brooks, Henry and Persand, 2002; Lien, Tse and Tsui, 2002; Alexander and Barbosa, 2006).

However, several studies suggest that sophisticated econometric models for estimating minimum-variance hedge ratios usually provide negligible economic benefits (Lence, 1995; Chen, Lee and Shrestha, 2003; Byström, 2003; Alexander and Barbosa 2007; Carbonez, Nguyen and Sercu, 2011; Cotter and Hanly, 2012). Moreover, more advanced econometric regression models result in much greater variability of the optimal hedge ratio and substantially larger transaction costs (Alexander and Barbosa, 2007). Nevertheless, particularly during crisis periods such as the sovereign debt crisis, characterized by volatility clustering, time-varying and clustered correlations, skewed and fat tailed bond portfolio returns, the MGARCH approach might lead to a superior hedging effectiveness compared to OLS. While earlier studies do not evaluate the performance of MGARCH hedging strategies during crisis periods, we contribute to the literature by evaluating hedging strategies during the European pre-crisis and recent sovereign debt crises period separately. Moreover, with the exception of Chen and Sutcliffe (2012), two-instrument (composite) MGARCH hedges have been neglect-ed in the literature. We also contribute to the literature by employing composite MGARCH hedges.

Bollerslev, Engel and Wooldridge's (1988) propose a flexible VECH MGARCH model which was employed for single instrument hedges by Myers (1991). As a shortcoming, this model requires a large number of estimation parameters. For a composite hedge with two hedging instruments, the number of coefficients to estimate amounts to 78. The large number of estimation parameters is associated with high estimation risk and requires large datasets for implementation. Therefore, the VECH MGARCH model seems inappropriate for computing composite hedges. For the same reason the BEKK-MGARCH model proposed by Engel and Kroner (1995) was only applied for single instrument hedges by Baillie and Myers (1991), Kroner and Sultan (1995), Koutmos and Pericil (1991) and Brooks, Henry and Persand (2002). More restrictive MGARCH models are applied by Cecchetti, Cumby and Flingwelisky (1988) based on a multivariate ARCH framework or by Baillie and Myers (1991), Bera, Garcia and Roh (1997), Yang and Allen (2005) and Cotter and Hanly (2012) who apply a diagonal VECH specification as developed by Bollerslev, Engel and Wooldridge (1988). Given the restrictions on the diagonal VECH model to ensure positive semidefinitness[2] as well as the inflexibility of the multivariate ARCH model to present higher order of volatility clustering, these models seem inappropriate for practical implementation for modeling unknown time varying covariance matrices. Brooks, Henry and Persand (2002) as well as Cotter and Hanly (2012) apply asymmetric MGARCH models in order to consider asymmetric properties of the return distribution when estimating the optimal hedge ratio. According to their studies the out-of-sample hedging effectiveness of this additional specification is rather limited.

[2] See Engel and Kroner (1995) and Attanasio (1991) on the exact model requirements for positive semidefiniteness and the difficulty to implement these during the estimation process.

Bollerslev (1990) proposes a constant conditional correlation (CCC) MGARCH specification which is employed for hedging by Kroner and Sultan (1993), Park and Schwitzer (1995), Bera, Garcia and Roh (1997), Lien, Tse and Tsui (2002), Byström (2003), Carbonez, Nguyen and Sercu (2011). Compared to the other MGARCH models the CCC model is more parsimonious and requires only 12 parameters for composite hedges. As a shortcoming, the CCC model assumes constant correlations of model residuals. A less restrictive, but also parsimonious model is the dynamic conditional correlation (DCC) MGARCH model (Engle, 2002). The DCC model requires 14 parameters for composite hedges, but has not been used for hedging in the literature so far. We contribute to the literature by evaluating single instrument and composite hedging strategies based on CCC- and DCC-MGARCH models.

A shortcoming of all MGARCH models is that the larger number of estimation parameters compared to OLS is associated with higher estimation errors. As a result, MGARCH models often involve a large level of futures turnover resulting in high implementation costs which might impede their practical implementation (Alexander and Barboza, 2007; Line, Tse and Tsui, 2002; Poomimars, Cadle and Theobald, 2003; Miffre 2004; Yang and Allen, 2004). Poomimars, Cadle and Theobald (2003) propose a Bayesian hedging strategy in order to control for estimation errors in MGARCH models. We extend this Bayesian hedging approach on composite hedges and to DCC-MGARCH models.

3. Hedging effectiveness measures

After having implemented a specific hedging strategy, the success or hedging effectiveness has to be evaluated. Several measures of the hedging effectiveness have been proposed in the academic literature. The most prominent approach is to measure the variance reduction (Ederington, 1976). However, this measure is downward biased, understating the benefits of hedging and, favoring OLS hedges especially in cases of small estimation windows, small out-of-sample periods, and small variations in the conditional variance and missing structural brakes (Lien, 2005, 2009). Cotter and Hanly (2012) employ alternative measures of risk reduction, including the reduction of a portfolio's value-at-risk and the reduction of a portfolio's lower partial moments (LPM). We employ the hedging effectiveness measures of Ederington (1976) and Cotter and Hanly (2012) and additionally measure the futures turnover to determine the associated costs of each hedging strategy.

III. Data

We analyze strategies for hedging EMU government bond portfolios during the time period from January 2000 to October 2013 and separate the full sample period into two sub-periods. The first sub-period ranges from January 2000 to December 2006 and covers the period after the introduction of the Euro but before the financial crisis. This period is characterized by low and relatively stable yield spreads between EMU government bond yields. The second sub-period ranges from January 2007 to October 2013 and includes the financial as well as the sovereign debt crisis period.

To represent EMU government bond portfolios we rely on the JP Morgan EMU Government Bond Index 1-10 years. This index reflects the development of the Euro denominated government bond market and is widely employed as benchmark for EMU fixed income portfolios. It contains market capitalization weighted government bonds of EMU member countries with maturities of 1-10 years and is rebalanced on a monthly basis. To compute optimal hedge ratios we rely on the price index. Hedge portfolio performance is computed based on the total return index which assumes that coupon payments are retained and reinvested.

As hedging instruments we employ German Bund and Italian BTP10 government bond future contracts. Bund and BTP10 futures are contracts on fictive bonds with 6% coupon and ten years maturity issued by the governments of Germany and Italy, respectively. While German bond futures should be better suited for hedging low sovereign risk bonds or during the pre-crisis period, Italian BTP contracts are expected to be more appropriate for hedging EMU government bonds with a higher level of sovereign risk during the crisis period. We assume that futures contracts are rolled forward to implement the hedging strategies. Specifically, at the last day of the month before delivery, futures contracts are rolled over to the futures with the second nearest maturity. Market prices of futures contracts are obtained from Thomson Reuters Datastream.[3]

IV. Methodology

1. Unconditional Minimum Variance Hedge Ratio Estimation (OLS)

For computing optimal hedge ratios we focus on the one dimensional minimum variance approaches as it is reasonable to assume that the daily expected returns of the applied hedging instruments are zero.[4] With zero returns for all hedging instruments the resulting optimal hedge ratios are identical for both the one and the two dimensional target functions. The minimum variance hedge ratio is derived by minimizing the return variance of the hedge portfolio (P), consisting of spot (S) and futures positions (F_i), with h_i being the hedge ratio for the hedging instrument (i). For a single hedge, in which only one futures contract is employed, the minimization problem is given by:

$$\min_{h} \; \mathrm{var}(r_P) = \mathrm{var}(r_S) + h_1^2 \, \mathrm{var}(r_{F_1}) - 2h \, \mathrm{cov}(S, F_1), \tag{1}$$

[3] BTP futures contracts were introduced and first traded in December 2009. To investigate the contribution of these futures contracts for hedging EMU bond portfolios throughout the entire financial crisis and sovereign debt crisis period, a longer time series is required. Therefore, we compute 'fair' BTP futures prices for the time period from January 2006 to December 2009 and use market prices when available. To calculate theoretic BTP futures prices, we compute the implied repo rate (IRR) in each quarter of the set of deliverable Italian government bonds, which is provided by 'Eurex'. From this we identify the cheapest-to-deliver bond and compute the theoretic futures price, taking into account accrued interest, financing costs, and the appropriate conversion factor. As robustness check, we compare the theoretically computed prices with the market prices for the period after introduction of the futures. The theoretic prices differ only marginally from the market prices, highlighting the accurateness of our calculations. As additional robustness check, we analyze the sub-periods with theoretic and actual market prices separately. The results do not differ qualitatively for both sub-periods confirming the robustness of our results.

[4] This aspect will be discussed and fortified in chapter 4: Data.

The first order condition leads to the minimum variance single hedge ratio:

$$h_{MV} = \frac{\text{cov}(S, F_1)}{\text{var}(F_1)}. \tag{2}$$

In the unconditional minimum variance hedge, the sample variances and covariances are employed. The hedge ratio is equivalent to the ordinary least squares (OLS) regression coefficient, regressing spot returns on futures returns (Houtthaker, 1959; Johnson, 1960; Stein, 1961; Ederington, 1979; Malliaris & Urrutia, 1991; Benet, 1992). Therefore, we refer to the unconditional approach as OLS hedge.

In a composite hedge with two hedging instruments, the hedge ratios are derived as (Chen and Sutcliffe, 2012):

$$h_1 = \frac{\text{cov}(r_s, r_{F1})\,\text{var}(r_{F2}) - \text{cov}(r_{F1}, r_{F2})\,\text{cov}(r_s, r_{F2})}{\text{var}(r_{F1})\,\text{var}(r_{F2}) - \text{cov}(r_{F1}, r_{F2})^2} \tag{3}$$

$$h_2 = \frac{\text{cov}(r_s, r_{F2})\,\text{var}(r_{F1}) - \text{cov}(r_{F1}, r_{F2})\,\text{cov}(r_s, r_{F1})}{\text{var}(r_{F1})\,\text{var}(r_{F2}) - \text{cov}(r_{F1}, r_{F2})^2} \tag{4}$$

In the unconditional (OLS) composite hedge, the sample variances and covariances are employed in equations (3) and (4), assuming that the return distribution and correlations are constant over time. This assumption is relaxed in the conditional MGARCH models discussed in the next section.

2. Conditional Minimum Variance Hedge Ratio Estimation

In the GARCH framework, the daily returns $r_i(t)$ of the spot and future contracts (i) are modeled as:

$$r_i(t) = m_i + u_i(t) \tag{5}$$

where $u_i(t)$ is the return residual capturing the deviation from the long run mean m_i on day t. The residual's mean is zero with variance $h_i(t)$. We follow this very simplified presentation of the assets conditional mean without a moving average or autoregressive component as the model selection analysis discussed in chapter 4 does not provide support for a moving average or autocorrelation effect in the return series. This model specification is in line with most of the previous studies (Myers, 1991; Cotter und Hanly, 2012) on MGARCH hedging approaches and supports the notion of short term unpredictability of spot and future returns. The volatility clustering of the spot and future returns is modeled in the residual term $u_i(t)$:

$$u_i(t) = \sqrt{h_i(t)} * \varepsilon_i(t) \tag{6}$$

The standardized residual ε_i is normally distributed with zero mean and a constant variance of one. The return variance $h_i(t)$ is modeled based on a GARCH framework depending on past estimations of the return variance $h_{i,t-1}$ and lagged squared residuals $u^2_{i,t-1}$ with the coefficients K, G and A and lag parameters P and Q[5].

$$\operatorname{var}(r_i(t) \mid \Theta) = h_i(t) = K + \sum_{n=1}^{P} G_n h_{t-n} + \sum_{j=1}^{Q} A_j u^2_{t-j} \tag{7}$$

In the MGARCH setting, $h_i(t)$ are the elements of the conditional covariance matrix H_t. In line with the hedging studies of Kroner and Sultan (1993), Park and Schwitzer (1995), Bera, Garcia and Roh (1997), Lien, Tse and Tsui (2002), Byström (2003), Carbonez, Nguyen and Sercu (2011), we adopt the constant conditional correlation (CCC) MGARCH specification proposed by Bollerslev (1990) to model the covariance matrix H_t. Based on information criteria for model selection, we employ a parsimonious CCC-MGARCH(1,1) model where the diagonal elements of the conditional covariance matrix H_t (conditional variance of the spot and future series) are dependent on their past estimations $h_{i,t-1}$ and the variance shock in the previous period $u^2_{i,t-1}$.

$$h_s(t) = K + G h_{S,t-1} + A u^2_{S,t-1} \tag{8a}$$

[5] The data analysis provided in chapter 3 supports our approach to employ a very parsimonious GARCH(1,1) model with P and Q equal to one which is line with Myers (1991); Baillie and Meyers (1991), Bera, Garia and Rho (1997), Miffre, (2004), Cotter and Hanly (2012).

$$h_{Fn}(t) = K + Gh_{Fn,t-1} + Au^2_{Fn,t-1} \qquad \text{with} \quad n \in \{1,2\} \quad (8b)$$

The conditional covariance between the spot and future returns is defined indirectly by the following variance correlation relationship:

$$h_{s,Fn}(t) = \sqrt{h_{s,}(t)} * \sqrt{h_{Fn}(t)} * \rho\ (\varepsilon_S(t), \varepsilon_{Fn}(t)) \qquad \text{with} \quad n \in \{1,2\} \quad (9)$$

The conditional variance covariance matrix H_t is thus defined as:

$$H_t = D_t R D_t \quad (10)$$

where D_t presents the diagonal matrix of conditional standard deviations and R is the correlation matrix of the standardized residuals $\varepsilon_i(t)$ which under the CCC-MGARCH specification is assumed to be constant over the estimation period. Given the assumption of constant correlation, the matrix R is equal to the sample correlation matrix (Bollerslev, 1990).

However, particularly during crisis periods, correlations might fluctuate over time and the assumption of constant correlations might be incorrect, resulting in severe hedging errors. Therefore, in addition to the CCC-MGARCH model, we relax the assumption of constant correlation and implement the dynamic conditional correlation (DCC) MGARCH model of Engle (2002). This is the first study that adopts this model for estimating minimum variance hedge ratios. In this setting, the correlations of standardized residuals are time varying and are modeled within a separate GARCH framework for the covariance of standardized residuals $q_{tS,Fn}$. In this framework the covariance is modeled as a function of past covariance estimations (q_{t-v}), contemporaneous covariance shocks $(\varepsilon_{s,t-w}\varepsilon_{Fn,t-w})$ and the sample covariance $\overline{q_{s,Fn}}$:

$$q_{t,S,Fn} = \text{cov}_t(\varepsilon_s(t), \varepsilon_{Fn}(t)) = \left(1 - \sum_{w=1}^{W} \alpha_w - \sum_{v=1}^{V} \beta_w\right)\overline{q_{s,Fn}} + \sum_{w=1}^{W} \alpha_m (\varepsilon_{s,t-w}\varepsilon_{Fn,t-w}) + \sum_{V=1}^{V} \beta_v q_{t-v} \quad (11)$$

The conditional covariance matrix H_t in the DCC MGARCH model is defined as:

$$H_t = D_t R_t D_t \tag{12}$$

with $\rho\ (\varepsilon_S(t), \varepsilon_{Fn}(t)) = q_{tS,Fn} * \sqrt{h_{S,t}} * \sqrt{h_{Fn,t}}$ being the elements of the time varying correlation matrix R_t.

We compute each unconditional (OLS) and conditional (CCC and DCC) hedging strategy out-of-sample for the period from 2000 to 2013. Out-of-sample means that we use daily return data available until day (t) to compute the hedge ratio employed on the next day (t+1). We employ rolling estimation windows of 250 days with equally weighted observations in the base case and implement different estimation windows as robustness check. Figure 3 provides an overview of the hedging approaches employed in this study including the required number of estimation parameters and the estimation procedure.

V. Empirical Results: OLS versus GARCH Hedging Strategies

1. Descriptive Statistics

Descriptive statistics of the daily return series for the JPM bond index (price index), the German Bund futures and the Italian BTP futures are provided in table 1. Panel A presents the descriptive statistics for the pre-crisis period from 2000 to 2006, and panel B includes the statistics for the financial and sovereign debt crisis period ranging from 2007 to 2013. The average annualized daily return of the JPM government bond price index is negative during the first period but positive for the second period. This observation can partly be explained by the changing interest rate environment. While interest rates in Europe slightly increased from 2000 to 2006, interest rates declined to very low levels between 2007 and 2013. All return series exhibit substantial excess kurtosis and the null hypothesis of normally distributed returns is rejected at the 1%-level. To account for non-normal returns and tail-risk, we compare the value-at-risk and the lower partial moments for the hedged and the unhedged portfolio as described in section 2. The augmented Dickey Fuller tests with lags from 1 to 30 do not indicate any sign of non-stationarity.

In Figure 4 we present the rolling correlation coefficients of the JPM bond index returns with the German Bund and Italian BTP10 futures returns. The figure illustrates that the correlations between the bond portfolio returns and the futures returns fluctuate substantially over time. Therefore, the constant conditional correlation (CCC) and dynamic conditional correlation (DCC) models might be better suited for hedging than the OLS approach, which assumes constant correlations within the sample period. However, by employing a rolling sample estimation method, the OLS model might partially capture the changing correlations. Moreover, figure 4 illustrates that the correlation between the Bund future and the JPM index declines sharply subsequent to March 2010, indicating that the Bund futures contract lost its advantages as an efficient hedging instrument for hedging EMU bond portfolios after 2010. Thus, an additional hedging instrument might be required such as the BTP futures to improve the hedging result during this crisis period.

[Table 1 about here]

2. GARCH Model Selection

To select the appropriate GARCH model, we rely on information criteria for model selection. We estimate different GARCH models with varying lag parameters for the conditional variance (P,Q) and the conditional mean (R,M). Based on the Schwarz-Bayes information criterion (BIC) for model selection, the ARMA (0,0), GARCH(1,1) model provides the best specification for capturing the conditional mean and variance returns for the JPM, Bund- and BTP10 futures. An additional likelihood ratio test validates this result. The model residuals do not show any sign of significant autocorrelation (Ljung-Box-Pierce-Q test) or heteroscedasticity (Engel-ARCH-test), confirming the accurateness of the selected model.

3. Analysis of optimal Hedge Ratios

In Figure 5 we present the time-varying optimal hedge ratios for the period from 2000 to 2013 for the single hedges with the Bund and BTP10 futures contracts, for the OLS (bold line), the CCC- (left-hand-side) and the DCC-GARCH (right-hand-side) models. Figure 6 shows the optimal hedge ratio for the composite hedges, simultaneously employing Bund and BTP10-futures contracts. The figure illustrates that the GARCH optimal hedge ratios fluctuate substantially over time, resulting in enormous futures turnovers and transaction costs. In contrast, the OLS hedge ratios exhibit only moderate variations over time. Moreover, it is evident that the hedge ratios based on MGARCH models particularly fluctuate during the period from 2010 to 2012 when the EMU sovereign debt crisis was at its summit. The hedging effectiveness measures in the next section provide further insights on the relative performance of the different hedging strategies.

4. Hedging Effectiveness Measures

Next, we compute several hedging effectiveness measures to evaluate the quality of the employed hedging strategies. First we compute the variance reduction of the hedged relative to the unhedged portfolio according to equation (13):

$$\text{VarianceReduction} = 1 - \frac{\text{var}(r_H)}{\text{var}(r_S)}, \tag{13}$$

where (r_H) is the return of the hedged portfolio and (r_S) is the return of the unhedged portfolio, i.e. the return of the bond portfolio (Ederington, 1979).

In line with Cotter and Hanly (2012), we additionally compute alternative measures of risk reduction including the reduction of a portfolio's value-at-risk (according to equation 14) and the reduction of a portfolio's lower partial moments (LPM) (equation 15), thereby accounting for non-normally distributed returns and the associated tail risk which particularly might exist during the sovereign debt crisis period:

$$\text{VaR-90\%-Reduction} = 1 - \frac{\text{VaR-90\%}_H}{\text{VaR-90\%}_S}, \tag{14}$$

$$\text{LPM} - \text{Reduction} = 1 - \frac{\text{LPM}_H}{\text{LPM}_S}. \tag{15}$$

VaR-90%$_H$ (LPM$_H$) is the 90%-value-at-risk (lower partial moment) of the empirical distribution of the realized hedged position's returns and (VaR-90%$_S$) (LPM$_S$) is the 90%-value-at-risk (lower partial moment) of the empirical distribution of the realized unhedged bond portfolio returns. VaR-90% is the 10%-quantile of the empirical return distribution. The lower partial moment (LPM) is computed as average loss where loss is defined as return smaller than zero. To estimate the transaction costs involved with each hedging strategy, we compute the futures trading volume of each hedging strategy. In line with Chen and Sutcliffe (2012), we assume a linear relationship between transaction costs and futures positions traded and proxy transaction costs by the value of futures positions traded each year. The required yearly futures trading (FT_i) to implement hedging strategy (i) is the average absolute change in daily hedge ratios (h) over the T rebalancing points in time and across the N implemented hedging instruments multiplied with the number of trading days per year (250):

$$FT_i = \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{N} \left(\left| h_{i,j,t+1} - h_{i,j,t} \right| \right) \cdot 250, \tag{16}$$

where $h_{i,j,t}$ denotes the hedge ratio of futures j at time t under hedging strategy i. If two hedging strategies achieve the same hedging effectiveness, the one with the lower turnover is preferably, because its implementation requires lower transaction costs.

Table 2 presents the hedging effectiveness measures for the OLS, CCC- and DCC-GARCH hedging strategies. Panel A includes the results for the first sub-period (2000-2006).

During this period the Bund futures was the only available futures contracts on EMU government bonds. The hedging effectiveness measures indicate that the OLS hedges with the Bund futures contract worked well. The portfolio variance was reduced by over 67% with the OLS approach. CCC- and DCC-GARCH models only marginally enhanced the hedging effectiveness, while the futures turnover increased dramatically. Thus, they perform inferior during the non-crises period.

Panel B of table 2 presents the hedging effectiveness measures for the second subperiod from 2007 to 2013 for single hedges, either using the Bund or the BTP10 futures contracts. The results reveal that the BTP10 futures achieve a higher variance and tail risk reduction compared to the Bund futures for all analyzed hedging strategies. The CCC-GARCH model works slightly better than the OLS hedging approach. Interestingly, the DCC-GARCH model achieves a substantially higher hedging effectiveness compared to OLS with the Bund futures, indicating that the DCC-GARCH model is better suited to capture the changing correlation structure during the sovereign debt crisis.

Panel C of table 2 includes the hedging effectiveness measures for the composite hedges with the Bund and the BTP10 futures. The results reveal that the composite hedges almost achieve the same hedging results during the crisis period (2007-2013) as the single hedge with the Bund during the pre-crisis (2000-2006) period (panel A). Surprisingly, for the composite hedges the CCC- and DCC-GARCH models perform slightly worse than the simple OLS approach. The likely explanation is that in the MGARCH models the estimation parameters and hence estimation errors rapidly increase with the number of hedging instruments. Consequently, in composite hedges the theoretical advantages of the MGARCH model compared to OLS are outweighed by the increase in estimation error.

To analyze the relative performance of the hedging approaches over time, we compute the rolling variance reduction which is presented in Figure 7. Evidently, there are some periods in which GARCH models substantially outperform OLS hedges. However, most of the time there are only marginal differences in the hedging performance of MGARCH compared to OLS hedging strategies.

Summarizing our results so far, is seems fair to conclude that, on the one hand, hedging strategies based on MGARCH models improve the hedging results of OLS single-hedges. However, on the other hand, MGARCH models have serious problems arising from estimation errors, resulting in inferior hedging results in composite hedges and a drastic increase in futures turnover. Moreover, the benefits of MGARCH hedging strategies are only observable in some short sub-periods, while mostly there are only marginal differences in the hedging effectiveness between the different hedging strategies. Implementing a Bayesian approach that controls for estimation errors in the MGARCH models might be able to improve the hedging performance. This approach is analyzed in the next section.

VI. Bayesian Approach for Model Selection

1. Methodology of Bayesian Hedges

The idea of the Bayesian approach to model selection is to improve the out-of-sample hedging performance by combining the strength of the OLS and MGARCH models endogenously. On the one hand, MGARCH models require a large number of estimation parameters which involve estimation errors and often lead to inefficient hedge ratios. Moreover, GARCH models are very sensitive to changes in market information processing and to market shocks. Examples are political announcements during the Euro crisis which resulted in a high futures turnover and transaction costs. On the other hand, the OLS method produces very stable hedge ratios which, however, might be too inaccurate particularly during crisis periods due to the assumption of constant correlations and homoscedastic returns. Overall, the OLS approach is not very adaptive to new distribution properties and the hedge ratio critically depends on the estimation window lengths.

Poomimars, Cadle, and Theobald (2003) propose a Bayesian hedging strategy to estimate the optimal hedge ratios. Bayesian estimations build on a prior and sample information in order to control for model uncertainty and parameter estimation risk. In the literature, Bayesian estimation approaches are widely established in the context of portfolio optimization for estimating returns or the covariance matrix of asset return (Alexander and Resnick, 1985; Jorion, 1985, 1986; Ledoit and Wolf, 2003a, 2003b). In the context of hedging, Bayesian estimation techniques were proposed for estimating optimal mean-variance hedge ratios (Lence and Hayes, 1994a, 1994b; Shi and Irwin, 2005).

The Bayesian minimum variance hedging approach by Poomimars, Cadle, and Theobald (2003) combines the static estimation method (OLS) and the dynamic estimation method (GARCH). In line with Vasicek (1973) for estimating CAPM betas, they compute the posterior hedge ratio as:

$$\mathrm{OHR(posterior)} = \frac{\hat{\mathrm{OHR}}_{\mathrm{GARCH}} * \left[\sigma^2_{\mathrm{OHR-GARCH}}\right]^{-1} + \hat{\mathrm{OHR}}_{\mathrm{OLS}} * \left[\sigma^2_{\mathrm{OHR-OLS}}\right]^{-1}}{\left[\sigma^2_{\mathrm{OHR-GARCH}}\right]^{-1} + \left[\sigma^2_{\mathrm{OHR-OLS}}\right]^{-1}} \tag{17}$$

This Bayesian hedging strategy employs the OLS hedge ratio as prior (shrinkage target) and the model precision of the GARCH relative to the OLS model as the shrinkage factor. While Poomimars, Cadle, and Theobald (2003) employ the Bayesian hedging approach for single instrument hedges for commodities, we employ composite hedges on bond portfolios during the sovereign debt crisis. The empirical results are provided in the next section.

2. Empirical Results of Bayesian Hedges

In Figure 8 we present the optimal hedge ratios of the Bayesian-CCC-GARCH and the Bayesian-DCC-GARCH composite hedge compared to the OLS hedge ratios. The figure indicates that the Bayesian hedge ratios are much less volatile compared to the pure MGARCH hedge ratios presented in figure 6. Hence, futures turnover and the costs of hedging are substantially reduced. The optimal hedge ratios in the Bayesian CCC and DCC approach seem to be very similar. The hedging effectiveness measures provided in table 3 provide further insights on the performance of the strategies. Table 3 shows that the Bayesian (CCC and DCC) hedging approaches dominate the respective CCC- and DCC-MGARCH hedges. The Bayesian strategies (CCC and DCC) achieve a larger level of risk reduction compared to pure MGARCH hedges presented in table 2. Simultaneously, the Bayesian (CCC and DCC) hedging approaches substantially reduce futures turnover by roughly 50% compared to the pure MGARCH models.

Compared to OLS, the Bayesian hedging approaches enhance the hedging effectiveness particularly during the crisis period. However, during the relatively stable pre-crisis period (2000-2006), the improvement of the hedging effectiveness is only marginal (below one percentage point). Consequently, it seems sufficient to employ the OLS model in the pre-crisis period, because the assumption of constant variances and correlations is not critically violated. In contrast, the benefits of the Bayesian hedge compared to the OLS hedge become more pronounced during the crisis period (2007-2013). This is in line with the observation that during the crisis period compared to the pre-crisis period changing correlations, volatility clustering, and market shocks due to political announcements are much more important. .

Figure 9 summarizes the results of the rolling variance reduction of the OLS, the DCC and the Bayesian-DCC composite hedge during the crisis period. The figure illustrates that the Bayesian hedging approach virtually always achieves a larger variance reduction compared to OLS and the DCC-MGARCH model. Therefore, it can be concluded that the Bayesian hedging approach successfully reduces estimation error in the MGARCH models and improves the out-of-sample hedging performance. However, compared to OLS, implementing the Bayesian approaches results in a much higher futures turnover and hence are more expensive to implement. Therefore, the Bayesian hedging approach does not dominate OLS. Consequently, there exists a trade-off between the additional benefits in risk reduction and the cost of the higher futures turnover. Hence, the decision whether to employ an OLS or a Bayesian hedging strategy depends on the investor's risk aversion and the variable transaction costs for trading futures. Thus, implementing statistically more demanding and sophisticated hedging approaches requires well functioning and efficiently organized futures markets, which offer market participants low transaction fees, minimal margin requirements, and basically no counterparty risk. Hence, introducing transaction taxes on futures positions as currently discussed in the European Union will not only result in lower market turnover and lower market liquidity, but possibly also in the choice of less sophisticated hedging approaches that involve lower futures turnover but may not optimally minimize risk for market participants. Overall this may result in an inferior risk situation.

3. Performance of Bayesian MGARCH vs. OLS hedges

To gain further insights under which conditions OLS or Bayesian hedging strategies are relatively more attractive, we compute the performance (Sharpe ratios) of the hedging strategies for different levels of transaction costs.[6] We assume a linear relationship between transaction costs and futures positions traded similar to Chen and Sutcliffe (2012). In Table 4 we present the Sharpe ratio measures for the OLS and the Bayesian hedging strategies. The Sharpe ratio measures are computed net of transaction costs for variable transaction costs for futures trading between 0 and 50 basis points. Panel A presents the results for the 2000 to 2006 non-crisis period for hedges with the Bund-futures. In Panel B we provide the results for the 2007 to 2013 crisis period for composite hedges with the Bund and BTP10 futures. The

[6] To compute Sharpe ratios we use a risk-free rate of zero. Usually the risk-free rate is approximated by government bond yields which does not seem plausible for evaluating EMU government bond portfolios.

results in Panel A indicate that the OLS approach was sufficient for hedging EMU bond portfolios during the non-crisis period. The Bayesian CCC MGARCH approach only marginally improved the Sharpe ratio of the OLS hedged portfolio for very low variable transaction costs (below 10 basis points). For higher transaction costs the optimal hedging strategy (based on the Sharpe ratio as selection criteria) was the OLS approach.

However, the ranking of optimal hedging strategies is different for the 2007 to 2013 crisis period. Panel B of table 4 reveals that during the crisis period the Bayesian-DCC strategy achieved the highest Sharpe ratio for variable transaction costs up to 30 basis points. For higher transaction costs the OLS approach was more lucrative due to its lower futures trading volume. The explanation for this finding is that the Bayesian MGARCH models do improve the performance of OLS hedging strategies. However, this improvement is offset by transaction costs if the variable transaction costs for futures trading exceed a certain level. During the non-crises period the critical level of transaction costs was 10 basis points, while it was 35 basis points in the crisis period.[7] This result confirms our finding that the benefits of more complex hedging strategies are much higher during crisis periods than during 'normal' non-crisis periods.

VII. Conclusion

In this study we analyze hedging strategies for EMU bond portfolios for non-crises and crisis periods. We analyze the improvement in hedging performance when the hedging framework is extended from one instrument hedges (single hedges) to two instrument-hedges (composite hedges), employing traditional (Bund-futures) and newly (re-)introduced futures contracts on Italian government bonds (BTP-futures). Moreover we evaluate the improvement in hedging effectiveness when moving from simple OLS to more complex CCC- and DCC-MGARCH hedging strategies (Bollerslev, 1990; Engle, 2002). To overcome the potential

[7] Critical value of transaction costs refers to the level of transaction costs at which two hedging strategies achieve the same Sharpe ratio.

problems of estimation errors and oversensitivity to market shocks, we additionally employ Bayesian hedging strategies based on Poomimars, Cadle and Theobald (2003) and extend the approach on composite hedges for hedging EMU bond portfolios.

Our empirical results suggest that while hedging with the Bund futures contract worked well during the pre-crisis period from 2000 to 2006, it performed poorly during the crisis period with hedging effectiveness measures dropping from almost 70% to below 40%. However, simultaneously employing Bund and BTP10 futures contracts in composite hedges almost achieved the pre-crisis hedging results. Comparing different hedging methods, we find that CCC- and DCC-GARCH hedges only marginally improve the hedging effectiveness of OLS for single hedges. For composite hedges, GARCH strategies are even inferior to OLS due to estimation errors. A Bayesian hedging approach, designed to control for estimation errors, produces superior hedging results and involves substantial lower futures turnover (transaction costs) than MGARCH models. Compared to OLS, Bayesian hedges achieve a larger level of risk reduction but involve higher futures turnover (transaction costs). Therefore, the decision whether to employ OLS or Bayesian-hedges depends on the individual risk aversion and the variable transaction costs for trading futures. Moreover, the Bayesian composite hedging strategy turns out to be particularly beneficial during the sovereign debt crisis period when risk reduction opportunities were mostly needed. Overall our results suggest that EMU bond portfolio managers should employ composite hedges with the Bund and BTP-Futures and rely on OLS or Bayesian hedging techniques.

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Figure 1: Yield Spreads to German Government Bond Yields – Maturity 10 y

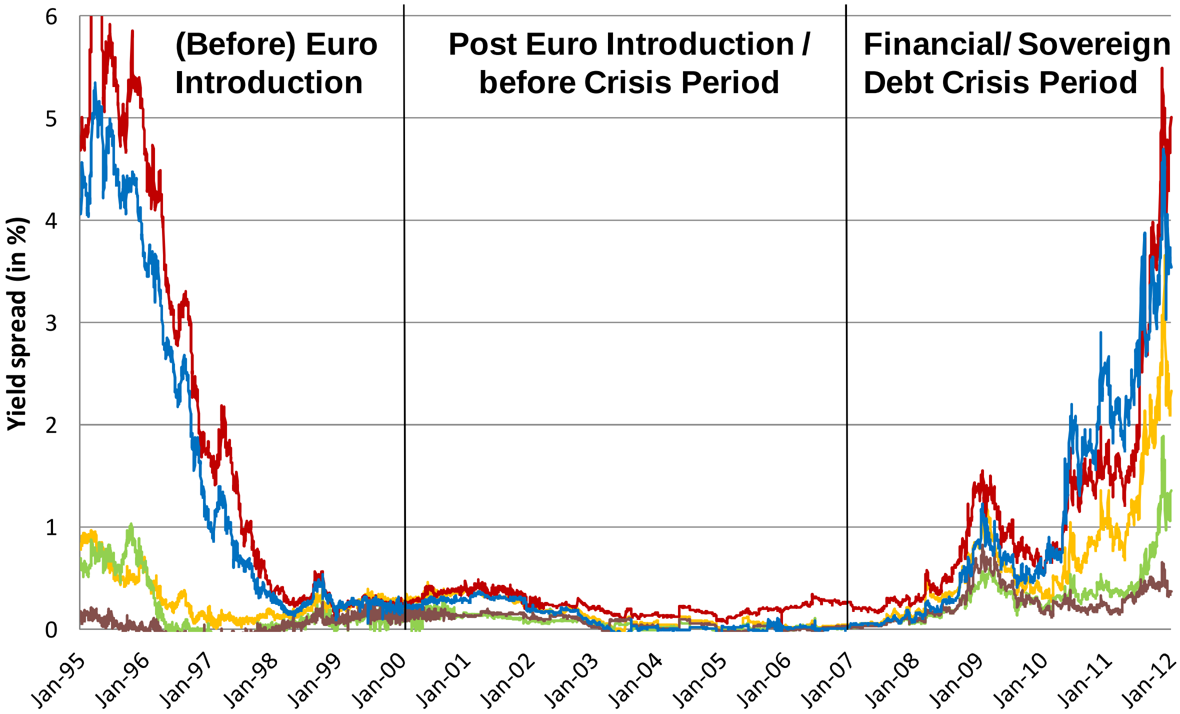

Figure 2: Political events during the sovereign debt crisis

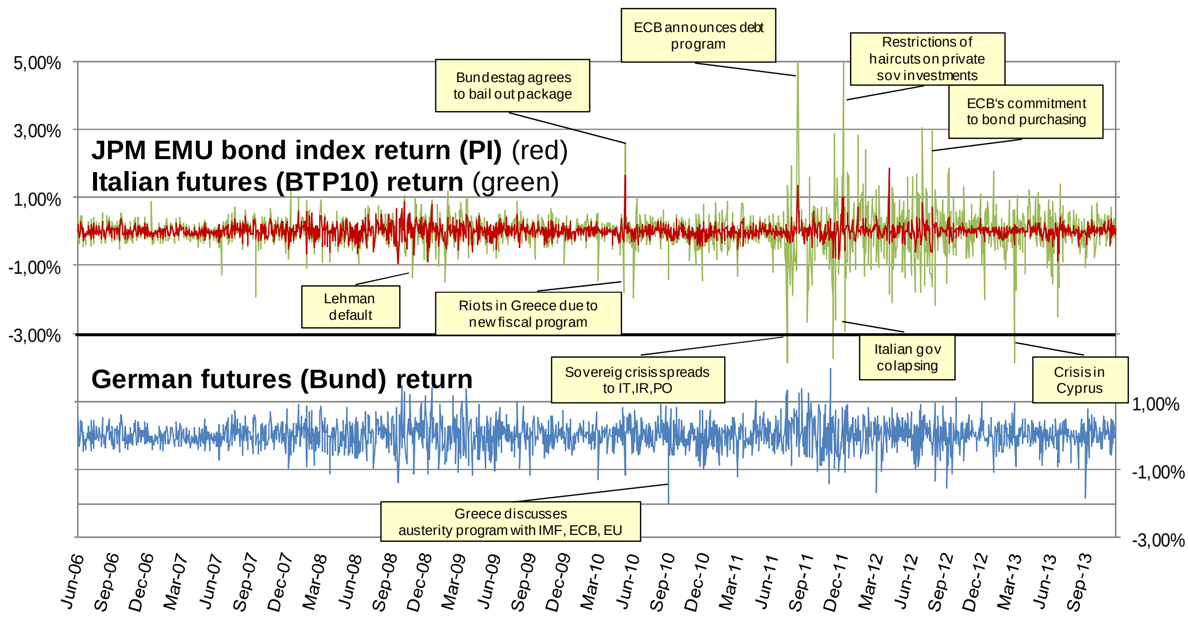

Figure 3: Overview of Hedging Approaches

	Bayesian models		
	OLS	GARCH	
		CCC	DCC
Variance Estimate	(Rolling) Sample Variance (unconditional)	Univariate GARCH estimate (conditional)	Univariate GARCH estimate (conditional)
Covariance Estimate	(Rolling) Sample Covariance (unconditional)	Assumption of Constant Correlation estimated as conditional Sample Correlation.	Modeling correlations using GARCH framework.
Number of estimation parameters in composite hedge (2-futures contracts)	2	12	14

Figure 4: Rolling correlation coefficients of EMU bond portfolio returns and futures returns.

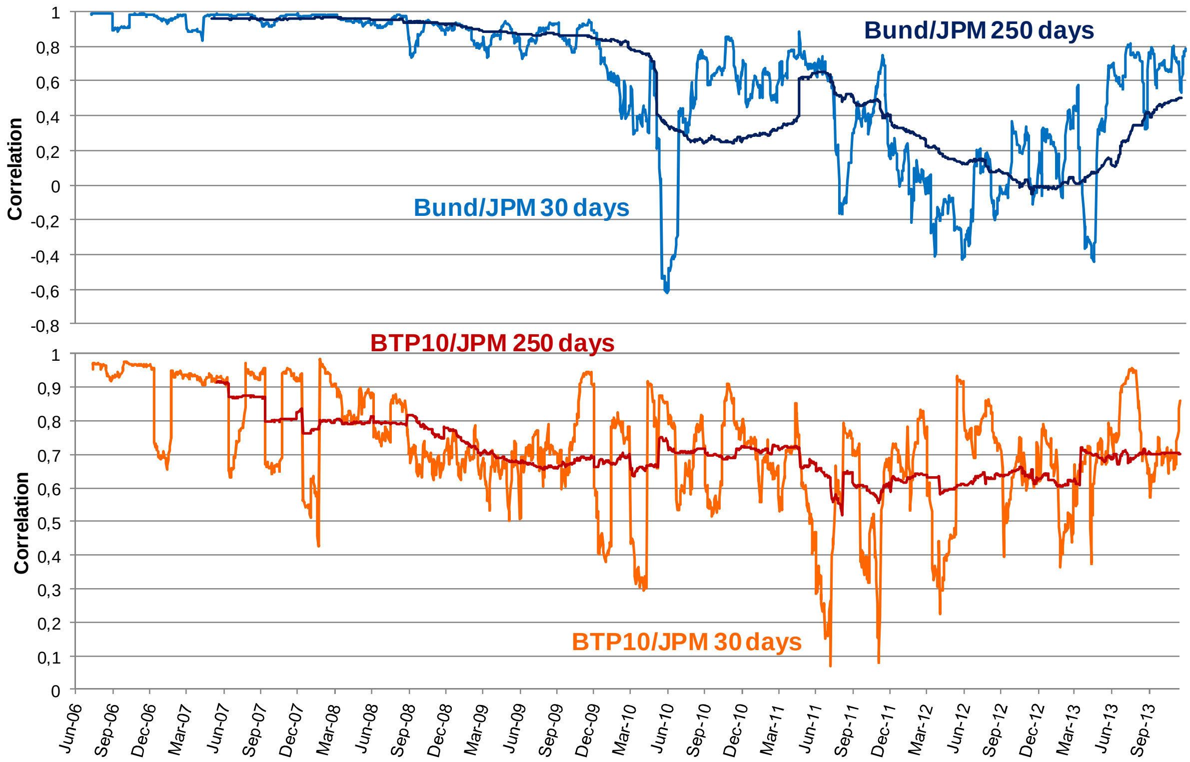

Figure 5: Analysis of Optimal Hedge Ratios: OLS vs. CCC/DCC-GARCH (Single Hedges).

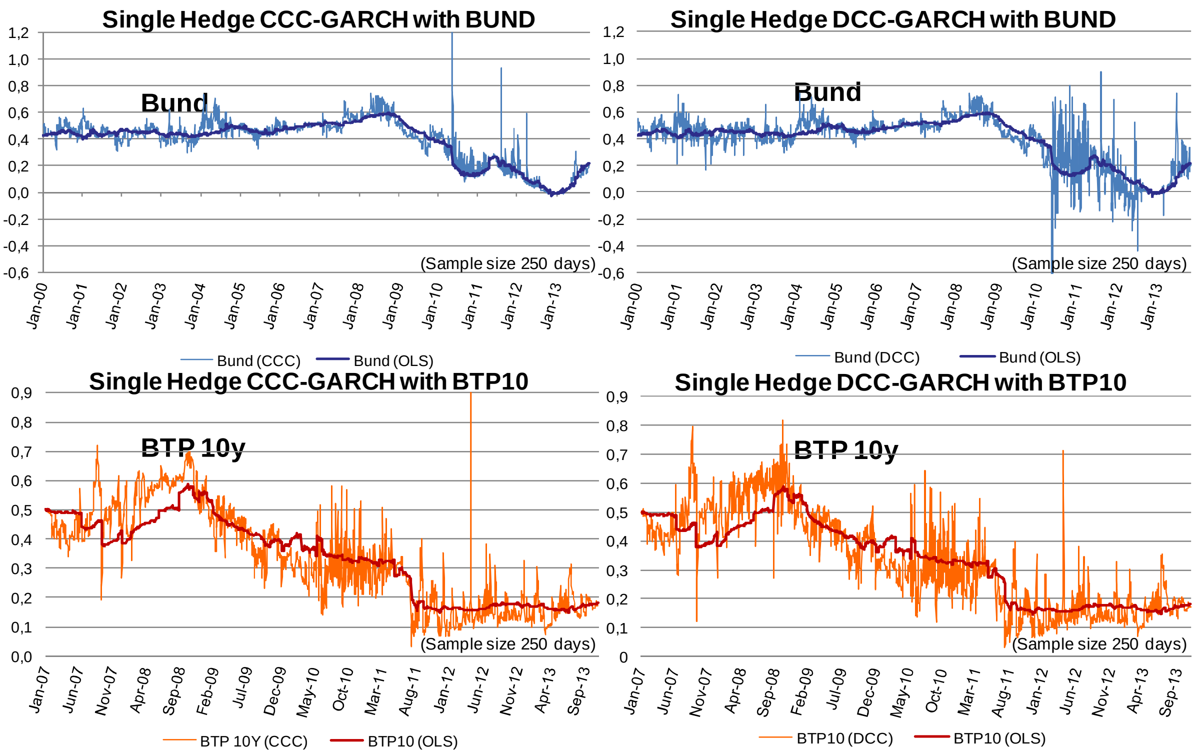

Figure 6: Analysis of Optimal Hedge Ratios: OLS vs. CCC/DCC-GARCH (Composite Hedges).

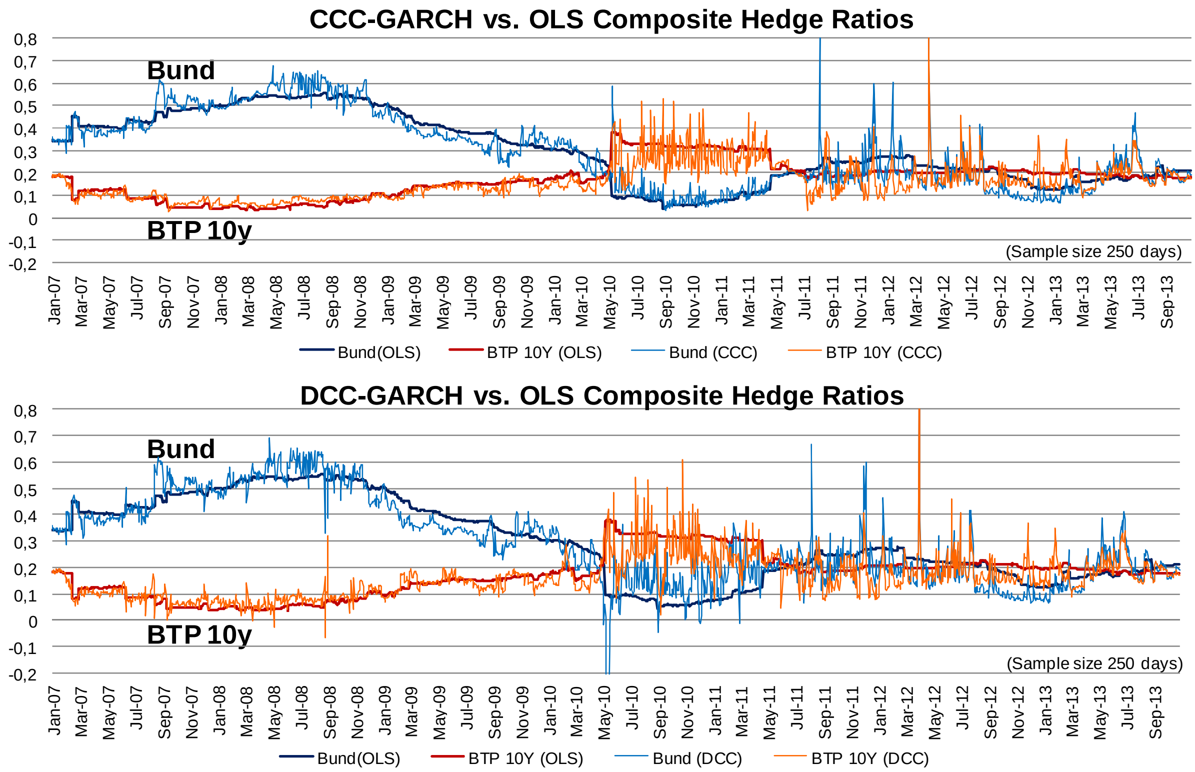

Figure 7: Rolling Hedging Effectiveness: OLS vs. CCC-GARCH and DCC-GARCH

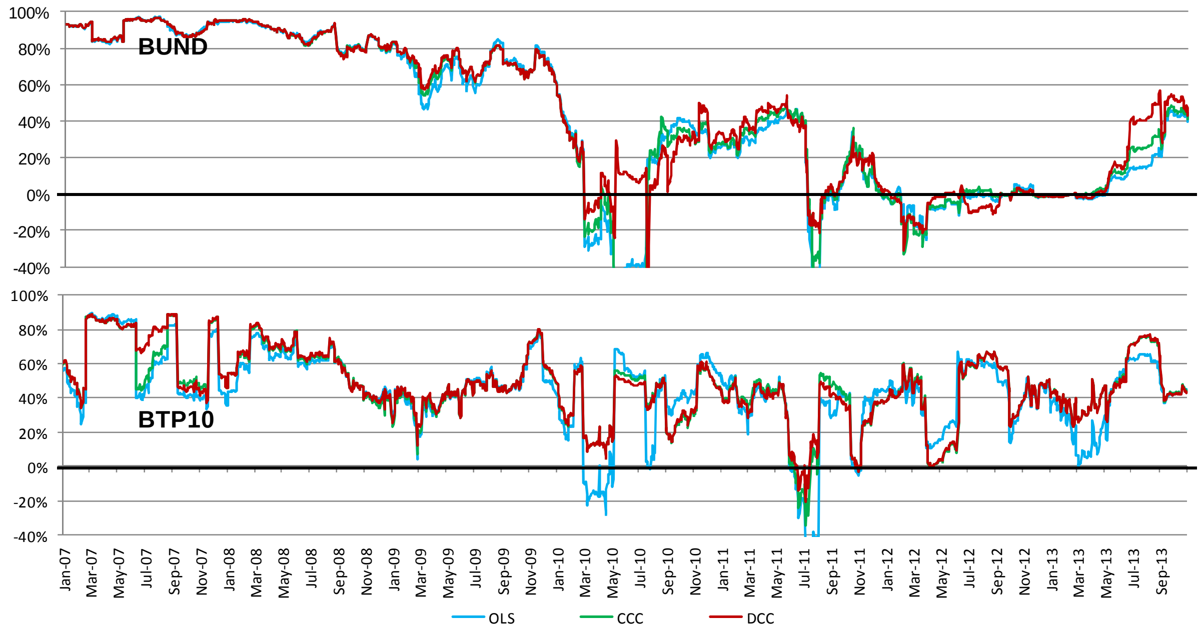

Figure 8: Analysis of Optimal Hedge Ratios: OLS vs. CCC/DCC-GARCH (Composite Hedges).

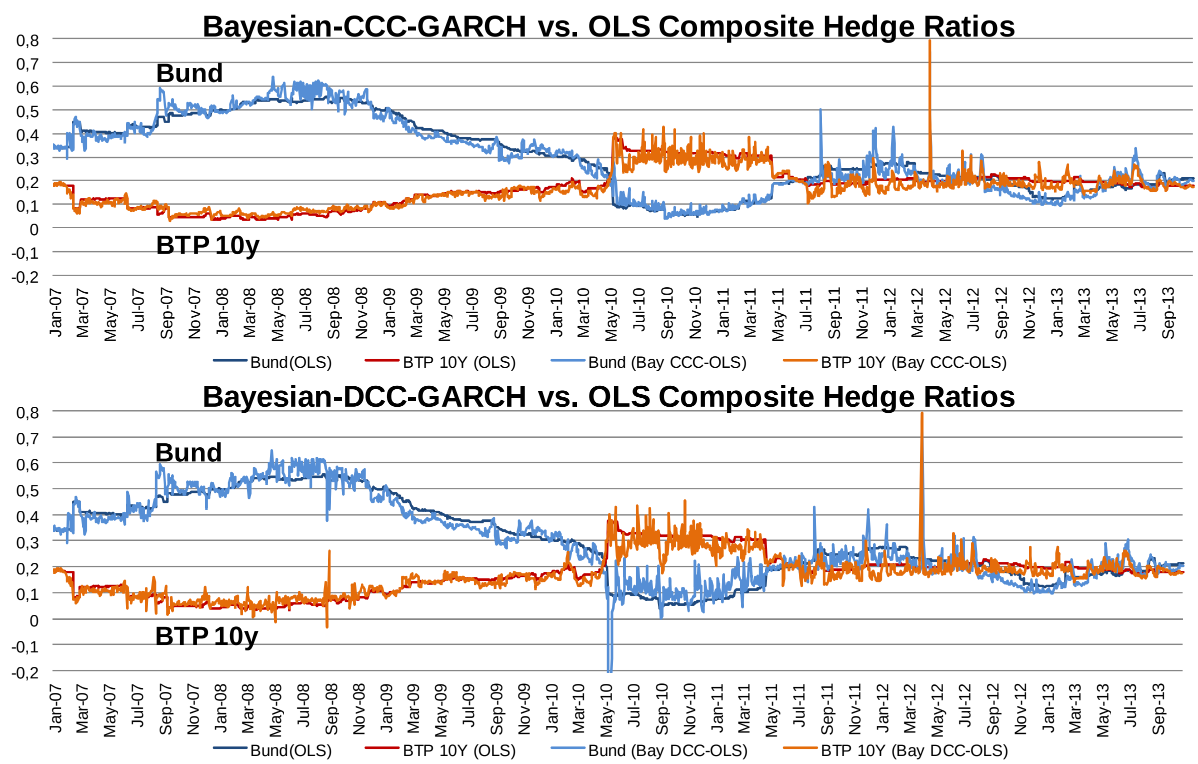

Figure 9: Rolling Hedging Effectiveness Composite OLS, CCC/DCC, and Bayesian

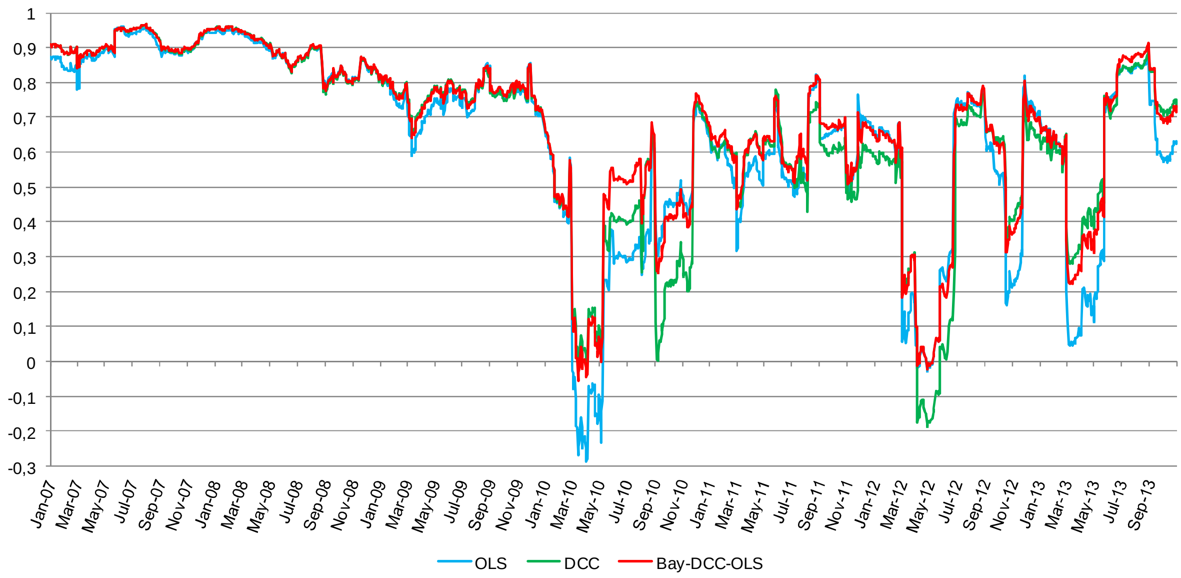

Table 1: Descriptive statistics of monthly return time series

Panel A: 2000-2006	JP Morgan PI	Bund	BTP 10Y
Sample Size	2087	2087	/
Mean (ann.)	-1,38%	0,03%	/
Median	0,00%	0,01%	/
Volatility (ann.)	2,76%	5,06%	/
Kurtosis	1,52	1,49	/
Skewness	-0,49	-0,48	/
Max	0,67%	1,23%	/
Min	-0,86%	-1,55%	/
Normal (JB-Test 99%)	no	no	/
Panel B: 2007-2013	**JP Morgan PI**	**Bund**	**BTP 10Y**
Sample Size	1784	1784	1784
Mean (ann.)	0,87%	2,83%	-0,01%
Median	0,00%	0,01%	0,00%
Volatility (ann.)	3,29%	6,40%	9,19%
Kurtosis	8,23	1,61	14,50
Skewness	0,75	-0,15	0,41
Max	1,85%	1,96%	5,68%
Min	-0,95%	-2,01%	-3,87%
Normal (JB-Test 99%)	no	no	no

Table 2: Hedge Effectiveness: OLS, CCC, DCC, single and composite hedges

Evaluation of Hedge Strategy	Hedging Strategy					
	OLS		CCC-GARCH		DCC-GARCH	
Panel A: 2000-2006: Single hedge	Bund		Bund		Bund	
Variance Reduction	67,68%		68,19%		68,15%	
VaR-99% Reduction	49,82%		50,25%		50,14%	
Lower Partial Moment Reduction	44,61%		44,77%		44,57%	
Turnover p.a.	0,38		4,02		6,46	
Panel B: 2007-2013: Single hedge	Bund	BTP10	Bund	BTP10	Bund	BTP10
Variance Reduction	37,66%	45,67%	38,37%	46,61%	42,39%	46,74%
VaR-99% Reduction	23,41%	34,36%	23,77%	33,51%	29,09%	31,29%
Lower Partial Moment Reduction	28,96%	30,83%	29,32%	32,45%	31,26%	32,03%
Turnover p.a.	0,46	0,45	5,74	7,75	12,19	9,59
Panel C: 2007-2013: Composite hedge	Bund & BTP10		Bund & BTP10		Bund & BTP10	
Variance Reduction	65,76%		64,12%		65,02%	
VaR-99% Reduction	50,70%		48,72%		51,36%	
Lower Partial Moment Reduction	49,08%		47,81%		48,71%	
Turnover p.a.	0,89		11,90		15,09	

Table 3: Analysis of Hedge Effectiveness for Bayesian Hedging Approaches

Evaluation of Hedge Strategy	Bayesian Composite hedge (Bund & BTP10)			
	OLS-CCC		OLS-DCC	
Panel A: 2000-2007: Single Hedge (Bund)	absolute	Δ OLS	absolute	Δ OLS
Variance Reduction	68,40%	0,72%	68,42%	0,74%
VaR-99% Reduction	49,69%	-0,12%	49,91%	0,10%
Lower Partial Moment Reduction	44,97%	0,36%	44,88%	0,27%
Turnover p.a.	2,07	1,69	3,28	2,90
Panel B: 2007-2013: Single hedge (BTP10)				
Variance Reduction	48,47%	2,80%	48,81%	3,13%
VaR-99% Reduction	37,30%	2,95%	37,16%	2,81%
Lower Partial Moment Reduction	32,48%	1,65%	32,43%	1,60%
Turnover p.a.	3,89	3,44	4,81	4,36
Panel B: 2007-2013: Composite hedge (Bund & BTP10)				
Variance Reduction	68,24%	2,48%	68,58%	2,81%
VaR-99% Reduction	52,84%	2,14%	53,18%	2,48%
Lower Partial Moment Reduction	49,91%	0,83%	50,37%	1,29%
Turnover p.a.	6,55	5,66	8,48	7,59

Table 4: Performance of Hedging Approaches net of Transaction Costs

	Hedging Strategy			
costs	unhedged	OLS	Bayesian OLS-CCC	Bayesian OLS-DCC
Panel A: 2000-2007: Single Hedge (Bund)				
0bp	1,91	2,92	***2,93***	2,91
5bp	1,91	2,92	***2,93***	2,90
10bp	1,91	***2,93***	***2,93***	2,90
15bp	1,91	***2,92***	2,92	2,89
20bp	1,91	***2,92***	2,91	2,88
25bp	1,91	***2,92***	2,90	2,86
30bp	1,91	***2,92***	2,90	2,85
35bp	1,91	***2,92***	2,89	2,84
40bp	1,91	***2,92***	2,88	2,83
45bp	1,91	***2,92***	2,88	2,82
50bp	1,91	***2,78***	2,14	1,65
costs	unhedged	OLS	Bayesian OLS-CCC	Bayesian OLS-DCC
Panel B: 2007-2013: Composite Hedge (Bund+BTP10)				
0bp	1,34	1,85	1,91	***1,98***
5bp	1,34	1,85	1,89	***1,96***
10bp	1,34	1,84	1,88	***1,94***
15bp	1,34	1,84	1,86	***1,92***
20bp	1,34	1,84	1,84	***1,89***
25bp	1,34	1,84	1,82	***1,87***
30bp	1,34	1,84	1,81	***1,85***
35bp	1,34	***1,83***	1,79	1,82
40bp	1,34	***1,83***	1,77	1,80
45bp	1,34	***1,83***	1,75	1,78
50bp	1,34	***1,83***	1,74	1,75