An Application of a Conditional Illiquidity Measure for U.S. Corporate Bond Yields*

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Abstract

The paper proposes an application of a conditional measure of illiquidity for U.S. dollars denominated corporate bond yields. The framework is based on the dynamic of the j-th order serial conditional covariance and accommodates for the changes of the unobserved yield related to a corporate bond, caused by the arrival of new information and the transaction costs incurred in making the exchange of it. The analysis derives: (i) several metrics of illiquidity risk management, able to identify the days characterized by a high level of conditional liquidity and illiquidity with the related values, based on different percentiles; (ii) the Conditional Illiquidity Market model and its influence on days of extreme liquidity and illiquidity, based on a quantile regression analysis; (iii) the implications, in terms of flight-to-liquidity and flight-to-quality phenomena that rely on the clientele effect, between the conditional illiquidity for U.S. dollars denominated corporate bond yields and financial as well as economic variables.

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1. Introduction

A number of market participants have recently raised concerns about the level of liquidity in the secondary corporate bond markets. In particular, the concerns pertain the role that changes in the market structure provide to liquidity and how these market structure changes are driven by variations in regulatory requirements. During the days of unexpected or significant market events, such as abrupt raises of interest rates or increases of the stock market volatility, investors could face possible rushes to crowded exits with the aim to sell their positions, creating a raise of the market illiquidity, with prices rapidly declining, as the markets seek a new equilibrium level for interest rates and credit risk transfers.

Therefore, managing the illiquidity risk constitutes a big challenge for investors and understanding the financial implications with the dynamics of several financial covariates, is a centerpiece for an investment decision. In light of these challenges posed by liquidity and the market structure effects, this paper applies a conditional illiquidity measure (Corvasce 2016), based on the dynamic of the $j-th$ order serial conditional covariance to U.S. dollars denominated corporate bond yields, with the aim to accommodate the limitations pointed out by Roll (1984), Glosten (1987b), Glosten and Harris (1988), Harris (1990). This measure of conditional illiquidity considers the changes of the unobserved yield for a corporate bond, caused by the arrival of new information and the transaction costs incurred in making the exchange of it, providing a deeper understanding of the interactions between the conditional illiquidity and the bond yield spreads, justifying the dynamics of the market liquidity, which have been especially detrimental to markets for fixed income securities and their derivatives, including the secondary corporate bond markets, during the financial crisis and the post-financial crisis.

The paper constructs several metrics of illiquidity risk, based on the distributions of the conditional illiquidity for corporate bond yields, with the aim to identify the dates of extreme liquidity and illiquidity, that might create implications in terms of losses and profits and evaluate the amount of capital that is at risk for an investment decision, provided a certain percentile. The identification of extreme days of (il)-liquidity allows to perform a quantile regression analysis, with the aim to study the statistical relations, based on the conditional illiquidity for a portfolio of U.S. corporate bond yields.

The estimation of the covariance matrix is based on the extension of Markov Chain Marginal Bootstrap algorithm (MCMB), proposed by He and Hu (2002) and called MCMB-A (Kocherginsky et al. 2005) and relies on the Mersenne Twister (Matsumoto and Nushimura 1997) algorithm, able to pseudo-random generate the bootstrapped replications. The sparsity method is based on the Epanechnikov kernel using the residuals, with a quantile function based on a Gumbel distribution and the Hall-Sheather (1998) bandwidth method, characterized by a parameter equals to 0.0041073.

The construction of a conditional illiquidity market model and the statistical relations with several financial covariates allow to describe the *flight-to-liquidity* and the *flight-to-quality* phenomena, in terms of general tendency for the investors, that are willing to allocate capital to safer and more valuable investments, due to a variation of the bid-ask spreads and a change of the quality for several U.S. dollars denominated corporate bonds. These phenomena create a change of the risk aversion perceived by the investors and are particularly evident during the financial crisis and the post-financial crisis.

The empirical study discusses the commonalities between the gold price (GOLD), the CBOE Volatility Index (VIX), the perception of the credit risk, broadly summarized by the TED spread, the CBOE SKEW Index (SKEW), the U.S. business cycle expansions and contractions (REC) variable as well as the Fama-French five factors, with the aim to interpret the implications with the conditional illiquidity for U.S. dollars denominated corporate bond yields, across all ratings.

The results corroborate the expectations of the readers in terms of *flight-to-liquidity* and *flight-to-quality* phenomena, providing a weak relation between the Fama-French factors (HML, SMB, RMW, CMA) and the conditional liquidity for U.S. corporate bond yields that increases during the recession periods, characterized by a tendency for accommodative monetary policies, making money cheaper for business to borrow and stimulating the economic growth by loosening money supply. Further, the conditional liquidity for U.S. corporate bond yields decreases during the expansionary periods, that represent those phases of the business cycles, when the economy moves from a period with a level of business activity surging and the gross domestic product expanding, until it reaches a peak.

The empirical findings provide a statistical support concerned about the combined effect regarding the increases in upside option call prices and the aggregate growth in downside put option premiums that occur when option buyers and sellers anticipate a likely sharp move to the downside, raising The CBOE Volatility Index (VIX) and decreasing the conditional illiquidity for U.S. corporate bond yields. Further, a raise of the slope for the implied volatility curve increases as this curve tends to steepen, determining an increase of the conditional illiquidity, for low risky U.S. corporate bonds and an increase of the conditional liquidity, for high risky U.S. corporate bonds, that is also related to an exacerbation of the counterparty risk, with interbank lenders demanding a higher rate of interest or willing to accept a lower return on safer investments, such as U.S. Treasury Bills, particularly during certain days of the financial crisis. This relation is statistically significant for A, AA, AAA and B categories of U.S. corporate bonds, but it is not significant for BB, BBB and CCC or below U.S. corporate bond yields.

The paper proceeds as follows. Section 2 overviews the literature on the topic. Section 3 revises the methodology and proposes some metrics of Illiquidity Risk Management (I-RM). Section 4 describes the

Data and reports the Descriptive statistics. Section 5 derives the econometric methodology. Section 6 discusses the empirical results. Section 7 offers final conclusions.

2. An Overview of the Literature

The literature around the topic of market liquidity for U.S. corporate bonds proposes alternative metrics (Schestag et al. (2016)), with the aim to study interesting features pertaining the bond characteristics. Mahanti et al. (2008) develop a new measure of liquidity known as "latent liquidity", defined as the weighted average turnover of investors who hold a bond, in which the weights are the fractional investor holdings, with the aim to evaluate the capacity for predicting the transaction costs and test the price impact of trading, over and above the trading activity and the bond-specific characteristics for the liquidity.

A regression based approach for estimating the transaction costs was developed by Schultz (2001) and Edwards et al. (2007). The authors estimate the transaction costs, as a difference of transaction prices and some reference prices. In particular, Edwards et al. (2007) estimate the average transaction costs as a function of the trade size for each bond that traded more than nine times, between January 2003 and January 2005, showing a negative statistical relation with the trade size. Hong and Warga (2000) and Chakravarthy and Sarkar (2003) propose a metric able to depict the difference between the average customer buy price and the average customer sell price; whereas, Feldhutter (2012) proposes the round-trip cost measure, where, dealers undertake round-trip trades to coordinate buy and sell orders, considered by investors.

The degree to which an asset can be quickly bought or sold in the market has implications for the provided expected return. Amihud and Mendelson (1991) show a negative relation between the liquidity of an asset and the expected return generated. The higher is the cost of trading an asset and the higher is the return that an investor requires for trading it. In particular, investors with long holding periods benefit from holding illiquid assets; whereas, liquid assets tend to be preferred by investors with short holding periods. The described effect, known as the *"clientele effect",* also takes in account the risk aversion for the investors as well as their utility functions. The positive relation between expected return and illiquidity explains the "*equity premium puzzle*" (Mehra and Prescott 1985), where, the risk aversion of the investors explains the large difference between the average expected return on equity and the expected return generated on less risky assets and the "*flight-to-liquidity*" phenomenon (Longstaff 2004).

In addition to the evidence proposed by Amihud and Mendelson (1986), Brennan and Subrahmanyam (1996), Datar et al. (1998) and Amihud (2002) show the consistency of the positive statistical relation between expected return and illiquidity, using other measures of illiquidity costs. Brennan and Subrahmanyam (1996) rely on the price impact order measure proposed by Kyle (1985), obtained from intraday estimates, using data on transactions and quotes; the illiquidity measure developed by Amihud (2002), that explains the price formation (Ho and Stoll 1981, 1983) and its impact in terms of how much trading activity is necessary for moving the price of an asset, is computed as the ratio of the absolute daily return to (dollar) volume; Datar et al. (1998) propose the stock turnover ratio (share trading volume relative to the number of shares outstanding) for supporting their empirical analysis, concerned about the declining of the stock expected return in stock's turnover; whereas, Jankowitsch et al. (2011) consider a measure of price dispersion that depends on the direct transaction costs, dealers' inventory risk and the costs paid by investors, for searching the counterparts.

For the corporate bonds, Chen et al. (2007) find a positive statistical relation between the bond yield and the illiquidity, with less liquid bonds, in particular the speculative-grade bonds, characterized by a higher yield. The results hold, after controlling for common bond and firm specific variables, macroeconomic variables and are robust to issuers' fixed effect and potential endogeneity biases.

In line with the findings proposed by Pastor and Stambaugh (2003), de Jong and Driessen (2007) as well as Lin et al. (2011) find a positive relation between expected return for corporate bonds and metrics of sensitivity to liquidity risk. Edwards et al. (2007) find lower transaction costs for highly rated bonds, recently issued bonds, and bonds close to maturity; whereas, Friewald et al. (2009) rely on daily trading data for n. 20000 corporate bonds, from Trade Reporting and Compliance Engine (TRACE), showing a deterioration of the liquidity for U.S. corporate bonds, during the onset of the crisis (3Q/2007), particularly consistent for U.S. corporate bonds with a high credit risk.

3. The Model

The framework assumes that the observed price of an asset $(\hat{p})$, at a certain time t, consists of two components $\tilde{p}$, that is the unobserved price of an asset also caused by the arrival of new information at a certain time t and p that represents the transaction costs incurred in making an exchange of a certain asset, at time t. Therefore,

$$\hat{p}_t = \tilde{p}_t + p_t, \tag{1}$$

where,

$$\tilde{p}_t = \tilde{p}_{t-1} + Q_t \cdot Z_t + e_t \tag{2}$$

whereas,

$$p_t = f\left(Q_t\,,\,C_t\right). \tag{3}$$

In particular, the quantity Q_t represents the unobserved indicator for the bid/ask classification, at time t, that takes a value equals to $+1$, if the transaction at time t was initiated by a buyer and a value equals to -1, if the transaction at time t was initiated by a seller. The quantity Z_t represents the adverse selection component that also depends on the order size, since, well informed traders maximize the return to their perishing information, impacting on the level of the asymmetric information, available at time t.

Therefore, the quantity $Q_t \cdot Z_t$ represents the adverse selection component, due to the revision in expectations, conditional on the arrival of a new order, at time t. A buy/sell order, assuming a positive quantity of Z_t, respectively creates a potential increase/decrease of the unobserved price $\left(\tilde{p}\right)$, at time t, with a size that is in absolute value equals to Z_t. The quantity $\tilde{p}_{t-1}$ represents the unobserved price of an asset, at time $t-1$; whereas, the quantity e_t represents the innovation for the unobserved price of an asset, that depends on the arrival of public information, from time $t-1$ to t and has a distribution $\left(G\right)$, with observations that are independent and identically distributed (i.i.d), with a mean equals to μ and a variance equals to v^2, at time t.

The component p is a function $f\left(\cdot\right)$ of the unobserved indicator for the bid/ask classification $\left(Q_t\right)$ and the unobserved transitory component $\left(C_t\right)$, that also depend on the order size, at time t[1]. As such, the equality n. (1) can be rewritten in the following way:

$$\hat{p}_t = \tilde{p}_{t-1} + Q_t \cdot Z_t + f\left(Q_t\,,\,C_t\right) + e_t. \tag{4}$$

Following Roll (1984), Kyle (1985), Easley and O'Hara (1987), Glosten (1987b), Glosten and Harris (1988) and Corvasce (2016), the observed price change $\left(\triangle\hat{p}\right)$, from $t-1$ to t, is equal to the following quantity:

$$\triangle\hat{p}_t = \triangle\tilde{p}_t + \triangle p_t, \tag{5}$$

where, $\triangle\tilde{p}_t$ is the unobserved price change, at time t and $\triangle p_t$ is the change of the transaction costs component, at time t. Therefore, the dynamic of the $j-th$ order serial conditional covariance for the quantity $\triangle\hat{p}_t$, able to consider the lag variations for the observed price $\hat{p}$, the asset information set F, at time $t-j-1$ and $j \geqslant 1$, can be computed in the following way:

$$Cov\left(\triangle\hat{p}_t\,,\,\triangle\hat{p}_{t-j}\,|\,F_{t-j-1}\right) = Cov\left(\triangle\tilde{p}_t + \triangle p_t\,,\,\triangle\tilde{p}_{t-j} + \triangle p_{t-j}\,|\,F_{t-j-1}\right). \tag{6}$$

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[1] The framework, proposed by Glosten and Harris (1998), considers the quantities Z_t and C_t, that respectively represent the adverse selection spread component and the transitory spread component, that are **linear** functions of the observed number of shares traded in a transaction, at time t.

Considering the equality n. 6, the dynamic of the $j-th$ order serial conditional covariance can be rewritten in the following way:

$$\begin{gathered}
Cov\left(\triangle\hat{p}_t\,,\,\triangle\hat{p}_{t-j}\,|\,F_{t-j-1}\right)=E\left[\left(\triangle\tilde{p}_t+\triangle p_t\right)\left(\triangle\tilde{p}_{t-j}+\triangle p_{t-j}\right)\,|\,F_{t-j-1}\right]+\\
-E\left[\left(\triangle\tilde{p}_t+\triangle p_t\right)\,|\,F_{t-j-1}\right]\cdot E\left[\left(\triangle\tilde{p}_{t-j}+\triangle p_{t-j}\right)\,|\,F_{t-j-1}\right],
\end{gathered}\tag{7}$$

or simply, the previous quantity can be re-compacted as follows:

$$\begin{gathered}
Cov\left(\triangle\hat{p}_t\,,\,\triangle\hat{p}_{t-j}\,|\,F_{t-j-1}\right)=E\left[\triangle\tilde{p}_t\cdot\triangle\tilde{p}_{t-j}+\triangle\tilde{p}_t\cdot\triangle p_{t-j}+\triangle p_t\cdot\triangle\tilde{p}_{t-j}+\triangle p_t\cdot\triangle p_{t-j}\,|\,F_{t-j-1}\right]+\\
-\left[E\left(\triangle\tilde{p}_t\,|\,F_{t-j-1}\right)+E\left(\triangle p_t\,|\,F_{t-j-1}\right)\right]\cdot\left[E\left(\triangle\tilde{p}_{t-j}\,|\,F_{t-j-1}\right)+E\left(\triangle p_{t-j}\,|\,F_{t-j-1}\right)\right].
\end{gathered}\tag{8}$$

The right hand side (RHS) of the equality can be decomposed in the following way:

$$\begin{gathered}
E\left[\triangle\tilde{p}_t\cdot\triangle\tilde{p}_{t-j}\,|\,F_{t-j-1}\right]-E\left[\triangle\tilde{p}_t\,|\,F_{t-j-1}\right]\cdot E\left[\triangle\tilde{p}_{t-j}\,|\,F_{t-j-1}\right]+\\
+E\left[\triangle\tilde{p}_t\cdot\triangle p_{t-j}\,|\,F_{t-j-1}\right]-E\left[\triangle\tilde{p}_t\,|\,F_{t-j-1}\right]\cdot E\left[\triangle p_{t-j}\,|\,F_{t-j-1}\right]+\\
+E\left[\triangle p_t\cdot\triangle\tilde{p}_{t-j}\,|\,F_{t-j-1}\right]-E\left[\triangle p_t\,|\,F_{t-j-1}\right]\cdot E\left[\triangle\tilde{p}_{t-j}\,|\,F_{t-j-1}\right]+\\
+E\left[\triangle p_t\cdot\triangle p_{t-j}\,|\,F_{t-j-1}\right]-E\left[\triangle p_t\,|\,F_{t-j-1}\right]\cdot E\left[\triangle p_{t-j}\,|\,F_{t-j-1}\right].
\end{gathered}\tag{9}$$

Therefore, the equality n. 8 can be rewritten in terms of contribution to the $j-th$ order serial conditional covariance. As such:

$$\begin{gathered}
Cov\left(\triangle\hat{p}_t\,,\,\triangle\hat{p}_{t-j}\,|\,F_{t-j-1}\right)=Cov\left(\triangle\tilde{p}_t\,,\,\triangle\tilde{p}_{t-j}\,|\,F_{t-j-1}\right)+Cov\left(\triangle\tilde{p}_t\,,\,\triangle p_{t-j}\,|\,F_{t-j-1}\right)+\\
+Cov\left(\triangle p_t\,,\,\triangle\tilde{p}_{t-j}\,|\,F_{t-j-1}\right)+Cov\left(\triangle p_t\,,\,\triangle p_{t-j}\,|\,F_{t-j-1}\right).
\end{gathered}\tag{10}$$

At time t, the dynamic of the $j-th$ order serial conditional level of illiquidity, considering the LAG variations $\left(LAG\,Illiquidity_t\right)$, is defined in the following way:

$$LAG\,Illiquidity_t=-Cov\left(\triangle\hat{p}_t\,,\,\triangle\hat{p}_{t-j}\,|\,F_{t-j-1}\right).\tag{11}$$

The quantity $Cov\left(\triangle\tilde{p}_t\,,\,\triangle\tilde{p}_{t-j}\,|\,F_{t-j-1}\right)$ represents the dynamic of the $j-th$ order serial conditional covariance for the unobserved price changes. This component depicts the conditional time-varying surprises that are possible to discover during the evolution of the observed prices, provided the information set for an asset that an investor receives at time $t-j-1$; the quantity $Cov\left(\triangle\tilde{p}_t\,,\,\triangle p_{t-j}\,|\,F_{t-j-1}\right)$ represents the time-varying level of the $j-th$ order serial conditional dependence, between the unobserved price variations and the changes in transaction costs, that an investor is willing to pay, with the aim to acquire information about the evolution of an asset, provided the information set (F), at time $t-j-1$.

The components $Cov\left(\triangle p_t\,,\,\triangle\tilde{p}_{t-j}\,|\,F_{t-j-1}\right)$ and $Cov\left(\triangle p_t\,,\,\triangle p_{t-j}\,|\,F_{t-j-1}\right)$ respectively represent the time-varying levels of the $j-th$ order serial conditional dependencies, between the variations of the transaction costs and the unobserved price changes, provided the information set F, acquired at time $t-j-1$, as well as the time-varying level of the $j-th$ order serial conditional covariance, between the variations of the transaction costs paid at time t and $t-j$. These quantities are able to incorporate the frictions and the market imperfections, for discovering the dynamics of the observed asset prices.

3.1. Some metrics of Illiquidity Risk Management (I-RM)

This section proposes several metrics of illiquidity risk management, that are based on the distribution of the conditional level of illiquidity for an asset. The main purpose of such metrics is to manage and control the liquidity risk and in particular the dates of extreme liquidity and illiquidity that might create implications, in terms of losses and profits.

The most well known measures of risk among risk managers are the Value at Risk (VaR) and the Expected Shortfall (ES). VaR has been introduced with the aim of answering to the following questions: What is the expected loss incurred by an asset, given a certain probability and a time horizon? What is the amount of capital that is at risk in an investment process? VaR is defined as the maximum loss incurred in a portfolio with a level of probability equals to $1-\alpha$, with $0<\alpha<1$[2].

Artzner et al. (1999) propose ES, as a valid alternative to VaR. The expected shortfall measures how much a financial institution can lose on average, in states beyond the Value at Risk, improving the reliability for its computation. For practical applications, risk managers have the aim to evaluate the risk contribution, coming from a particular exposure on a certain class of assets.

This section applies the metrics for measuring the market risk to the evaluation and control of the liquidity risk. As such, the conditional level of illiquidity for a portfolio of assets, at time t $(LAG\,Illiquidity\,Portfolio_t)$, can be decomposed as a weighted average of the conditional level of illiquidity for each asset i, in which a company can be involved and it is written in the following way:

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[2] From a mathematical standpoint, VaR is the $\alpha-quantile$ of the inverse distribution function $F_X^{-1}$of a random variable X, taken with a negative sign, if it is defined in terms of a loss function. If the inverse distribution function does not exist, VaR can be defined as the $\alpha-quantile$ of the generalized inverse distribution function F_X. Several studies among others (Artzner et al. 1997, 1999), Jorion (2001), Rockafellar and Uryasev (2002) show the inadequacy of VaR, as a coherent measure of risk and point out how VaR used by regulators and banking supervisors “can destabilize an economy and induce crashes, when they would not otherwise occur” (Danielsson et al. 2002).

$$LAG\,Illiquidity\,Portfolio_t = \sum_{i=1}^{N} \omega_{i,t} \cdot LAG\,Illiquidity_{i,t} \tag{12}$$

or simply,

$$LAG\,Illiquidity\,Portfolio_t = -\sum_{i=1}^{N} \omega_{i,t} \cdot Cov\left(\triangle\hat{p}_{i,t}\,, \,|\, \triangle\hat{p}_{i,t-j} \,|\, F_{i,t-j-1}\right), \tag{13}$$

where, $Cov\left(\triangle\hat{p}_{i,t}\,, \,|\, \triangle\hat{p}_{i,t-j} \,|\, F_{i,t-j-1}\right)$ represents the dynamic for the $j-th$ order serial conditional covariance, considering the change for the observed prices of an asset i at time t $(\triangle\hat{p}_{i,t})$, the *lag* variations for the observed price and the information set (F), at time $t-j-1$, with $j \geq 1$; whereas, $\omega_{i,t}$ is the percentage amount of capital that a company can allocate, for each asset i at time t and N is the number of assets that composes a portfolio.

In formula, the Expected Shortfall (ES) for the distribution of the conditional level of illiquidity can be computed in the following way:

$$ES_\alpha\left(LAG\,Illiquidity\,Portfolio_t\right) = \tag{14}$$

$$= E\left[LAG\,Illiquidity\,Portfolio_t \,|\, LAG\,Illiquidity\,Portfolio_t \leq VaR_\alpha\left(LAG\,Illiquidity\,Portfolio_t\right)\right] =$$

$$= \sum_{i=1}^{N} \omega_{i,t} \cdot E\left[LAG\,Illiquidity_{i,t} \,|\, LAG\,Illiquidity\,Portfolio_t \leq VaR_\alpha\left(LAG\,Illiquidity\,Portfolio_t\right)\right].$$

The operator $E(\cdot)$ represents the expected value and the quantity α represents the percentile for the distribution of the conditional level of illiquidity incurred in a portfolio, with probability $1-\alpha$, provided $0 < \alpha < 1$.

The quantity $ES_\alpha\left(LAG\,Illiquidity\,Portfolio_t\right)$ is the average of the conditional levels of illiquidity, on days when the conditional level of illiquidity for a portfolio of assets, at time $t(LAG\,Illiquidity\,Portfolio_t)$, is below or equal the quantity $VaR_\alpha\left(LAG\,Illiquidity\,Portfolio_t\right)$[3]. It can be decomposed in the following way:

$$ES_\alpha\left(LAG\,Illiquidity\,Portfolio_t\right) = \sum_{i=1}^{N} \frac{\partial ES_\alpha\left(LAG\,Illiquidity\,Portfolio_t\right)}{\partial\omega_{i,t}} \cdot \omega_{i,t}\,. \tag{15}$$

This alternative specification relies on Euler's decomposition of the Expected Shortfall (ES), computed on the distribution for the conditional level of illiquidity, into individual exposures (Tasche 1999, Yamai and Yoshiba (2002a, 2002b, 2002c, 2005)). The quantity $\frac{\partial ES_\alpha(LAG\,Illiquidity\,Portfolio_t)}{\partial\omega_{i,t}}$ is the sensitivity of the distribution for the conditional level of illiquidity for a portfolio of assets, with respect to each individual exposure[4]. For the purpose of the paper, this quantity is called the *Conditional Illiquidity - Marginal Expected*

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[3] The equality n. (14) contains a definition of the Expected Shortfall (ES), where, the conditional expectation is computed with the conditional level of illiquidity for a portfolio of assets $(LAG\,Illiquidity\,Portfolio_t)$, that is below or equal the quantity $VaR_\alpha\left(LAG\,Illiquidity\,Portfolio_t\right)$.

[4] The expected shortfall (ES) can also be written as a *distortion risk* measure, based on the distortion function.

Shortfall (MES) and depicts the first derivative of the expected value of the conditional level of illiquidity for a portfolio of assets $(LAG\, Illiquidity\, Portfolio_t)$, with respect to the quantity $\omega_{i,t}$, provided a certain level of the percentile α.

4. Data and descriptive statistics

The paper relies on a data-set of U.S. dollars denominated corporate bonds that allow the creation of indexes able to track the performance of investment grade rated corporate debt, publicly issued in the U.S. domestic market, provided the following ratings: A, AA, AAA, B, BB, BBB and CCC or below.

[Please Insert Table 1 around here]

Table 1 summarizes the descriptive statistics, expressed in percentage values, pertaining the yields for U.S. corporate bond indexes elaborated by Bank of America Merrill Lynch. In particular, the mean values of the yields for U.S. dollars denominated B, BB and CCC or below corporate bonds are respectively equal to 9.270%, 7.434% and 15.148%; the mean values of the yields decrease below 6.00% for U.S. dollars denominated A, AA, AAA and BBB corporate bonds, pointing out a lower yield for corporate bonds characterized by low default risk.

[Please Insert Figure 1 around here]

For the period from January 1997 to February 2016, the values of the standard deviations are greater for the yields of U.S. dollars denominated corporate bonds, with a high default risk and decrease below 2.00%, for the yields of U.S. dollars denominated A, AA, AAA, BBB corporate bonds, characterized by low default risk. Figure 1 reports the evolution of the yields for all ratings of U.S. corporate bonds and the descriptive statistics show an increase of the yields, during the financial crisis period (July 2007 - December 2008) and a further decrease that characterizes the first and the second phases of the sovereign debt crisis. The *flight-to-liquidity* and the *flight-to-quality* phenomena are particularly relevant during the periods characterized by an increase of the volatility for the stock markets, with a general tendency of the investors to allocate capital to safer investments and more liquid U.S. dollars denominated corporate bonds.

[Please Insert Table 2 around here]

Table 2 reports the auto-correlation and the partial auto-correlation of the indexes, for the first 5 lags of U.S. corporate bond yields, across all ratings. The Bank of America Merrill Lynch indexes, for high yield U.S. corporate bonds (B, BB and CCC or below), report values of the first order serial correlations that are greater than 0.200, with highly significant Q-statistical values; whereas, the values of the first order serial correlations are below 0.030, for U.S. corporate bonds, with ratings A, AA, AAA and BBB.

The empirical analysis studies the behavior of several financial and economic covariates and the implications with the conditional illiquidity for U.S. corporate bond yields, also during the days of extreme liquidity and illiquidity. These episodes might be coupled by a *flight-to-liquidity* phenomenon, with U.S. corporate bonds characterized by a higher liquidity, becoming relatively more valuable and with U.S. corporate bonds characterized by a lower liquidity, becoming less valuable.

The dataset considers the dynamics of the London Bullion Market Association (LBMA) gold price, usually regarded by investors as a source of value for diversifying risk, especially through the use of future contracts and derivatives. An increase of the gold price is usually related to a dramatic decline of the liquidity, for the U.S. equity market and corporate bonds.

Other financial variables able to depict interesting features for the capital markets and describe the mechanisms, connected to the *flight-to-liquidity* and the *flight-to-quality* phenomena (Ericsson and Renault 2006, Beber et al. 2009), that also rely on the *clientele effect* (Amihud and Mendelson 1986, 1991, Amihud et al. 2005), are the following: the TED spread, the CBOE Volatility Index (VIX), the CBOE Skew Index (Skew) and the U.S. Business Cycle Expansions and Contractions (REC) variable, provided by the National Bureau of Economic Research (NBER), able to consider recessionary and expansionary periods[5].

The dynamics of the TED spread, defined as the difference between 3-month LIBOR based on U.S. dollars and 3-month U.S. Treasury Bills, is able to depict the perceived credit risk for the U.S. economy. During the period from July 2007 to March 2009, the TED spread increases to an average level of 1.366%, implying an exacerbation of the counterparty risk, with interbank lenders respectively demanding a higher rate of interest for certain days or willing to accept a lower return on safer investments, such as U.S. Treasury Bills, in other days. Further, the TED spread declines to an average level of 0.258%, due to a dramatic reduction of the counterparty risk, from March 2009 to June 2016.

The CBOE Volatility Index (VIX) and the CBOE Skew Index (SKEW), also called the *Black Swan* Index, respectively measure the market expectations of near term volatility conveyed by the S&P500 Index option prices and the probability of getting outliers, from the prices of S&P500 out-of-money options, implying a demand for low strike puts. Therefore, the *Black Swan* Index measures the overall steepening of the curve

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[5] In particular, the expansions (recessions) periods begin at the peak (trough) of the cycles and end at the trough (peak), with the business cycle dates available from the NBER website (www.nber.org/cycles.html).

for implied volatilities. The financial crisis period (from July 2007 to March 2009) reports an average value of the VIX and SKEW indexes respectively equal to 30.998 and 115.189; whereas, the probability of outlier returns becomes more significant, due to a higher perceived tail risk and a higher chance of black swan events, reaching an average level of 123.430, during the period from March 2009 to February 2016.

The dataset also considers the Fama-French five factors, with the aim to analyze the implications and depict the co-movements between the dynamics of the conditional illiquidity for A, AA, AAA, B, BB, BBB and CCC or below U.S. corporate bonds and the dynamics of the market portfolios. In particular, the dataset considers the following covariates: (i) MKT-BR that represents the excess return of the market portfolio (MKT), computed as the value-weighted return for all CRSP firms incorporated in the U.S. and listed on the NYSE, AMEX and NASDAQ, with respect to the benchmark interest rate (BR); (ii) High Minus Low (HML) is the average return on the two "value" portfolios, minus the average return on the two "growth" portfolios; (iii) Small Minus Big (SMB) is the average return on the nine "small stock" portfolios, minus the average return on the nine "big stock" portfolios; (iv) Robust Minus Weak (RMW) is the average return on the two "robust operating profitability" portfolios, minus the average return on the two "weak operating profitability" portfolios; (v) Conservative Minus Aggressive (CMA) is the average return on the two "conservative investment" portfolios, minus the average return on the two "aggressive investment" portfolios. The portfolios are based on the stock performance for all CRSP firms incorporated in the U.S. and listed on the NYSE, AMEX and NASDAQ.

4.1 Interpolating the Data

This subsection discusses the *Cardinal* splines (Schoenberg 1973), as an interpolation technique that also allows the creation of continuous time series for the covariates. The *Cardinal* splines (CS) are a subset of the cubic Hermitian splines that are a third degree piece-wise parameterized curves, characterized by $n-3$ third degree polynomials that define the curves, among n vertices, with the first and the last vertices that are not included[6]. The proposed technique builds on interpolating the spline curves for the covariates, through a set of control points, relying on two levels of continuity:

$$C^0, meaning\, that\, two\, segments\, match\, values\, at\, the\, join;$$

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[6] The main issue with a Hermite spline is to find the coefficients for n–3 third degree polynomials.

C^1, meaning that two segments match slopes at the join.

The technique is composed of cubic Biezer splines that are joined via C^1continuity. The cubic Biezer splines are determined by four control points, such as two endpoints and two derivatives at the points, creating the basic intuition for the *Cardinal* splines that generalize the *Catmull-Rom* splines, providing a shape parameter(c) that represents the *tension* defined within the interval$(0\,,\,1)$. A representation of the *Cardinal* splines is defined, as a function of the successive control points p_i and p_{i+1}, in the following way:

$$t_i = \frac{1}{2} \cdot (1-c) \cdot (p_{i+1} - p_{i-1}) \tag{16}$$

and

$$t_{i+1} = \frac{1}{2} \cdot (1-c) \cdot (p_{i+2} - p_i)\,. \tag{17}$$

The tangent vectors t_i and t_{i+1} represent the direction vectors. In particular, the values of c between $0\,and\,1(0 < c < 1)$, modulate a higher curvature for the splines; whereas, the values of $c < 0$ and $c = 0$ respectively allow to lower the curvature and originate the *Catmull-Rom* splines. The previous control point (p_{i-1}) and the subsequent control point (p_{i+2}) allow to define the slopes, in terms of vectors between control points and test the level of continuity C^1, in correspondence of the control points p_i and p_{i+1}.

For the purpose of the analysis, the data are interpolated with a *Cardinal* spline, based on two previous non-missing values and the next two non-missing values, assuming a parameter of the tension c equals to 0.1 and a parameter λ, able to depict the relative position of the missing value divided by the total number of missing values, within a time series. As such, the interpolated values (IV_{CS}), based on this extended technique, rely on the following formula:

$$IV_{CS} = \left(2\lambda^3 - 3\lambda^2 + 1\right) \cdot p_{i-1} + (1-c) \cdot \left(\lambda^3 - 2\lambda^2 + \lambda\right) \cdot (p_{i+1} - p_{i-2}) - \left(2\lambda^3 - 3\lambda^2\right) \cdot p_{i+1} + (1-c) \cdot \left(\lambda^3 - \lambda^2\right) \cdot (p_{i+2} - p_{i-1})\,. \tag{18}$$

Therefore, if the time series regarding a covariate has a single missing value, then the interpolated value will be half way, between the previous value and the next value. If the time series of a covariate has two missing values, the first value will be interpolated as 1/3 of the distance between the previous value and the next; whereas, the second value will be interpolated as 2/3 of the distance, influencing the value of the parameter λ.

5. The Econometric Methodology

This section proposes a simple econometric framework for estimating the time-varying level of the conditional illiquidity for U.S. corporate bond yields[7]. The analysis discusses the case with $j = 1$ and aims to compute the dynamics of the first order serial conditional covariance, considering arithmetic and natural logarithmic changes for U.S. corporate bond yields, with the following ratings: A, AA, AAA, B, BB, BBB and CCC & below. The arithmetic and the natural logarithmic variations for the daily observed U.S. corporate bond yields $(\hat{y})$, at time $t-1$, can be computed in the following way:

$$\triangle\hat{y}_{A,t-1} = \left(\frac{\hat{y}_{A,t-1} - \hat{y}_{A,t-2}}{\hat{y}_{A,t-2}}\right) \simeq log\left(\frac{\hat{y}_{A,t-1}}{\hat{y}_{A,t-2}}\right) = \alpha + \epsilon_{A,t-1} \tag{19}$$

$$\triangle\hat{y}_{AA,t-1} = \left(\frac{\hat{y}_{AA,t-1} - \hat{y}_{AA,t-2}}{\hat{y}_{AA,t-2}}\right) \simeq log\left(\frac{\hat{y}_{AA,t-1}}{\hat{y}_{AA,t-2}}\right) = \delta + \epsilon_{AA,t-1} \tag{20}$$

$$\triangle\hat{y}_{AAA,t-1} = \left(\frac{\hat{y}_{AAA,t-1} - \hat{y}_{AAA,t-2}}{\hat{y}_{AAA,t-2}}\right) \simeq log\left(\frac{\hat{y}_{AAA,t-1}}{\hat{y}_{AAA,t-2}}\right) = \zeta + \epsilon_{AAA,t-1} \tag{21}$$

$$\triangle\hat{y}_{B,t-1} = \left(\frac{\hat{y}_{B,t-1} - \hat{y}_{B,t-2}}{\hat{y}_{B,t-2}}\right) \simeq log\left(\frac{\hat{y}_{B,t-1}}{\hat{y}_{B,t-2}}\right) = \upsilon + \epsilon_{B,t-1} \tag{22}$$

$$\triangle\hat{y}_{BB,t-1} = \left(\frac{\hat{y}_{BB,t-1} - \hat{y}_{BB,t-2}}{\hat{y}_{BB,t-2}}\right) \simeq log\left(\frac{\hat{y}_{BB,t-1}}{\hat{y}_{BB,t-2}}\right) = \lambda + \epsilon_{BB,t-1} \tag{23}$$

$$\triangle\hat{y}_{BBB,t-1} = \left(\frac{\hat{y}_{BBB,t-1} - \hat{y}_{BBB,t-2}}{\hat{y}_{BBB,t-2}}\right) \simeq log\left(\frac{\hat{y}_{BBB,t-1}}{\hat{y}_{BBB,t-2}}\right) = \xi + \epsilon_{BBB,t-1} \tag{24}$$

$$\triangle\hat{y}_{CCC,t-1} = \left(\frac{\hat{y}_{CCC,t-1} - \hat{y}_{CCC,t-2}}{\hat{y}_{CCC,t-2}}\right) \simeq log\left(\frac{\hat{y}_{CCC,t-1}}{\hat{y}_{CCC,t-2}}\right) = \varpi + \epsilon_{CCC,t-1}. \tag{25}$$

where, the quantities α, δ, ζ, υ, λ, ξ, ϖ, are respectively the coefficients of the mean equations that describe the evolution of the daily observed U.S. corporate bond yields, at time $t-1$. The arithmetic and the natural logarithmic variations of the daily observed U.S. corporate bond yields, at time t, are computed in the following way:

$$\triangle\hat{y}_{A,t} = \left(\frac{\hat{y}_{A,t} - \hat{y}_{A,t-1}}{\hat{y}_{A,t-1}}\right) \simeq log\left(\frac{\hat{y}_{A,t}}{\hat{y}_{A,t-1}}\right) = \beta + \epsilon_{A,t} \tag{26}$$

$$\triangle\hat{y}_{AA,t} = \left(\frac{\hat{y}_{AA,t} - \hat{y}_{AA,t-1}}{\hat{y}_{AA,t-1}}\right) \simeq log\left(\frac{\hat{y}_{AA,t}}{\hat{y}_{AA,t-1}}\right) = \epsilon + \epsilon_{AA,t} \tag{27}$$

$$\triangle\hat{y}_{AAA,t} = \left(\frac{\hat{y}_{AAA,t} - \hat{y}_{AAA,t-1}}{\hat{y}_{AAA,t-1}}\right) \simeq log\left(\frac{\hat{y}_{AAA,t}}{\hat{y}_{AAA,t-1}}\right) = \eta + \epsilon_{AAA,t} \tag{28}$$

$$\triangle\hat{y}_{B,t} = \left(\frac{\hat{y}_{B,t} - \hat{y}_{B,t-1}}{\hat{y}_{B,t-1}}\right) \simeq log\left(\frac{\hat{y}_{B,t}}{\hat{y}_{B,t-1}}\right) = \iota + \epsilon_{B,t} \tag{29}$$

$$\triangle\hat{y}_{BB,t} = \left(\frac{\hat{y}_{BB,t} - \hat{y}_{BB,t-1}}{\hat{y}_{BB,t-1}}\right) \simeq log\left(\frac{\hat{y}_{BB,t}}{\hat{y}_{BB,t-1}}\right) = \mu + \epsilon_{BB,t} \tag{30}$$

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[7] It is important to remark the difference between the *price* and the associated *yield* for a corporate bond. The econometric methodology studies the conditional level of illiquidity for U.S. corporate bond yields, relying on the theoretical framework developed in section 3, where, the observed yield for a U.S. corporate bond $(\hat{y})$, at a certain time t, consists of two components: $\tilde{y}$ is the unobserved yield, related to the unobserved price of the U.S. corporate bond, caused by the arrival of new information at a certain time t; whereas, y represents the transaction costs incurred in making an exchange of the U.S. corporate bond, at time t.

$$\triangle\hat{y}_{BBB,t} = \left(\frac{\hat{y}_{BBB,t} - \hat{y}_{BBB,t-1}}{\hat{y}_{BBB,t-1}}\right) \simeq log\left(\frac{\hat{y}_{BBB,t}}{\hat{y}_{BBB,t-1}}\right) = o + \epsilon_{BBB,t} \tag{31}$$

$$\triangle\hat{y}_{CCC,t} = \left(\frac{\hat{y}_{CCC,t} - \hat{y}_{CCC,t-1}}{\hat{y}_{CCC,t-1}}\right) \simeq log\left(\frac{\hat{y}_{CCC,t}}{\hat{y}_{CCC,t-1}}\right) = \rho + \epsilon_{CCC,t} \tag{32}$$

where, the quantities β, ϵ, η, ι, μ, $o\, and\, \rho$ are respectively the coefficients of the mean equations that describe the evolution of the daily observed U.S. corporate bond yields, at time t. The innovations of the residuals follow a Panel Diagonal BEKK(1,1) (Baba et al. 1985), with a multivariate *t-student* distribution and an unknown parameter, for quantifying the degrees of freedom. This assumption, regarding the distribution of the disturbances, allows to depict the "stylized facts" (i.e. asymmetry and fat tails), concerned about the distributions for U.S. corporate bond yields.

Therefore, the conditional variance processes for the observed corporate bond yields $(\hat{y})$, at time $t-1$ and provided the information set $\left(F\right)$ at time $t-2$, are computed in the following way:

$$E\left[\epsilon^2_{A,t-1}\,|\,F_{t-2}\right] = \sigma^2_{A,t-1} = m1\_1 + a1\_1 \cdot \epsilon^2_{A,t-2} + b1\_1 \cdot \sigma^2_{A,t-2} \tag{33}$$

$$E\left[\epsilon^2_{AA,t-1}\,|\,F_{t-2}\right] = \sigma^2_{AA,t-1} = m4\_4 + a4\_4 \cdot \epsilon^2_{AA,t-2} + b4\_4 \cdot \sigma^2_{AA,t-2} \tag{34}$$

$$E\left[\epsilon^2_{AAA,t-1}\,|\,F_{t-2}\right] = \sigma^2_{AAA,t-1} = m7\_7 + a7\_7 \cdot \epsilon^2_{AAA,t-2} + b7\_7 \cdot \sigma^2_{AAA,t-2} \tag{35}$$

$$E\left[\epsilon^2_{B,t-1}\,|\,F_{t-2}\right] = \sigma^2_{B,t-1} = m10\_10 + a10\_10 \cdot \epsilon^2_{B,t-2} + b10\_10 \cdot \sigma^2_{B,t-2} \tag{36}$$

$$E\left[\epsilon^2_{BB,t-1}\,|\,F_{t-2}\right] = \sigma^2_{BB,t-1} = m13\_13 + a13\_13 \cdot \epsilon^2_{BB,t-2} + b13\_13 \cdot \sigma^2_{BB,t-2} \tag{37}$$

$$E\left[\epsilon^2_{BBB,t-1}\,|\,F_{t-2}\right] = \sigma^2_{BBB,t-1} = m16\_16 + a16\_16 \cdot \epsilon^2_{BBB,t-2} + b16\_16 \cdot \sigma^2_{BBB,t-2} \tag{38}$$

$$E\left[\epsilon^2_{CCC,t-1}\,|\,F_{t-2}\right] = \sigma^2_{CCC,t-1} = m19\_19 + a19\_19 \cdot \epsilon^2_{CCC,t-2} + b19\_19 \cdot \sigma^2_{CCC,t-2} \tag{39}$$

where, $m1\_1$, $m4\_4$, $m7\_7$, $m10\_10$, $m13\_13$, $m16\_16$, $m19\_19$ are respectively the diagonal coefficients, that depict the long term components of the variances and covariances for U.S. dollars denominated corporate bonds, with ratings A, AA, AAA, B, BB, BBB, CCC or below; the diagonal coefficients $a1\_1$, $a4\_4$, $a7\_7$, $a10\_10$, $a13\_13$, $a16\_16$, $a19\_19$ respectively depict the influence of the squared residuals, at time $t-2$; whereas, the diagonal coefficients $b1\_1, b4\_4, b7\_7, b10\_10, b13\_13\,, b16\_16, b19\_19$ depict the persistence of the conditional variance components, at time $t-2$.

The conditional variance processes for the observed U.S. corporate bond yields, at time t, and provided the information set (F), at time $t-1$, are computed in the following way:

$$E\left[\epsilon^2_{A,t}\,|\,F_{t-1}\right] = \sigma^2_{A,t} = m2\_2 + a2\_2 \cdot \epsilon^2_{A,t-1} + b2\_2 \cdot \sigma^2_{A,t-1} \tag{40}$$

$$E\left[\epsilon^2_{AA,t}\,|\,F_{t-1}\right] = \sigma^2_{AA,t} = m5\_5 + a5\_5 \cdot \epsilon^2_{AA,t-1} + b5\_5 \cdot \sigma^2_{AA,t-1} \tag{41}$$

$$E\left[\epsilon^2_{AAA,t}\,|\,F_{t-1}\right] = \sigma^2_{AAA,t} = m8\_8 + a8\_8 \cdot \epsilon^2_{AAA,t-1} + b8\_8 \cdot \sigma^2_{AAA,t-1} \tag{42}$$

$$E\left[\epsilon^2_{B,t}\,|\,F_{t-1}\right] = \sigma^2_{B,t} = m11\_11 + a11\_11 \cdot \epsilon^2_{B,t-1} + b11\_11 \cdot \sigma^2_{B,t-1} \tag{43}$$

$$E\left[\epsilon^2_{BB,t}\,|\,F_{t-1}\right] = \sigma^2_{BB,t} = m14\_14 + a14\_14 \cdot \epsilon^2_{BB,t-1} + b14\_14 \cdot \sigma^2_{BB,t-1} \tag{44}$$

$$E\left[\epsilon^2_{BBB,t}\,|\,F_{t-1}\right] = \sigma^2_{BBB,t} = m17\_17 + a17\_17 \cdot \epsilon^2_{BBB,t-1} + b17\_17 \cdot \sigma^2_{BBB,t-1} \tag{45}$$

$$E\left[\epsilon^2_{CCC,t}\,|\,F_{t-1}\right] = \sigma^2_{CCC,t} = m20\_20 + a20\_20 \cdot \epsilon^2_{CCC,t-1} + b20\_20 \cdot \sigma^2_{CCC,t-1}\,. \tag{46}$$

The econometric methodology allows the joint estimation for the *contemporaneous* and the *lag* variations, related to the observed U.S. corporate bond yields. For simplicity of the notation, this section only reports the first order serial conditional covariances, for each category of U.S. corporate bonds.

Therefore, the first order serial conditional covariances between the *lag* and the *contemporaneous* observed yields for U.S. corporate bonds, provided the information set (F) at time $t-2$, can be represented in the following way: $\sigma_{A\_A,t-2}$, $\sigma_{AA\_AA,t-2}$, $\sigma_{AAA\_AAA,t-2}$, $\sigma_{B\_B,t-2}$, $\sigma_{BB\_BB,t-2}$, $\sigma_{BBB\_BBB,t-2}$, $\sigma_{CCC\_CCC,t-2}$.

The conditional covariances are computed in terms of conditional expectations:

$$E\left[\epsilon_{A,t-1} \cdot \epsilon_{A,t}\,|\,F_{t-2}\right] = \sigma_{A\_A,t-2} = \rho_{A\_A,t-2} \cdot \sigma_{A,t-1} \cdot \sigma_{A,t} \tag{47}$$

$$E\left[\epsilon_{AA,t-1} \cdot \epsilon_{AA,t}\,|\,F_{t-2}\right] = \sigma_{AA\_AA,t-2} = \rho_{AA\_AA,t-2} \cdot \sigma_{AA,t-1} \cdot \sigma_{AA,t} \tag{48}$$

$$E\left[\epsilon_{AAA,t-1} \cdot \epsilon_{AAA,t}\,|\,F_{t-2}\right] = \sigma_{AAA\_AAA,t-2} = \rho_{AAA\_AAA,t-2} \cdot \sigma_{AAA,t-1} \cdot \sigma_{AAA,t} \tag{49}$$

$$E\left[\epsilon_{B,t-1} \cdot \epsilon_{B,t}\,|\,F_{t-2}\right] = \sigma_{B\_B,t-2} = \rho_{B\_B,t-2} \cdot \sigma_{B,t-1} \cdot \sigma_{B,t} \tag{50}$$

$$E\left[\epsilon_{BB,t-1} \cdot \epsilon_{BB,t}\,|\,F_{t-2}\right] = \sigma_{BB\_BB,t-2} = \rho_{BB\_BB,t-2} \cdot \sigma_{BB,t-1} \cdot \sigma_{BB,t} \tag{51}$$

$$E\left[\epsilon_{BBB,t-1} \cdot \epsilon_{BBB,t}\,|\,F_{t-2}\right] = \sigma_{BBB\_BBB,t-2} = \rho_{BBB\_BBB,t-2} \cdot \sigma_{BBB,t-1} \cdot \sigma_{BBB,t} \tag{52}$$

$$E\left[\epsilon_{CCC,t-1} \cdot \epsilon_{CCC,t}\,|\,F_{t-2}\right] = \sigma_{CCC\_CCC,t-2} = \rho_{CCC\_CCC,t-2} \cdot \sigma_{CCC,t-1} \cdot \sigma_{CCC,t}\,. \tag{53}$$

where, the quantities $\rho_{A\_A,t-2}$, $\rho_{AA\_AA,t-2}$, $\rho_{AAA\_AAA,t-2}$, $\rho_{B\_B,t-2}$, $\rho_{BB\_BB,t-2}$, $\rho_{BBB\_BBB,t-2}$, $\rho_{CCC\_CCC,t-2}$, describe the conditional cross-correlations between the *lag* and the *contemporaneous* observed yields for U.S. dollars denominated corporate bonds, with ratings A, AA, AAA, B, BB, BBB and CCC or below, provided the information set, at time $t-2$.

6. Empirical Results

This section reports the estimates of the econometric methodology proposed in Section 4 and tests the implications with some financial and economic covariates, also in terms of extreme events of liquidity and illiquidity for U.S. corporate bond yields. The estimation procedure relies on a Panel Diagonal BEKK (1,1), with a disturbance assumption based on a multivariate *t-student* distribution, able to depict the dynamics of the first order serial conditional covariances. The evaluation technique relies on the *Berndt-Hall-Hall-*

Hausman (B-H-H-H) algorithm (Berndt et al. 1974), with more than 2.7 degrees of freedom, for both arithmetic and natural logarithmic changes.

[Please Insert Table 3 around here]

The estimated values for the *constants* of the mean equations are negative and high statistically significant. The magnitude of these values is greater for U.S. dollars denominated B, BB and CCC or below corporate bond yields, characterized by high default risk and it is lower for U.S. dollars denominated corporate bond yields, with low default risk. Table 3 also reports the estimated diagonal coefficients, able to depict the influence of the long period conditional variances and covariances, the related levels of persistence as well as the influence of the residuals. These values are positive, high statistically significant and are greater for U.S. dollars denominated corporate bond yields, with greater default risk and characterized by higher levels of conditional volatility.

The estimated values that lead to the description of the conditional levels of illiquidity, also depend on the computation of the changes. The empirical results discuss both arithmetic and natural logarithmic variations. The estimated *constants* of the mean equations and the coefficients able to depict the persistence of the conditional variances and covariances are generally lower for the empirical framework based on logarithmic changes than the estimated coefficients based on arithmetic changes; whereas, the estimated coefficients able to describe the significativity of the residuals and the estimated constants able to characterize the long period variances and covariances are greater for the framework based on arithmetic changes than the framework based on natural logarithmic variations.

The logarithmic changes of the corporate bond yields allow to standardize the values along a logarithmic function, implying a different effect, in case of variations that are not small in terms of size, on the dynamics of the conditional volatilities and covariances. The *time-additivity* property of the logarithmic changes allows the possibility to consider an order sequence of n trades, that impacts on the levels of the first order serial conditional correlations and on the estimated levels of the conditional illiquidity.

Therefore, the computation of the logarithmic changes imply a different effect, in terms of price formation for the corporate bonds (Kyle 1985, Lo and MacKinley 1990) and the market structure, pertaining the prediction of the subsequent changes for the last available bid or ask quotes, available for the next period. Further, the estimated values of the coefficients able to depict the *persistence* of the conditional variances and covariances are greater for U.S. dollars denominated corporate bonds with lower yields, than for U.S. corporate bonds with higher yields; whereas, the estimated values that describe the influence of the *residuals* and the estimated values able to depict the effects of the conditional volatility processes report a lower magnitude for U.S. dollars denominated corporate bonds with lower yields than for corporate bonds with higher yields.

[Please Insert Figure 2 and Figure 3 around here]

The dynamic of the conditional level of illiquidity, for the equally weighted index of U.S. dollars denominated corporate bond yields, with ratings A, AA, AAA, B, BB, BBB and CCC or below, based on arithmetic and logarithmic changes, is reported in Figure 2. The arithmetic variations of the corporate bond yields allow the derivation of lower values for the conditional levels of illiquidity, during those days characterized by a higher level of the conditional volatility for the S&P 500 Index, showing a negative correlation with its dynamic; whereas, the logarithmic changes derive a higher value of the conditional illiquidity, across ratings of U.S. dollars denominated corporate bonds, due to the assumptions of the logarithmic changes that provide an alternative financial implication, in case of variations that are not small in terms of size.

Figure 3 reports the dynamics of the conditional level of illiquidity for U.S. dollars denominated corporate bond yields, across all ratings. The index is computed as the equally weighted average for the conditional levels of illiquidity, across U.S. corporate bond yields and the changes are based on arithmetic and logarithmic variations. The methodology, for computing the changes of the bond yields, does not drastically influence the dynamic for the conditional illiquidity of the index, provided its construction that smooths the financial implications, concerned about the formation of the U.S. dollars denominated corporate bond prices.

[Please Insert Figure 4 and Figure 5 around here]

The statistical implications regarding the computation of the conditional level of illiquidity, based on arithmetic and logarithmic changes of the corporate bond yields, are also reported in Figure 4 and Figure 5. The figures show the evolution of the conditional level of illiquidity for the sub-periods, before the financial crisis (January 1997 - June 2007), during the financial crisis (July 2007 - March 15th 2009) and after the financial crisis (March 16th, 2009 - February 19th, 2016). A comparison of the figures shows how trading days, characterized by a higher level of the conditional volatility and variations for U.S. dollars denominated corporate bond yields, that are not small in terms of size, report lower levels of the conditional illiquidity, in case of computations based on continuously compounded variations.

A dramatic decline of the conditional measure of illiquidity for U.S. dollars denominated corporate bond yields is observed around the U.S. financial crisis period, with a particular decrease around the Bankruptcy of Lehman Brothers (September 15th, 2008) and a further reduction in the last quarter of 2008, following a further deterioration of the conditional level of illiquidity, around the first quarter of 2009, for all U.S. dollars denominated corporate bond yields, with the exclusion of those with ratings AA, AAA. The dynamics of the conditional level of illiquidity fluctuate during the great sovereign debt crisis and decline around the third and the last quarters of 2011 as well as around the second and the third quarters of 2013.

The *flight-to-liquidity* and the *flight-to-quality* effects explain the general tendency of the investors to allocate capital to safer investments, due to a variation of the bid-ask spreads and a change of the quality for several U.S. dollars denominated corporate bonds. These phenomena create a change of the risk aversion and are particularly evident, during the financial crisis and the post-financial crisis. The dynamic of the conditional illiquidity for corporate bond yields also explains the *clientele effect* (Amihud and Mendelson 1986, 1991), with *more liquid and risky* U.S. dollars denominated corporate bonds allocated to short-term investors and *less liquid and risky* U.S. dollars denominated corporate bonds allocated to long-term investors.

In general, the concave function between the expected return on U.S. dollars denominated corporate bonds and the bid-ask spreads, is also related to the dynamic of the conditional serial covariance. This relation reflects the *clientele effect*, with illiquid corporate bonds that are less frequently traded and transaction costs that are amortized over longer periods. The longer is the holding period, the smaller is the compensation required by investors for a given increase of the bid-ask spread. In equilibrium, higher spreads for U.S dollars denominated corporate bonds tend to be acquired by investors with longer horizons, with the added return required for a given increase of the spread, becoming smaller.

6.1 Metrics of Market Illiquidity Risk: an empirical discussion

This sub-section discusses the estimation of the metrics, proposed in sub-section 3.1, for managing the illiquidity risk, based on the distributions of the conditional level of illiquidity for U.S. dollars denominated corporate bond yields. The main purpose of such metrics is to identify the dates of extreme liquidity and illiquidity, that might create implications in terms of losses and profits and evaluate the amount of capital that is at risk in an investment decision as well as the loss incurred, provided a certain percentile. Table 4 respectively reports the estimated VaR and ES, for the 1st and 5th percentiles (the left tail of the distributions for the conditional illiquidity), as well as for the 99th and 50th percentiles (the right tail of the distributions for the conditional illiquidity).

[Please Insert Table 4 around here]

The value at risk and the expected shortfall, evaluated at the 1st and the 5th percentiles of the distributions, respectively identify the dates with lower values of conditional liquidity, for high risky U.S. dollars denominated corporate bonds and higher values of conditional liquidity, for low risky U.S. corporate bonds, considering the period before the financial crisis (BC); whereas, the 99th and the 50th percentiles of the distributions identify higher levels of conditional illiquidity, for low risky U.S. corporate bonds and lower levels of conditional illiquidity, for high risky U.S. corporate bonds.

During the financial crisis (FC), the metrics of risk computed for the *left (right)* tails of the distributions, respectively identify higher levels of conditional (il)-liquidity for A, AAA, B and CCC or below U.S. corporate bonds and lower levels of conditional (il)-liquidity for AA, BB and BBB U.S. dollars denominated corporate bond yields; whereas, the period after the financial crisis reports mixed results, concerned about the VaR and ES, for the conditional illiquidity of U.S. corporate bond yields.

[Please Insert Table 5 around here]

Table 5 respectively reports the estimates of the *Conditional Illiquidity - Marginal Expected Shortfall (MES)* for the left and the right tails of the distributions, related to the conditional illiquidity for U.S. dollars denominated corporate bond yields. The construction of the distributions, realized before the financial crisis, allows to respectively identify a greater level of conditional (il)-liquidity for low risky U.S. corporate bonds and a lower level of conditional (il)-liquidity for high risky U.S. corporate bonds, during the days of extreme conditional (il)-liquidity of the index.

During the financial crisis, the *flight-to-liquidity* phenomenon allows to identify a greater level of conditional liquidity for AAA, B and CCC or below U.S. dollars denominated corporate bonds and a lower level of conditional liquidity for A, AA, BB and BBB U.S. corporate bonds; whereas, the period from March 16th, 2009 to February 19th, 2016 shows a higher level of conditional liquidity for A, AA, AAA, B and CCC or below U.S. dollars denominated corporate bonds, for those days of extreme conditional liquidity.

6.2 The Conditional Illiquidity Market Model

This subsection discusses the conditional illiquidity market model, showing the statistical relation between the conditional illiquidity for each category of U.S. dollars denominated corporate bond yields and the conditional illiquidity for the portfolio of U.S. corporate bond yields (LAG_Illiquidity_Portfolio). Table 6 reports a positive and statistically significant value of the coefficient, concerned about LAG_Illiquidity_Portfolio, implying a raise of the conditional illiquidity for each category of U.S. corporate bond yields. The estimation procedure relies on the Newey-West estimator (Newey and West 1987) of the covariance matrix that considers the auto-correlation and the heteroskedasticity of the error terms in the regression framework.

[Please Insert Table 6 around here]

The estimated values of the unknown parameter for LAG_Illiquidity_Portfolio are respectively equal to 2.352 and 0.973, for AAA and CCC or below U.S. dollars denominated corporate bond yields; whereas, the constants of the regressions are positive and statistically significant, for low risky U.S. corporate bonds, turning to negative and statistically significant, for high risky U.S. corporate bonds.

Table 7 corroborates the estimation results, reporting the statistics of the quantile regressions, with the aim to relate the extreme days of conditional liquidity and illiquidity, respectively identified by the 1st and the 99th percentiles of the distributions for the conditional illiquidity of U.S. dollars denominated corporate bond yields, with the conditional illiquidity for the portfolio of U.S. corporate bond yields. The estimation of the covariance matrix relies on the extention of Markov Chain Marginal Bootstrap algorithm (MCMB), proposed by He and Hu (2002) and called MCMB-A (Kocherginsky et al. 2005).

The estimation procedure of the quantile regressions relies on n. 100000 replications, pseudo-random generated with the Mersenne Twister (Matsumoto and Nushimura 1997) algorithm. The sparsity method relies on the Epanechnikov kernel using the residuals, with a quantile function based on a Gumbel distribution and the Hall-Sheather (1998) bandwidth method, characterized by a parameter equals to 0.0041073.

[Please Insert Table 7 around here]

The estimated values of the coefficients for the covariate LAG_Illiquidity_Portfolio, related to the quantile regressions for the left tails of the distributions, are greater for low risky U.S. corporate bonds, ranging from 3.733 for AAA U.S. corporate bond yields to 1.429 for BBB U.S. corporate bond yields and are smaller than 1.22, for high risky U.S. corporate bonds. The quantile regressions, performed on the right tails of the distributions for the conditional illiquidity, report a higher value of the coefficient for A, AA, AAA and BBB U.S. dollars denominated corporate bonds and a lower value, for high risky U.S. corporate bonds, characterized by higher default risk.

6.3 The implications of financial and economic variables with the conditional illiquidity for U.S. corporate bond yields

This subsection describes the statistical findings between the financial as well as the economic variables and the conditional illiquidity for U.S. corporate bond yields, with the aim to test some stylized facts. For the period from January 1997 to February 2016, Table 8 and Table 9 report the estimated values of the regressions, showing a positive relation between the variable GOLD and the conditional illiquidity for U.S. corporate bond yields, with a low default risk and a negative relation with the conditional illiquidity, for high risky U.S. corporate bond yields.

The levels of statistical significance are greater, in absolute terms, for B, BB and CCC or below U.S. corporate bonds and smaller for A, AA, AAA and BBB U.S. corporate bonds, respectively reflecting the standard deviations for U.S. corporate bond yields, that are greater for high risky corporate bonds and smaller for low risky ones and the computation of the t-statistical values, related to the unknown parameters.

[Please Insert Table 8 and Table 9 around here]

GOLD as well as many precious metals represent one of the most popular investment for diversifying risk and also used by many investors for hedging purposes. An increase of the gold price generally describes periods of dramatic declines for the stock markets and the tendency of the investors to allocate capital to more liquid investments, such as high yield U.S. corporate bonds, rather than low yield U.S. corporate bonds. The empirical findings seem counter-intuitive, since a reader would expect a greater allocation of capital to U.S. corporate bonds, that are safer and characterized by a low default risk rather than high yield U.S. corporate bonds. The signs of the statistical relations are mainly driven by the financial crisis period, with the *flight-to-liquidity* and the *flight-to-quality* phenomena particularly relevant, during the extreme days of conditional illiquidity for U.S. stock exchanges, with investors allocating capital to high risky U.S. corporate bonds.

The analysis also reports the statistical implications for the “fear index” that is a measure of market perceived volatility, for the downside and the upside of the S&P500 Index. The behavior of the investors to anticipate large upside volatility means they are generally unwilling to sell upside call options unless they receive a large premium; whereas, option buyers are willing to pay such high premiums, only if, similarly anticipating a large upside move, with the aim to compensate the risk allocated to it. The combined effect, concerned about the increases in upside option call prices and the aggregate growth in downside put option premiums that occur when option buyers and sellers anticipate a likely sharp move to the downside, raises The CBOE Volatility Index (VIX). This financial implication, related to a general decrease of the conditional illiquidity for U.S. corporate bond yields, is significant and characterized by higher t-statistic values, for low risky U.S. corporate bonds and smaller t-statistic values, for high risky U.S. corporate bonds, implying a stronger statistical implication for the first category of U.S. corporate bonds and a weaker implication for the second one.

The perception of the credit risk for the U.S. economy, broadly summarized by the TED spread, is negatively related with the dynamics of the conditional illiquidity, for all categories of U.S. dollars denominated corporate bond yields. This relation is statistically significant for A, AA, AAA and B categories of U.S. corporate bonds and it is not significant for BB, BBB and CCC or below U.S. corporate bonds. An exacerbation of the counterparty risk, with interbank lenders demanding a higher rate of interest or willing to accept a lower return on safer investments, such as U.S. Treasury Bills, is related to a raise of the conditional liquidity for U.S. corporate bonds.

Considering several factors of risk for an allocation of capital, determined by the application of several covariates, a general raise of the credit risk perception for the U.S. economy creates an increase of the conditional illiquidity for U.S. corporate bond yields, characterized by high default risk; conversely, a reduction of the credit risk, measured as a function of the counterparty risk, determines a deterioration of the conditional illiquidity, for high risky U.S. corporate bond yields. The change of the statistical signs, mainly for the conditional illiquidity related to high risky U.S. corporate bond yields, can be explained in terms of the *flight-to-quality* phenomenon, with investors preferring to allocate capital to financial securities with a better quality, moving their capital away from riskier investments, particularly during the financial crisis.

The empirical analysis also reports the statistical results concerned about the relation between the CBOE Skew Index (SKEW) and the conditional illiquidity for U.S. corporate bond yields. An increase of the slope for the implied volatility curve increases as this curve tends to steepen, determining an increase of the conditional illiquidity, for low risky U.S. corporate bonds and an increase of the conditional liquidity, for high risky U.S. corporate bonds. In particular, the relation is strongly significant with the conditional illiquidity for high risky U.S. corporate bonds. Dramatic variations experienced by the S&P 500 Index, particularly during the days of extreme (il)-liquidity, increase the weights of the tails and the potential associated asymmetric jumps skew the distribution for the arithmetic (or natural logarithmic) variations of the stock market prices. A value of SKEW equals to 100 identifies a normal distribution for the S&P 500 Index changes; whereas, a value of SKEW equals to 150 identifies a non-normal distribution, with comparable probabilities that are approximated by adding a skewness term to the normal distribution. The S&P 500 Index variations identify unimodal distributions with a negative skewness, indicating a longer or a fatter tail on the left side of the probability densities than on the right side, along several periods of time; whereas, a positive skewness indicates a longer or a fatter tail on the right side, than on the left side of the distributions.

In cases, where one tail is long but the other tail is fat, the level of the skewness is computed accordingly.

The empirical findings also discuss the relations between the conditional illiquidity for U.S. dollars denominated corporate bond yields with the covariate REC, able to identify U.S. business cycle expansions and contractions. The results corroborate the expectations of the readers, with the conditional liquidity for U.S. corporate bond yields that increases during recession periods, characterized by the tendency of the Federal Open Market Committee (FOMC) and the Federal Reserve Bank to decide accommodative monetary policies, with the aim to decrease the Federal funds rates, making money cheaper for business to borrow and stimulating the economic growth by loosening money supply. The conditional liquidity for U.S. corporate bond yields decreases during expansionary periods, that represent those phases of the business cycles, when the economy moves from a period with the level of business activity surging and gross domestic product expanding, until it reaches a peak. This relation is statistically significant across all ratings and vanishes for A, BBB U.S. dollars denominated corporate bonds, when the statistical relations also consider other financial covariates.

The analysis also discusses the Fama-French five factors, with the aim to explain the conditional illiquidity for U.S. dollars denominated corporate bond yields, across all ratings. The empirical findings are weak and unable to justify the statistical relations with the factors HML, SMB, RMW, CMA as well as with the difference between the excess return of the market (MKT), computed as the value-weighted return for all CRSP firms incorporated in the U.S. and listed on the NYSE, AMEX and NASDAQ, with respect to the benchmark interest rate (BR); whereas, the constants of the regressions related to the conditional illiquidity for high risky and BBB U.S. corporate bonds are negative and statistically significant.

7. Concluding remarks

The remarks proposed in the academic literature and in consultation reports, with bid-ask spreads narrowing and the impact of trades on prices of U.S. corporate bonds continuing to fall, are time-varying and depend on the examined period. The paper enriches the recent debate pertaining the liquidity for U.S. corporate bonds in the secondary markets, proposing a framework based on the dynamics of the j-th order serial conditional covariances. The methodology, accommodating for the changes of the unobserved yield for a corporate bond, caused by the arrival of new information and the transaction costs incurred in making the exchange of it, derives several metrics of illiquidity risk management, able to identify the days and the related values, characterized by a high level of conditional liquidity and illiquidity.

The construction of a conditional illiquidity market model, corroborated by a quantile regression analysis, shows the adequacy of the empirical findings proposed by the financial literature and allows to study the implications pertaining the *flight-to-liquidity* and the *flight-to-quality* phenomena, particularly relevant during the periods characterized by an increase of the volatility for the stock markets, with a general tendency of the investors to allocate capital to safer investments and more liquid U.S. dollars denominated corporate bonds, due to a variation of the bid-ask spreads and a change of the quality for several U.S. dollars denominated corporate bonds. These phenomena create a change of the risk aversion for the investors and are particularly evident, during the financial crisis and the post-financial crisis.

The dynamics of the conditional illiquidity for U.S. corporate bond yields also explain the *clientele effect*, with *more liquid and risky* U.S. dollars denominated corporate bonds allocated to short-term investors and *less liquid and risky* U.S. dollars denominated corporate bonds, allocated to long-term investors.

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Table 1.

Summary and Descriptive Statistics

The table reports the summary and descriptive statistics for the following U.S. corporate and high yield bonds rated: **A, AA, AAA, B, BB, BBB** as well as **CCC or below**. In particular, the table shows the mean, median, min., max. and the standard deviations for the following (sub)-periods: (i) The time period from **01/01/1997** to **02/19/2016**, defined as the entire period (**ALL**); (ii) The time period that spans from 01/01/1997 to 06/29/2007, defined as the period before the financial crisis (**BC**); (iii) The time period that spans from 07/01/2007 to 03/15/2009, defined as the financial crisis period (**FC**); (iv) The time period that spans from 03/16/2009 to 02/19/2016, defined as the post-financial crisis (**AFC**). The data are downloaded from *The Bank of America Merrill Lynch* database. The statistics are reported in percentage values.

CREDIT RATING	**Mean**				**Median**				**Min.**				**Max.**				**Std. Dev.**			
	ALL	**BC**	**FC**	**AFC**	**ALL**	**BC**	**FC**	**AFC**	**ALL**	**BC**	**FC**	**AFC**	**ALL**	**BC**	**FC**	**AFC**	**ALL**	**BC**	**FC**	**AFC**
A	5.050	5.856	6.631	3.439	5.150	5.920	6.075	3.070	2.300	3.310	5.320	2.300	9.630	8.250	9.630	8.050	1.641	1.121	1.096	1.019
AA	4.576	5.481	5.834	2.894	4.670	5.510	5.675	2.650	1.930	2.730	4.810	1.930	8.020	8.020	7.970	6.760	1.648	1.234	0.668	0.788
AAA	4.473	5.515	5.297	2.691	4.620	5.460	5.150	2.610	1.500	3.350	4.290	1.500	8.250	7.760	8.250	6.950	1.623	1.042	0.688	0.723
B	9.270	9.896	12.333	7.569	8.640	9.550	10.635	7.150	5.030	6.670	7.950	5.030	23.070	15.440	23.070	18.240	2.783	2.179	4.101	2.064
BB	7.434	7.960	9.836	6.046	7.330	7.820	8.500	5.650	4.130	5.520	7.200	4.130	16.410	11.590	16.410	13.580	2.001	1.320	2.729	1.678
BBB	5.828	6.548	7.255	4.385	5.865	6.710	6.560	4.090	3.160	4.430	5.920	3.160	10.230	8.930	10.230	9.700	1.565	1.040	1.373	1.136
CCC or below	15.148	16.512	18.778	12.192	12.720	13.730	14.770	11.240	7.910	9.050	9.820	7.910	45.020	29.200	45.020	34.160	5.949	5.624	9.084	3.836

Table 2.

Auto-Correlation and Partial Auto-Correlation

The table reports the Auto-Correlation (**AC**) and the Partial Auto-Correlation (**PAC**) for U.S. corporate and high yield bonds rated **A, AA, AAA, B, BB, BBB, CCC or below** for the period that spans from **01/01/1997 to 02/19/2016**. The table shows the first 5 lags for computing the **AC** and the **PAC** as well as the corresponding Q-statistics that lead to the Probability values.

CREDIT RATING		Arithmetic Variations			Logarithmic Variations		
	LAGS	AC	PAC	Q-stat.	AC	PAC	Q-stat.
A	**1**	0.014	0.014	0.980	0.013	0.013	0.841
	2	-0.036	-0.037	7.604	-0.037	-0.037	7.602
	3	0.023	0.025	10.364	0.023	0.024	10.321
	4	0.005	0.003	10.483	0.004	0.002	10.422
	5	0.003	0.005	10.530	0.003	0.004	10.461
AA	**1**	-0.002	-0.002	0.012	-0.003	-0.003	0.032
	2	-0.044	-0.044	9.683	-0.045	-0.045	10.015
	3	0.018	0.018	11.318	0.018	0.018	11.619
	4	0.005	0.003	11.421	0.004	0.002	11.693
	5	-0.005	-0.003	11.538	-0.005	-0.004	11.829
AAA	**1**	0.022	0.022	2.357	0.019	0.019	1.842
	2	-0.047	-0.047	13.252	-0.047	-0.047	12.940
	3	0.022	0.024	15.728	0.022	0.024	15.297
	4	-0.016	-0.019	16.941	-0.016	-0.019	16.531
	5	0.005	0.008	17.061	0.006	0.009	16.699
B	**1**	0.300	0.300	451.39	0.301	0.301	452.45
	2	0.189	0.109	630.37	0.188	0.107	629.15
	3	0.151	0.075	744.57	0.150	0.075	741.29
	4	0.127	0.054	824.65	0.125	0.053	819.77
	5	0.086	0.015	861.75	0.085	0.015	855.83
BB	**1**	0.238	0.238	283.64	0.238	0.238	282.14
	2	0.117	0.063	351.56	0.114	0.061	347.43
	3	0.069	0.030	375.47	0.066	0.028	369.32
	4	0.075	0.049	403.30	0.073	0.049	396.24
	5	0.062	0.030	422.67	0.061	0.029	414.61
BBB	**1**	0.029	0.029	4.203	0.028	0.028	4.015
	2	-0.009	-0.009	4.569	-0.009	-0.010	4.415
	3	0.022	0.022	6.979	0.021	0.022	6.719
	4	0.005	0.004	7.117	0.005	0.004	6.844
	5	0.011	0.012	7.769	0.011	0.011	7.448
CCC or below	**1**	0.227	0.227	257.62	0.227	0.227	258.74
	2	0.145	0.099	362.83	0.144	0.098	363.15
	3	0.123	0.075	438.18	0.123	0.076	438.83
	4	0.097	0.047	485.28	0.096	0.046	485.34
	5	0.072	0.025	511.43	0.072	0.025	511.25

Table 3.
Estimation Results: Mean Equation and Conditional Variance/Covariance Matrix

The table shows the estimated coefficients for the **Panel Diagonal BEKK(1,1)** specification, with a disturbance assumption based on a multivariate ***t-student***, able to depict the dynamics of the first order serial covariances. **Panel 3.1** shows the coefficients $\beta , \varepsilon , \eta , \iota , \mu , o , \rho$ that represent the constants of the mean equations for the **CONTEMPORANEOUS** variations. The diagonal coefficients for the long period conditional variance/covariance matrix are equal to: **m2_2, m5_5, m8_8, m11_11, m14_14, m17_17** and **m20_20.** The diagonal estimated coefficients for the residuals are equal to: **a2_2, a5_5, a8_8, a11_11, a14_14, a17_17** and **a20_20.** The diagonal estimated coefficients for the persistence of the conditional variance/covariance matrix are equal to **b2_2, b5_5, b8_8, b11_11, b14_14, b17_17** and **b20_20. Panel 3.2** shows the coefficients $\alpha , \delta , \zeta , \upsilon , \lambda , \xi , \varpi$ that represent the constants of the mean equations for the **LAG** variations. The diagonal coefficients for the long period conditional variance/covariance matrix are equal to: **m1_1, m4_4, m7_7, m10_10, m13_13, m16_16** and **m19_19.** The estimated diagonal coefficients for the residuals are equal to: **a1_1, a4_4, a7_7, a10_10, a13_13, a16_16** and **a19_19.** The estimated diagonal coefficients for the persistence of the conditional variance/covariance matrix are equal to **b1_1, b4_4, b7_7, b10_10, b13_13, b16_16** and **b19_19. t** is the estimated number for the degrees of freedom. The optimization algorithm relies on the *Berndt-Hall-Hall-Hausman* (B-H-H-H) procedure and the jointly estimated coefficients consider the period that spans from **01/01/1997** to **02/19/2016**. The last column reports the Probability value **(Prob.)**, related to the z-statistics.

Panel 3.1**:** Estimation Results for the *CONTEMPORANEOUS* variations

Coefficient	**Estimated Value**		**Standard Error**		**Prob.**	
	ARITH.	**LOG.**	**ARITH.**	**LOG.**	**ARITH.**	**LOG.**
β	-0.000186	-0.000168	6.55E-05	6.56E-05	0.0045	0.0107
ε	-0.000172	-0.000155	6.96E-05	6.98E-05	0.0135	0.0262
η	-0.000190	-0.000173	6.88E-05	6.90E-05	0.0059	0.0123
ι	-0.000363	-0.000360	4.95E-05	4.94E-05	0.0000	0.0000
μ	-0.000344	-0.000332	4.72E-05	4.72E-05	0.0000	0.0000
o	-0.000190	-0.000169	5.77E-05	5.78E-05	0.0010	0.0034
ρ	-0.000289	-0.000291	6.40E-05	6.40E-05	0.0000	0.0000
m2_2	1.90E-08	1.85E-08	2.22E-09	2.18E-09	0.0000	0.0000
m5_5	1.80E-08	1.74E-08	2.71E-09	2.67E-09	0.0000	0.0000
m8_8	1.89E-08	1.85E-08	3.16E-09	3.08E-09	0.0000	0.0000
m11_11	2.17E-08	2.14E-08	4.76E-09	4.75E-09	0.0000	0.0000
m14_14	3.87E-08	3.88E-08	5.40E-09	5.38E-09	0.0000	0.0000
m17_17	1.31E-08	1.27E-08	3.00E-09	2.95E-09	0.0000	0.0000
m20_20	3.45E-08	3.45E-08	8.66E-09	8.64E-09	0.0001	0.0001
a2_2	0.184703	0.182604	0.004108	0.004066	0.0000	0.0000
a5_5	0.192230	0.190814	0.004312	0.004266	0.0000	0.0000
a8_8	0.189924	0.188079	0.004236	0.004187	0.0000	0.0000
a11_11	0.106768	0.106980	0.002709	0.002711	0.0000	0.0000
a14_14	0.121079	0.121197	0.002983	0.002985	0.0000	0.0000
a17_17	0.175762	0.174383	0.004074	0.004036	0.0000	0.0000
a20_20	0.092911	0.092954	0.002401	0.002397	0.0000	0.0000
b2_2	0.992025	0.992093	0.000177	0.000176	0.0000	0.0000
b5_5	0.991340	0.991406	0.000195	0.000194	0.0000	0.0000
b8_8	0.991485	0.991586	0.000189	0.000189	0.0000	0.0000
b11_11	0.996203	0.996181	0.000129	0.000130	0.0000	0.0000
b14_14	0.995234	0.995206	0.000145	0.000147	0.0000	0.0000
b17_17	0.992713	0.992773	0.000182	0.000182	0.0000	0.0000
b20_20	0.996754	0.996725	0.000119	0.000120	0.0000	0.0000

Panel 3.2: Estimation Results for the *LAG* variations

Coefficient	**Estimated Value**		**Standard Error**		**Prob.**	
	ARITH.	**LOG.**	**ARITH.**	**LOG.**	**ARITH.**	**LOG.**
α	-0.000225	-0.000208	6.56E-05	6.58E-05	0.0006	0.0016
δ	-0.000241	-0.000226	7.00E-05	7.02E-05	0.0006	0.0013
ζ	-0.000233	-0.000218	6.93E-05	6.95E-05	0.0008	0.0017
υ	-0.000436	-0.000435	5.08E-05	5.07E-05	0.0000	0.0000
λ	-0.000385	-0.000374	4.84E-05	4.84E-05	0.0000	0.0000
ξ	-0.000237	-0.000217	5.80E-05	5.81E-05	0.0000	0.0002
ϖ	-0.000360	-0.000362	6.46E-05	6.46E-05	0.0000	0.0000
m1_1	2.07E-08	2.01E-08	2.32E-09	2.27E-09	0.0000	0.0000
m4_4	1.94E-08	1.87E-08	2.86E-09	2.81E-09	0.0000	0.0000
m7_7	2.32E-08	2.26E-08	3.44E-09	3.34E-09	0.0000	0.0000
m10_10	2.28E-08	2.23E-08	4.87E-09	4.85E-09	0.0000	0.0000
m13_13	3.58E-08	3.58E-08	5.44E-09	5.41E-09	0.0000	0.0000
m16_16	1.09E-08	1.04E-08	3.15E-09	3.09E-09	0.0005	0.0007
m19_19	2.64E-08	2.63E-08	8.36E-09	8.31E-09	0.0016	0.0015
a1_1	0.189027	0.187066	0.004225	0.004173	0.0000	0.0000
a4_4	0.195820	0.193869	0.004375	0.004317	0.0000	0.0000
a7_7	0.193186	0.190836	0.004330	0.004273	0.0000	0.0000
a10_10	0.104741	0.104826	0.002663	0.002660	0.0000	0.0000
a13_13	0.119818	0.119804	0.002929	0.002826	0.0000	0.0000
a16_16	0.178963	0.177155	0.004159	0.004110	0.0000	0.0000
a19_19	0.092650	0.092564	0.002384	0.002378	0.0000	0.0000
b1_1	0.991707	0.991809	0.000183	0.000182	0.0000	0.0000
b4_4	0.991112	0.991217	0.000197	0.000196	0.0000	0.0000
b7_7	0.991270	0.991408	0.000196	0.000195	0.0000	0.0000
b10_10	0.996372	0.996359	0.000126	0.000127	0.0000	0.0000
b13_13	0.995450	0.995431	0.000139	0.000140	0.0000	0.0000
b16_16	0.992573	0.992661	0.000186	0.000185	0.0000	0.0000
b19_19	0.996886	0.996865	0.000114	0.000115	0.0000	0.0000
t	2.721339	2.731359	0.035647	0.036060	0.0000	0.0000

Table 4.

Metrics of Market Illiquidity Risk for U.S. Corporate Bond Yields

The table respectively reports the Value at Risk (**VaR**) and the Expected Shortfall (**ES**) for the conditional distributions related to the illiquidity for the following U.S. corporate bond yields: **A, AA, AAA, B, BB, BBB, CCC or below** as well as an **Index** for U.S. Corporate Bond Yields (**CB**), estimated at 1st and 5th percentiles and based on natural ***logarithmic variations***. The **Index for CB** is the equally weighted average of the values for U.S. corporate bond yields. The values are reported for the following sub-periods: (i) The period before the financial crisis (**BC**) that spans from 01/01/1997 to 06/29/2007; (ii) The financial crisis period (**FC**) that spans from 07/01/2007 to 03/15/2009; (iii) the period after the financial crisis (**AFC**) that spans from 03/16/2009 to 02/19/2016. The data are downloaded from *The Bank of America Merrill Lynch* database. **Panel 4.1** respectively depicts the **VaR** and the **ES** for the 1st and 5th percentiles, concerned about the **LEFT** tail of the conditional distributions; whereas, **Panel 4.2** respectively depicts the **VaR** and the **ES** for the 99th and 50th percentiles (**RIGHT** tail of the conditional distributions).

Panel 4.1: The LEFT tail of the conditional distributions

CREDIT RATING	**VaR**						**ES**					
	1%			**5%**			**1%**			**5%**		
	BC	**FC**	**AFC**	**BC**	**FC**	**AFC**	**BC**	**FC**	**AFC**	**BC**	**FC**	**AFC**
A	-3.993E-5	-1.503E-4	-7.076E-5	-3.359E-5	-1.265E-4	-5.952E-5	-6.003E-5	-1.563E-4	-1.035E-4	-5.260E-5	-1.481E-5	-7.849E-5
AA	-4.926E-5	-1.070E-4	-8.634E-5	-4.143E-5	-9.005E-5	-7.263E-5	-7.086E-5	-1.248E-4	-1.223E-4	-6.470E-5	-1.023E-4	-1.033E-4
AAA	-4.726E-5	-2.114E-4	-2.685E-4	-3.976E-5	-1.778E-4	-2.259E-4	-6.425E-5	-5.633E-4	-4.645E-4	-5.835E-5	-5.633E-4	-4.496E-4
B	-2.433E-5	-1.407E-4	-6.269E-5	-2.047E-5	-1.184E-4	-5.273E-5	-3.080E-5	-1.635E-4	-9.008E-5	-2.896E-5	-1.625E-4	-7.945E-5
BB	-2.534E-5	-1.005E-4	-4.571E-5	-2.131E-5	-8.453E-5	-3.845E-5	-3.312E-5	-1.146E-4	-5.993E-5	-3.008E-5	-1.110E-4	-5.557E-5
BBB	-3.099E-5	-7.645E-4	-4.998E-5	-2.607E-5	-6.431E-5	-4.204E-5	-4.006E-5	-7.645E-5	-7.586E-5	-3.836E-5	-6.998E-5	-6.347E-5
CCC or below	-1.765E-5	-1.276E-4	-9.450E-5	-1.485E-5	-1.073E-4	-7.949E-5	-2.186E-5	-1.761E-4	-1.395E-4	-2.041E-5	-1.631E-4	-1.257E-4
Index for CB	-2.462E-5	-1.209E-4	-7.750E-5	-2.071E-5	-1.017E-4	-6.520E-5	-3.522E-5	-1.414E-4	-1.277E-4	-3.227E-5	-1.161E-4	-1.226E-4

Panel 4.2: The RIGHT tail of the conditional distributions

CREDIT RATING	**VaR**						**ES**					
	99%			**50%**			**99%**			**50%**		
	BC	**FC**	**AFC**	**BC**	**FC**	**AFC**	**BC**	**FC**	**AFC**	**BC**	**FC**	**AFC**
A	3.993E-5	1.503E-4	7.076E-5	1.155E-5	4.349E-5	2.047E-5	5.539E-5	**1.503E-4**	**7.076E-5**	2.219E-5	5.605E-5	3.454E-5
AA	4.926E-5	1.070E-4	8.634E-5	1.425E-5	3.097E-5	2.497E-5	6.016E-5	**1.070E-4**	**8.634E-5**	2.900E-5	4.320E-5	4.860E-5
AAA	4.726E-5	2.114E-4	2.685E-4	1.367E-5	6.115E-5	7.768E-5	5.438E-5	**2.114E-4**	**2.685E-4**	2.487E-5	6.893E-5	1.021E-4
B	2.433E-5	1.407E-4	6.269E-5	7.039E-6	4.070E-5	1.813E-5	**2.433E-5**	**1.407E-4**	**6.269E-5**	**7.039E-6**	**4.070E-5**	**1.813E-5**
BB	2.534E-5	1.005E-4	4.571E-5	7.329E-6	2.907E-5	1.322E-5	**2.534E-5**	**1.005E-4**	**4.571E-5**	**7.329E-6**	**2.907E-5**	**1.322E-5**
BBB	3.099E-5	7.645E-5	4.998E-5	8.964E-6	2.212E-5	1.446E-5	3.546E-5	**7.645E-5**	**4.998E-5**	1.754E-5	3.350E-5	2.485E-5
CCC or below	1.765E-5	1.276E-4	9.450E-5	5.107E-6	3.690E-5	2.734E-5	1.765E-5	**1.276E-4**	**9.450E-5**	**5.107E-6**	**3.690E-5**	**2.734E-5**
Index for CB	2.462E-5	1.209E-4	7.750E-5	7.121E-6	3.497E-5	2.242E-5	2.626E-5	1.209E-4	7.750E-5	1.281E-5	3.497E-5	2.569E-5

Table 5.
The Conditional Illiquidity-MES for U.S. Corporate Bond Yields

The table reports the *Conditional Illiquidity-Marginal Expected Shortfall* (MES) for the following U.S. corporate bond yields: **A, AA, AAA, B, BB, BBB** as well as **CCC or below**, estimated for the 1st and 5th percentiles and based on ***logarithmic variations***. The values are reported for the following sub-periods: (i) The period before the financial crisis (**BC**) that spans from 01/01/1997 to 06/29/2007; (ii) The financial crisis period (**FC**) that spans from 07/01/2007 to 03/15/2009; (iii) the period after the financial crisis (**AFC**) that spans from 03/16/2009 to 02/19/2016. The data are downloaded from *The Bank of America Merrill Lynch* database. **Panel 5.1** respectively depicts The Conditional Illiquidity-MES for the 1st and the 5th percentiles (**LEFT** tail of the conditional distributions); whereas, **Panel 5.2** respectively depicts the *Conditional Illiquidity-MES* for the 99th and the 50th percentiles (**RIGHT** tail of the conditional distributions).

Panel 5.1: The LEFT tail of the conditional distributions

CREDIT RATING	**The Conditional Illiquidity-MES**					
	1%			**5%**		
	BC	**FC**	**AFC**	**BC**	**FC**	**AFC**
A	-4.573E-5	-1.453E-4	-1.172E-4	-4.197E-5	-1.249E-4	-9.046E-5
AA	-4.812E-5	-1.303E-4	-1.123E-4	-4.548E-5	-1.248E-4	-9.228E-5
AAA	-4.950E-5	-2.233E-4	-2.551E-4	-4.697E-5	-1.569E-4	-2.369E-4
B	-3.094E-5	-1.629E-4	-1.029E-4	-2.906E-5	-1.615E-4	-9.250E-5
BB	-3.243E-5	-1.251E-4	-8.227E-5	-2.980E-5	-1.146E-4	-7.605E-5
BBB	-3.742E-5	-1.209E-4	-9.994E-5	-3.504E-5	-1.017E-4	-8.803E-5
CCC or below	-2.654E-5	-1.761E-4	-1.243E-4	-2.365E-5	-1.535E-4	-1.103E-4

Panel 5.2: The RIGHT tail of the conditional distributions

CREDIT RATING	**The Conditional Illiquidity-MES**					
	99%			**50%**		
	BC	**FC**	**AFC**	**BC**	**FC**	**AFC**
A	3.404E-5	**1.209E-4**	**7.750E-5**	1.704E-5	4.951E-5	3.560E-5
AA	3.901E-5	**1.209E-4**	9.315E-5	1.956E-5	4.944E-5	4.661E-5
AAA	3.513E-5	**1.209E-4**	1.017E-4	1.722E-5	4.986E-5	5.672E-5
B	**2.462E-5**	**1.209E-4**	**7.750E-5**	**7.121E-6**	**3.497E-5**	**2.242E-5**
BB	**2.462E-5**	**1.209E-4**	**7.750E-5**	**7.121E-6**	**3.497E-5**	**2.242E-5**
BBB	3.317E-5	**1.209E-4**	**7.750E-5**	1.482E-5	4.435E-5	3.003E-5
CCC or below	**2.462E-5**	**1.209E-4**	**7.750E-5**	**7.121E-6**	**3.497E-5**	**2.242E-5**

Table 6.
The Conditional Illiquidity Market Model

The table shows the relation between the conditional illiquidity for U.S. corporate bond yields rated **A**, **AA**, **AAA** and **BBB** and the conditional illiquidity for an index of U.S. Corporate Bond Yields (**LAG_Illiquidity_Portfolio**), computed as the equally weighted average of conditional illiquidity measures for U.S. Corporate Bond Yields, with ratings **A, AA, AAA, B, BB, BBB** and **CCC or below**. Monday, Tuesday, Wednesday, Thursday and Friday represent the control variables of the framework. The estimation procedure considers the period from 01/01/1997 to 02/19/2016. The statistical significances at 1%, 5% and 10%, are respectively indicated with *, **, ***. The values of the t- statistics consider infinite degrees of freedom.

Panel 6.1: The Conditional Illiquidity Market Model for U.S. Corporate Bond Yields, rated **A**, **AA**, **AAA**, **BBB**

Covariates	**U.S. Corporate Bond Effective Yields**			
	A	**AA**	**AAA**	**BBB**
c	9.41E-06*** (4.141)	1.38E-05*** (5.411)	3.30E-05*** (5.637)	1.07E-06 (0.579)
LAG_Illiquidity_Portfolio	0.899*** (12.226)	0.871*** (15.463)	2.352*** (10.308)	0.481*** (9.384)
Monday	5.26E-07 (0.259)	-3.40E-07 (-0.140)	-7.05E-06 (-1.412)	1.24E-06 (0.728)
Tuesday	5.44E-07 (0.268)	-3.14E-07 (-0.129)	-6.82E-06 (-1.377)	1.26E-06 (0.739)
Wednesday	4.89E-07 (0.241)	-4.22E-07 (-0.173)	-6.80E-06 (-1.380)	1.39E-06 (0.812)
Thursday	4.83E-07 (0.236)	-5.75E-07 (-0.236)	-7.53E-06 (-1.505)	1.28E-06 (0.749)
Friday	3.72E-07 (0.184)	-5.05E-07 (-0.208)	-6.93E-06 (-1.394)	1.22E-06 (0.718)
Adj-R^2	**66.20%**	**55.99%**	**70.16%**	**45.98%**

Panel 6.2: The Conditional Illiquidity Market Model for U.S. Corporate Bond Yields, rated **B**, **BB**, **CCC or below**

Covariates	**U.S. Corporate Bond Effective Yields**		
	B	**BB**	**CCC or below**
c	-2.10E-05*** (-7.859)	-1.82E-05*** (-7.491)	-1.81E-05*** (-5.963)
LAG_Illiquidity_Portfolio	0.870*** (16.808)	0.555*** (13.061)	0.973*** (17.294)
Monday	2.43E-06 (0.942)	2.04E-06 (0.899)	1.15E-06 (0.390)
Tuesday	2.37E-06 (0.922)	1.90E-06 (0.841)	1.06E-06 (0.360)
Wednesday	2.37E-06 (0.923)	1.92E-06 (0.853)	1.05E-06 (0.358)
Thursday	2.78E-06 (1.080)	2.26E-06 (1.004)	1.30E-06 (0.439)
Friday	2.52E-06 (0.982)	2.06E-06 (0.914)	1.26E.06 (0.429)
Adj-R^2	**60.02%**	**41.55%**	**54.74%**

Table 7.

The days of extreme conditional LIQUIDITY and ILLIQUIDITY for U.S. Corporate Bond Yields

The table shows the relation between the conditional illiquidity for U.S. Corporate Bond Yields, during days of extreme conditional liquidity, with the conditional illiquidity for an Index of U.S. Corporate Bond Yields (**LAG_Illiquidity_Portfolio**), computed as the equally weighted average of conditional illiquidity measures for U.S. Corporate Bond Yields, with the following ratings: **A**, **AA**, **AAA**, **B**, **BB**, **BBB** and **CCC or below**. The empirical analysis is developed for the period between 01/01/1997 and 02/19/2016. The statistical significances, at **1%**, **5%** and **10%**, are respectively indicated with *, **, ***. The values of the t-statistics consider infinite degrees of freedom. The estimation of the covariance matrix relies on an extension of the **Markov Chain Marginal Bootstrap** (MCMB-A) algorithm, proposed by Kocherginsky et al. (2005), with n. 100000 replications and a pseudo-random generator that relies on The Mersenne Twister (Matsumoto and Nushimura 1997). The sparsity method relies on the Epanechnikov kernel, using the residuals and a Gumbel distribution quantile function. The bandwith method is Hall-Sheather (1998), with a parameter that is equal to 0.0041073.

Panel 7.1: The days of extreme conditional **LIQUIDITY** for U.S. Corporate Bond Yields, rated **A, AA, AAA, B, BB, BBB** and **CCC or below (Quantile regression, 0.01)**

	Rating of U.S. Corporate Bonds						
Covariates	**A**	**AA**	**AAA**	**B**	**BB**	**BBB**	**CCC or below**
c	6.87E-07 (0.356)	1.93E-06 (0.282)	-1.41E-05 (-0.174)	-7.18E-05 (-1.482)	-6.21E-05*** (-17.980)	-9.97E-06 (-0.317)	-9.73E-05 (-1.620)
LAG_Illiquidity_Portfolio	1.734*** (64.119)	2.026*** (165.260)	3.733*** (124.815)	1.026*** (60.706)	0.579*** (32.777)	1.429*** (72.914)	1.220*** (43.502)
Monday	-8.25E-07 (-0.402)	-1.37E-06 (-0.199)	-1.27E-05 (-0.156)	2.36E-06 (0.049)	-3.99E-06 (-1.013)	2.25E-07 (0.007)	1.08E-05 (0.179)
Tuesday	-6.79E-07 (-0.328)	-1.23E-06 (-0.179)	-1.14E-05 (-0.140)	3.06E-06 (0.063)	-4.97E-06 (-1.209)	4.98E-07 (0.016)	1.00E-05 (0.167)
Wednesday	-4.68E-07 (-0.230)	-1.25E-06 (-0.182)	-9.49E-06 (-0.117)	3.52E-06 (0.073)	-3.74E-06 (-0.945)	1.23E-06 (0.039)	1.21E-05 (0.202)
Thursday	-5.49E-07 (-0.262)	-1.14E-06 (-0.166)	-1.11E-05 (-0.137)	4.42E-06 (0.091)	-1.80E-07 (-0.049)	1.03E-06 (0.033)	1.06E-05 (0.176)
Friday	-5.47E-07 (-0.277)	-9.59E-07 (-0.136)	-1.27E-05 (-0.152)	5.40E-06 (0.108)	-2.91E-06 (-0.785)	1.39E-06 (0.043)	1.16E-05 (0.187)
Adj-R^2	**76.03%**	**73.04%**	**85.62%**	**62.53%**	**45.13%**	**61.73%**	**50.71%**

Panel 7.2: The days of extreme conditional **ILLIQUIDITY** for U.S. Corporate Bond Yields, rated **A, AA, AAA, B, BB, BBB** and **CCC or below (Quantile regression, 0.99)**

	Rating of U.S. Corporate Bond Yields						
Covariates	**A**	**AA**	**AAA**	**B**	**BB**	**BBB**	**CCC or below**
c	4.01E-05 (0.891)	5.24E-05*** (9.054)	1.23E-04 (0.914)	-2.52E-06 (-0.712)	3.53E-06*** (3.753)	2.19E-05*** (5.086)	2.21E-06 (0.343)
LAG_Illiquidity_Portfolio	0.662*** (70.639)	0.840*** (71.797)	1.780*** (33.935)	0.247*** (25.074)	0.369*** (36.145)	0.303*** (35.655)	0.355*** (42.657)
Monday	7.56E-06 (0.168)	1.73E-05*** (2.653)	-7.98E-06 (-0.059)	7.07E-07 (0.200)	2.06E-07 (0.200)	1.17E-05** (2.500)	-5.41E-07 (-0.084)
Tuesday	7.48E-06 (0.166)	1.78E-05*** (2.704)	-8.92E-06 (-0.066)	7.08E-07 (0.200)	1.97E-07 (0.192)	1.13E-05** (2.481)	-2.43E-07 (-0.038)
Wednesday	7.18E-06 (0.160)	1.71E-05** (2.632)	-8.29E-06 (-0.062)	7.32E-07 (0.207)	8.06E-09 (0.008)	1.11E.05** (2.284)	-3.71E-07 (-0.058)
Thursday	7.85E-06 (0.174)	1.80E-05*** (2.834)	-1.15E-05 (-0.085)	7.47E-07 (0.211)	5.08E-07 (0.531)	1.28E-05*** (2.662)	-3.74E-07 (-0.058)
Friday	6.41E-06 (0.138)	1.45E-05** (2.144)	-9.30E-06 (-0.067)	7.03E-07 (0.193)	2.28E-07 (0.252)	9.88E-06** (2.188)	-3.76E-07 (-0.056)
Adj-R^2	**34.20%**	**27.95%**	**21.03%**	**6.85%**	**12.82%**	**14.80%**	**13.01%**

Table 8.

The conditional illiquidity measure for U.S. Corporate Bond Yields vs. Financial and Economic variables

The table shows the statistical relation between the conditional illiquidity measure for U.S. Corporate Bond Yields, rated **A**, **AA**, **AAA** and **BBB** with financial and economic variables, such as: (i) **GOLD** is the London Bullion Market Association Gold Price; (ii) **TED** is the spread between 3-month LIBOR based on US dollars and 3-month U.S. Treasury Bills, multiplied by 100; (iii) **VIX** measures the implied volatility of S&P500 index options, calculated and published by the Chicago Board Options Exchange (CBOE); (iv) **SKEW** is The CBOE SKEW Index derived from the prices of S&P500 out-of-money options and ranges from 100 to 150; (v) **REC** is an interpretation of US Business Cycle Expansions and Contractions Data. A value of 1 is a recession period; whereas, a value of 0 is an expansionary period; (iv) **MKT-BR** is the excess return of the market (**MKT**), computed as the value-weighted return for all CRSP firms incorporated in the U.S. and listed on the NYSE, AMEX or NASDAQ, with respect to the benchmark interest rate (**BR**); (v) **HML** (High Minus Low) is the average return on the two "**value**" portfolios minus the average return on the two "**growth**" portfolios; (vi) **SMB** (Small Minus Big) is the average return on the nine "**small stock**" portfolios minus the average return on the nine "**big stock**" portfolios; (vii) **RMW** (Robust Minus Weak) is the average return on the two "**robust operating profitability**" portfolios minus the average return on the two "**weak operating profitability**" portfolios; (viii) **CMA** (Conservative Minus Aggressive) is the average return on the two "**conservative investment**" portfolios minus the average return on the two "**aggressive investment**" portfolios. The portfolios are based on the stock performance for all CRSP firms incorporated in the U.S. and listed on the NYSE, AMEX and NASDAQ. (ix) **Cal. Effect** takes a value equals to **Y**, if the dummy variables are considered for the trading days. The statistical significances, at **1%**, **5%** and **10%**, are respectively indicated with *, **, ***. The values of the t- statistics consider infinite degrees of freedom.

Panel 8.1**:** The conditional illiquidity measure for **A** U.S. Corporate Bonds Effective Yield

Cov.	**A**							
c	-1.15E-05** (-2.568)	1.06E-06 (0.213)	2.20E-05*** (3.386)	5.14E-05* (1.803)	5.48E-05* (1.917)	-5.65E-06 (-1.285)	-5.65E-06 (-1.286)	-5.63E-06 (-1.281)
GOLD	7.87E-09*** (3.563)	4.07E-09* (1.670)	3.09E-09 (1.213)	4.44E-09 (1.581)	4.97E-09* (1.777)			
TED		-2.16E-05*** (-3.828)	-1.08E-05** (-2.534)	-1.10E-05*** (-2.626)	-9.47E-06** (-2.334)			
VIX			-1.17E-06*** (-4.961)	-1.21E-06*** (-4.912)	-1.12E-06*** (-4.598)			
SKEW				-2.47E-07 (-1.137)	-2.94E-07 (-1.347)			
REC					-8.06E-06 (-1.442)			
MKT-BR						1.45E-05 (0.264)	2.46E-05 (0.413)	1.95E-05 (0.340)
HML						1.39E-04 (1.194)	1.33E-04 (1.081)	1.49E-04 (0.981)
SMB						-4.15E-05 (-0.508)	-2.65E-05 (-0.300)	-2.29E-05 (-0.254)
RMW							5.84E-05 (0.458)	6.63E-05 (0.536)
CMA								-5.40E-05 (-0.353)
Cal. Effect	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**
Adj-R^2	**1.40%**	**10.12%**	**18.12%**	**18.32%**	**18.80%**	**0.00%**	**0.00%**	**0.00%**

Panel 8.2**:** The conditional illiquidity measure for **AA** U.S. Corporate Bonds Effective Yield

Cov.	**AA**							
c	-1.48E-05*** (-3.223)	-9.95E-06** (-1.989)	2.98E-06 (0.438)	4.54E-06 (0.159)	1.09E-05 (0.380)	-6.65E-07 (-0.151)	-6.64E-07 (-0.151)	-6.21E-07 (-0.141)
GOLD	1.87E-08*** (6.773)	1.73E-08*** (5.938)	1.67E-08*** (5.423)	1.68E-08*** (4.993)	1.77E-08*** (5.261)			
TED		-8.30E-06** (-1.980)	-1.65E-06 (-0.487)	-1.66E-06 (-0.488)	1.20E-06 (0.362)			
VIX			-7.24E-07*** (-3.102)	-7.26E-07*** (-3.023)	-5.52E-07** (-2.399)			
SKEW				-1.31E-08 (-0.059)	-9.99E-08 (-0.440)			
REC					-1.49E-05*** (-2.713)			
MKT-BR						-4.60E-05 (-1.014)	-5.33E-05 (-1.109)	-6.52E-05 (-1.409)
HML						2.43E-05 (0.242)	2.88E-05 (0.278)	6.68E-05 (0.542)
SMB						-1.68E-04** (-2.279)	-1.79E-04** (-2.237)	-1.71E-04** (-2.103)
RMW							-4.23E-05 (-0.412)	-2.39E-05 (-0.228)
CMA								-1.27E-04 (-0.901)
Cal. Effect	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**
Adj-R^2	**7.64%**	**8.79%**	**11.53%**	**11.52%**	**13.04%**	**0.00%**	**0.00%**	**0.04%**

Panel 8.3: The conditional illiquidity measure for **AAA** U.S. Corporate Bonds Effective Yield

Cov.	**AAA**							
c	-2.27E-05*** (-2.946)	-6.67E-06 (-0.722)	4.09E-05** (2.225)	1.77E-05 (0.433)	4.93E-05 (1.181)	-5.93E-06 (-0.776)	-5.93E-06 (-0.776)	-6.02E-06 (-0.788)
GOLD	2.20E-08*** (4.382)	1.71E-08*** (3.052)	1.49E-08** (2.349)	1.38E-08* (1.835)	1.87E-08*** (2.728)			
TED		-2.76E-05*** (-3.244)	-3.10E-06 (-0.595)	-2.92E-06 (-0.560)	1.12E-05 (1.636)			
VIX			-2.66E-06*** (-3.730)	-2.63E-06*** (-3.812)	-1.77E-06*** (-3.485)			
SKEW				1.95E-07 (0.502)	-2.34E-07 (-0.648)			
REC					-7.35E-05*** (-3.286)			
MKT-BR						-1.90E-04 (-1.391)	-2.02E-04 (-1.367)	-1.76E-04 (-1.365)
HML						-2.79E-04 (-0.800)	-2.71E-04 (-0.767)	-3.53E-04 (-0.787)
SMB						-3.83E-04* (-1.845)	-4.00E-04 (-1.600)	-4.19E-04 (-1.640)
RMW							-6.92E-05 (-0.285)	-1.09E-04 (-0.450)
CMA								2.73E-04 (0.723)
Cal. Effect	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**
Adj-R^2	**1.71%**	**3.91%**	**10.30%**	**10.30%**	**16.74%**	**0.09%**	**0.08%**	**0.07%**

Panel 8.4**:** The conditional illiquidity measure for **BBB** U.S. Corporate Bonds Effective Yield

Cov.	**BBB**							
c	-1.15E-05*** (-3.907)	-1.18E-05*** (-3.479)	-2.73E-07 (-0.068)	2.58E-05 (1.340)	2.39E-05 (1.245)	-7.00E-06** (-2.458)	-7.00E-06** (-2.460)	-6.98E-06** (-2.450)
GOLD	6.14E-09*** (3.688)	6.23E-09*** (3.534)	5.69E-09*** (3.169)	6.88E-09*** (3.285)	6.58E-09*** (3.161)			
TED		5.13E-07 (0.153)	6.47E-06** (2.510)	6.27E-06** (2.472)	5.42E-06** (2.188)			
VIX			-6.49E-07*** (-4.877)	-6.84E-07*** (-4.865)	-7.37E-07*** (-5.167)			
SKEW				-2.19E-07 (-1.467)	-1.93E-07 (-1.299)			
REC					4.44E-06 (1.474)			
MKT-BR						-4.07E-05 (-1.350)	-2.87E-05 (-0.896)	-3.42E-05 (-1.119)
HML						1.02E-04* (1.695)	9.46E-05 (1.499)	1.12E-04 (1.469)
SMB						-4.98E-05 (-1.078)	-3.19E-05 (-0.634)	-2.80E-05 (-0.539)
RMW							6.96E-05 (0.968)	7.82E-05 (1.049)
CMA								-5.88E-05 (-0.603)
Cal. Effect	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**
Adj-R^2	**2.14%**	**2.13%**	**8.08%**	**8.47%**	**8.82%**	**0.07%**	**0.07%**	**0.06%**

Table 9.

The conditional illiquidity measure for U.S. HIGH YIELD Corporate Bonds vs. Financial and Economic variables

The table shows the statistical relation between the conditional illiquidity measure for U.S. High Yield Corporate Bonds, rated **B**, **BB** and **CCC or below** with financial and economic variables, such as: (i) **GOLD** is the London Bullion Market Association Gold Price; (ii) **TED** is the spread between 3-month LIBOR based on US dollars and 3-month Treasury Bills; (iii) **VIX** measures the implied volatility of S&P500 index options, calculated and published by the Chicago Board Options Exchange (CBOE); (iv) **SKEW** is The CBOE SKEW Index derived from the prices of S&P500 out-of-money options and ranges from 100 to 150; (v) **REC** is an interpretation of US Business Cycle Expansions and Contractions Data. A value of 1 is a recession period; whereas, a value of 0 is an expansionary period; (iv) **MKT-BR** is the excess return of the market (**MKT**), computed as the value-weighted return of all CRSP firms incorporated in the US and listed on the NYSE, AMEX or NASDAQ, with respect to the benchmark interest rate (**BR**); (v) **HML** (High Minus Low) is the average return on the two "**value**" portfolios minus the average return on the two "**growth**" portfolios; (vi) **SMB** (Small Minus Big) is the average return on the nine "**small stock**" portfolios minus the average return on the nine "**big stock**" portfolios; (vii) **RMW** (Robust Minus Weak) is the average return on the two "**robust operating profitability**" portfolios minus the average return on the two "**weak operating profitability**" portfolios; (viii) **CMA** (Conservative Minus Aggressive) is the average return on the two "**conservative investment**" portfolios minus the average return on the two "**aggressive investment**" portfolios. The portfolios are based on the stock performance for all CRSP firms incorporated in the US and listed on the NYSE, AMEX and NASDAQ. (ix) (ix) **Cal. Effect** takes a value equals to **Y**, if the dummy variables for the trading days are considered. The statistical significances, at 1%, 5% and 10%, are respectively indicated with *, **, ***. The values of the t- statistics considers infinite degrees of freedom.

Panel 9.1**:** The conditional illiquidity measure for **B U.S. HIGH YIELD Corporate Bonds**

Cov.	**B**							
c	-1.76E-05*** (-4.092)	-6.81E-06 (-1.592)	3.64E-05*** (7.345)	7.42E-05*** (4.351)	8.64E-05*** (5.332)	-3.56E-05*** (-7.787)	-3.56E-05*** (-7.784)	-3.56E-05*** (-7.775)
GOLD	-2.37E-08*** (-13.185)	-2.70E-08*** (-13.199)	-2.90E-08*** (-15.004)	-2.73E-08*** (-13.098)	-2.54E-08*** (-13.081)			
TED		-1.86E-05*** (-3.706)	3.60E-06* (1.825)	3.30E-06* (1.708)	8.75E-06*** (4.156)			
VIX			-2.42E-06*** (-11.812)	-2.47E-06*** (-11.846)	-2.14E-06*** (-12.881)			
SKEW				-3.18E-07** (-2.402)	-4.84E-07*** (-3.789)			
REC					-2.84E-05*** (-5.961)			
MKT-BR						-1.10E-06 (-0.019)	-5.78E-06 (-0.093)	-1.11E-05 (-0.194)
HML						1.97E-04 (1.362)	2.00E-04 (1.343)	2.17E-04 (1.187)
SMB						-4.31E-05 (-0.534)	-5.01E-05 (-0.572)	-4.63E-05 (-0.521)
RMW							-2.72E-05 (-0.218)	-1.90E-05 (-0.157)
CMA								-5.61E-05 (-0.354)
Cal. Effect	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**
Adj-R^2	**13.22%**	**19.54%**	**52.65%**	**52.98%**	**58.99%**	**0.04%**	**0.02%**	**0.01%**

Panel 9.2: The conditional illiquidity measure for **BB U.S. HIGH YIELD Corporate Bonds**

Cov.	**BB**							
c	-9.34E-06*** (-3.011)	-5.21E-06 (-1.515)	2.22E-05*** (5.650)	6.72E-05*** (4.078)	7.06E-05*** (4.313)	-2.76E-05*** (-8.250)	-2.76E-05*** (-8.250)	-2.76E-05*** (-8.239)
GOLD	-2.41E-08*** (-13.898)	-2.54E-08*** (-13.337)	-2.66E-08*** (-15.406)	-2.46E-08*** (-13.399)	-2.41E-08*** (-13.165)			
TED		-7.11E-06* (-1.762)	7.03E-06*** (3.990)	6.68E-06*** (3.941)	8.17E-06*** (4.609)			
VIX			-1.54E-06*** (-10.827)	-1.60E-06*** (-10.914)	-1.51E-06*** (-10.995)			
SKEW				-3.79E-07*** (-2.962)	-4.24E-07*** (-3.338)			
REC					-7.79E-06** (-2.418)			
MKT-BR						-1.90E-05 (-0.489)	-1.08E-05 (-0.264)	-1.30E-05 (-0.339)
HML						1.72E-04* (1.843)	1.67E-04* (1.739)	1.74E-04 (1.489)
SMB						-3.21E-06 (-0.051)	9.01E-06 (0.137)	1.05E-05 (0.158)
RMW							4.76E-05 (0.494)	5.09E-05 (0.540)
CMA								-2.28E-05 (-0.200)
Cal. Effect	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**
Adj-R^2	**23.30%**	**24.85%**	**47.55%**	**48.36%**	**49.12%**	**0.10%**	**0.09%**	**0.07%**

Panel 9.3: The conditional illiquidity measure for **CCC or below U.S. HIGH YIELD Corporate Bonds**

Cov.	**CCC or below**							
c	-1.06E-05** (-2.357)	-3.77E-06 (-0.854)	3.90E-05*** (7.101)	2.78E-05 (1.560)	3.78E-05** (2.073)	-3.44E-05*** (-6.864)	-3.44E-05*** (-6.865)	-3.44E-05*** (-6.865)
GOLD	-3.16E-08*** (-15.949)	-3.36E-08*** (-16.351)	-3.56E-08*** (-17.415)	-3.62E-08*** (-15.904)	-3.46E-08*** (-15.513)			
TED		-1.17E-05*** (-3.057)	1.03E-05*** (4.205)	1.04E-05*** (4.189)	1.49E-05*** (5.089)			
VIX			-2.40E-06*** (-9.857)	-2.38E-06*** (-9.704)	-2.11E-06*** (-10.567)			
SKEW				9.42E-08 (0.654)	-4.15E-08 (-0.275)			
REC					-2.33E-05*** (-3.362)			
MKT-BR						-3.16E-05 (-0.514)	-3.68E-05 (-0.542)	-4.44E-05 (-0.700)
HML						1.73E-04 (1.092)	1.77E-04 (1.097)	2.01E-04 (1.048)
SMB						-5.86E-05 (-0.637)	-6.64E-05 (-0.651)	-6.10E-05 (-0.594)
RMW							-3.02E-05 (-0.229)	-1.85E-05 (-0.140)
CMA								-8.06E-05 (-0.478)
Cal. Effect	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**	**Y**
Adj-R^2	**17.14%**	**18.93%**	**42.58%**	**42.59%**	**45.54%**	**0.00%**	**0.00%**	**0.00%**

Figure 1.

The dynamics of U.S. Corporate Bond Yields

The figure shows the dynamics of the yields for the following categories of U.S. corporate bond yields: **A, AA, AAA, B, BB, BBB** as well as **CCC or below**. The data are downloaded from *The Bank of America Merrill Lynch* database and the figure reports the values from **01/01/1997** to **02/19/2016**.

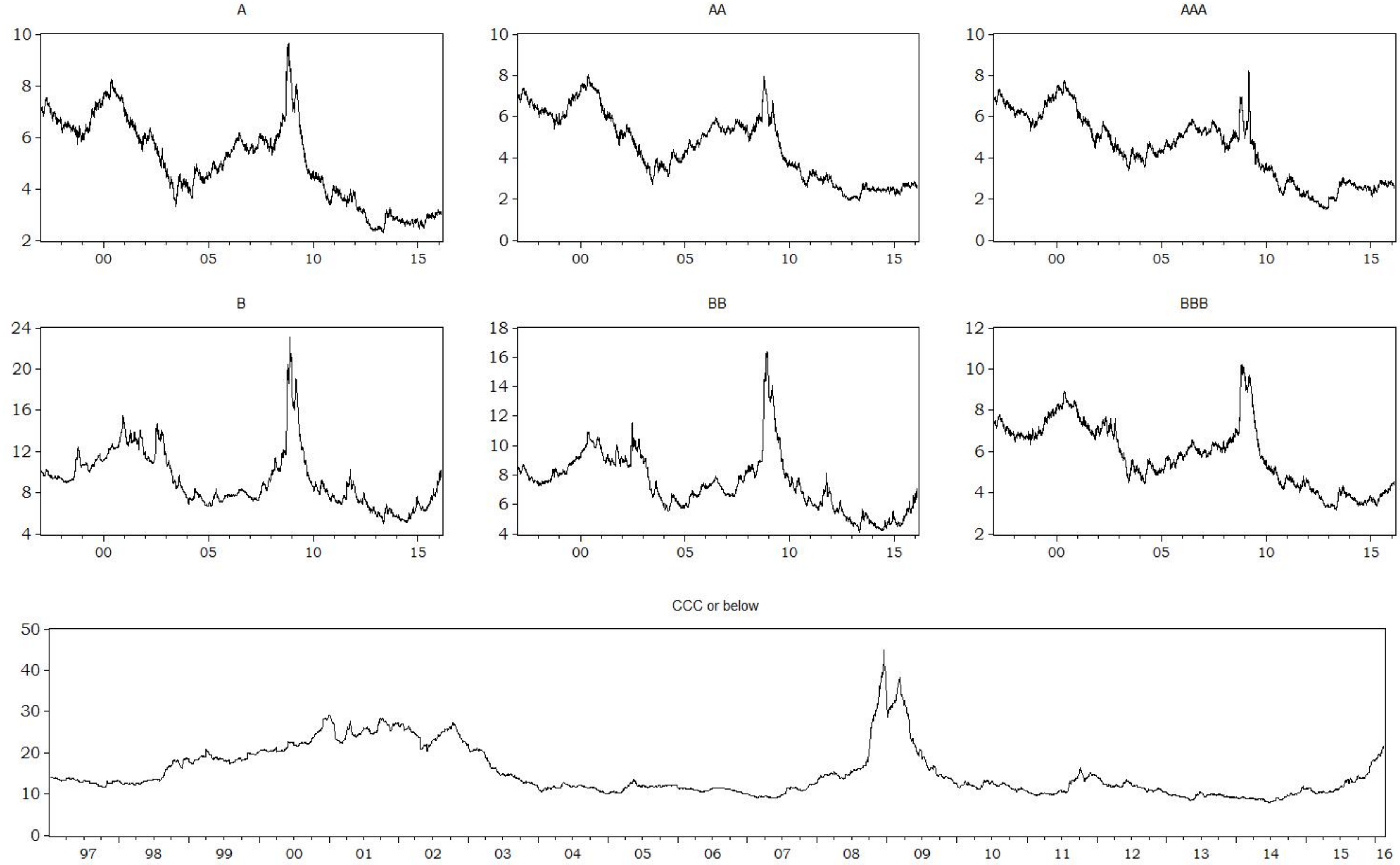
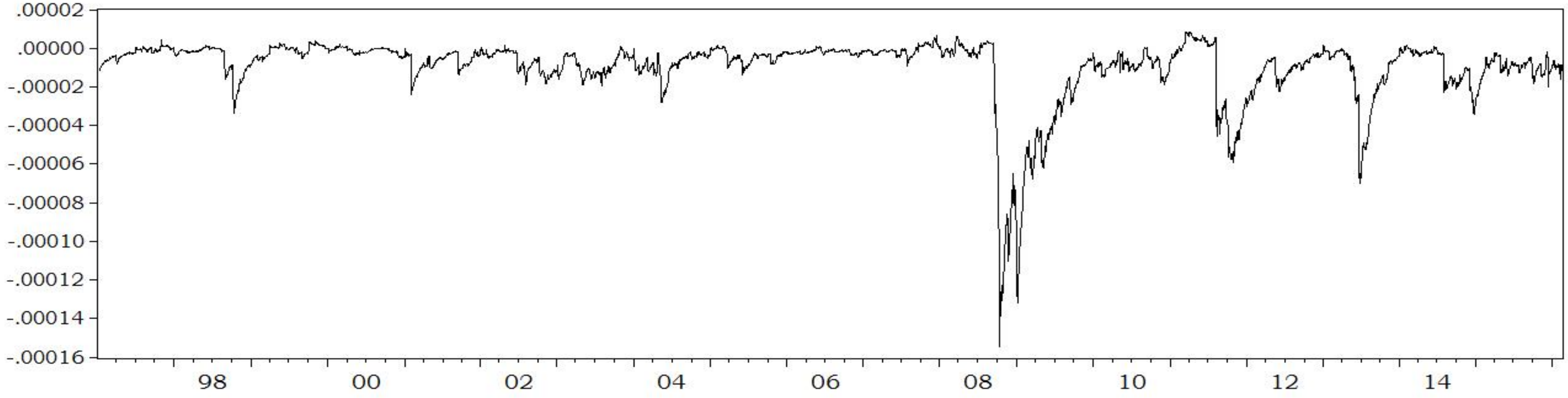
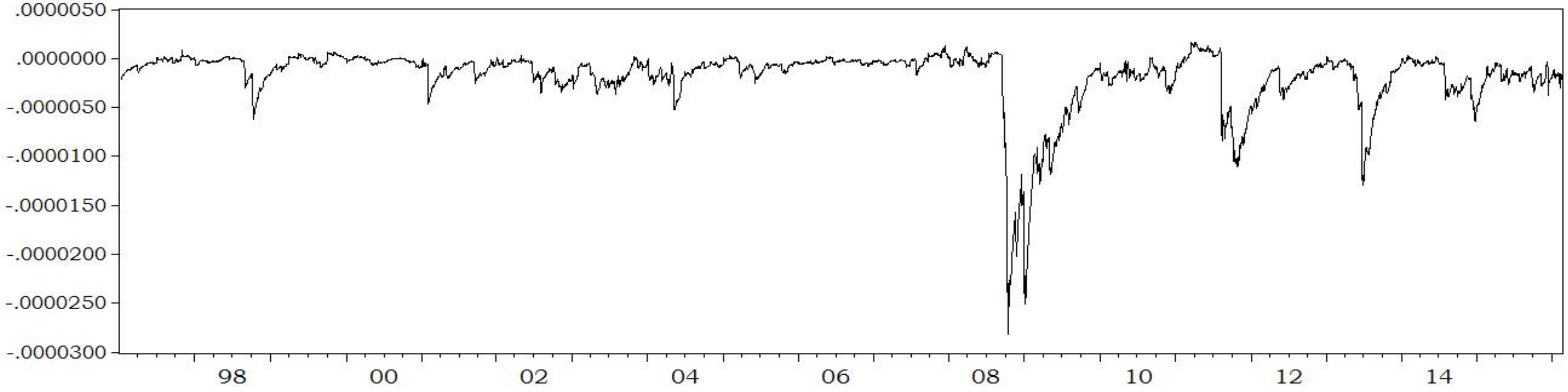


Figure 2.
The Conditional Illiquidity Index for U.S. Corporate Bond Yields

The figure shows the dynamics of **The Conditional Illiquidity Index for U.S. Corporate Bond Yields**. The Index is the equally weighted average across U.S. Corporate Bond Yields with the following credit ratings: **A, AA, AAA, B, BB, BBB** as well as **CCC or below.** The data are downloaded from *The Bank of America Merrill Lynch* database and the figure reports the estimated values from **01/01/1997** to **02/19/2016**.



The Conditional Illiquidity Index for Corporate Bonds
(arithmetic variations - LAG)

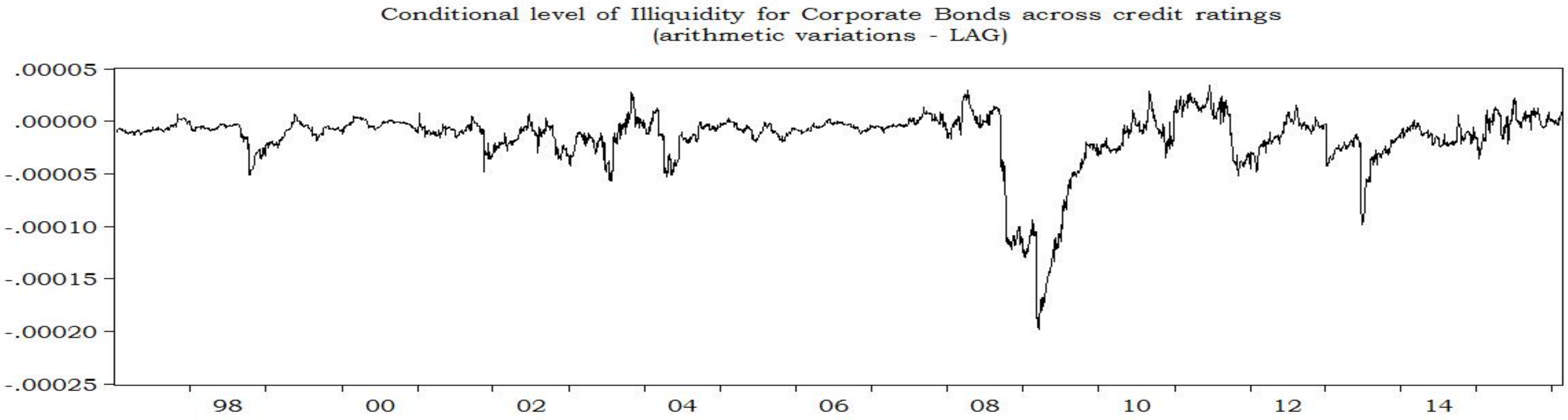


The Conditional Illiquidity Index for Corporate Bonds
(logarithmic variations - LAG)

Figure 3.

Conditional level of Illiquidity for U.S. Corporate Bond Yields, across credit ratings

The figure shows the dynamics of the conditional level of illiquidity for U.S. Corporate Bond Yields, across the following credit ratings: **A, AA, AAA, B, BB, BBB** as well as **CCC or below**. The index is computed as the equally weighted average related to the conditional illiquidity for each category of U.S. corporate bond yields. The data are downloaded from *The Bank of America Merrill Lynch* database and the figure reports the estimated values from **01/01/1997** to **02/19/2016**.



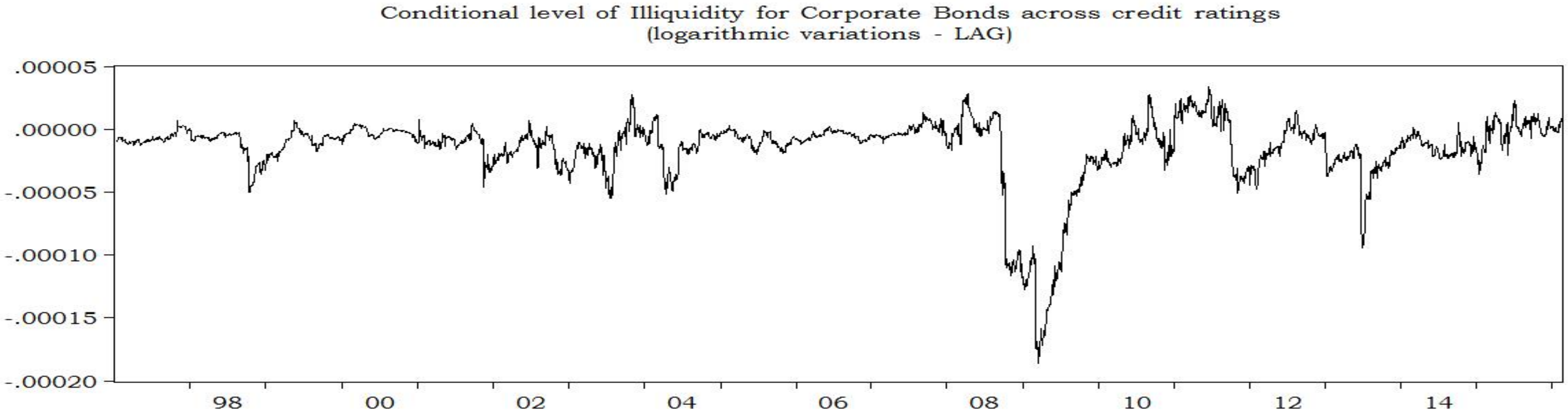


Figure 4.

Conditional level of Illiquidity for U.S. Corporate Bond Yields (arithmetic variations)

The figure shows the dynamics of the conditional levels of illiquidity for the following categories of U.S. corporate bond yields: **A, AA, AAA, B, BB, BBB** as well as **CCC or below**. **Figure 4.1** shows the conditional level of illiquidity for U.S. corporate bond yields, BEFORE THE FINANCIAL CRISIS (**BC**). **Figure 4.2** shows the conditional level of illiquidity for U.S. corporate bond yields, DURING THE FINANCIAL CRISIS (**FC**). **Figure 4.3** shows the conditional level of illiquidity for U.S. corporate bond yields, AFTER THE FINANCIAL CRISIS (**PFC**). The data are downloaded from *The Bank of America Merrill Lynch* database and the figures report the estimated values from **01/01/1997** to **02/19/2016**.

Figure 4.1: The conditional level of illiquidity, based on *ARITHMETIC* variations (BEFORE THE FINANCIAL CRISIS)

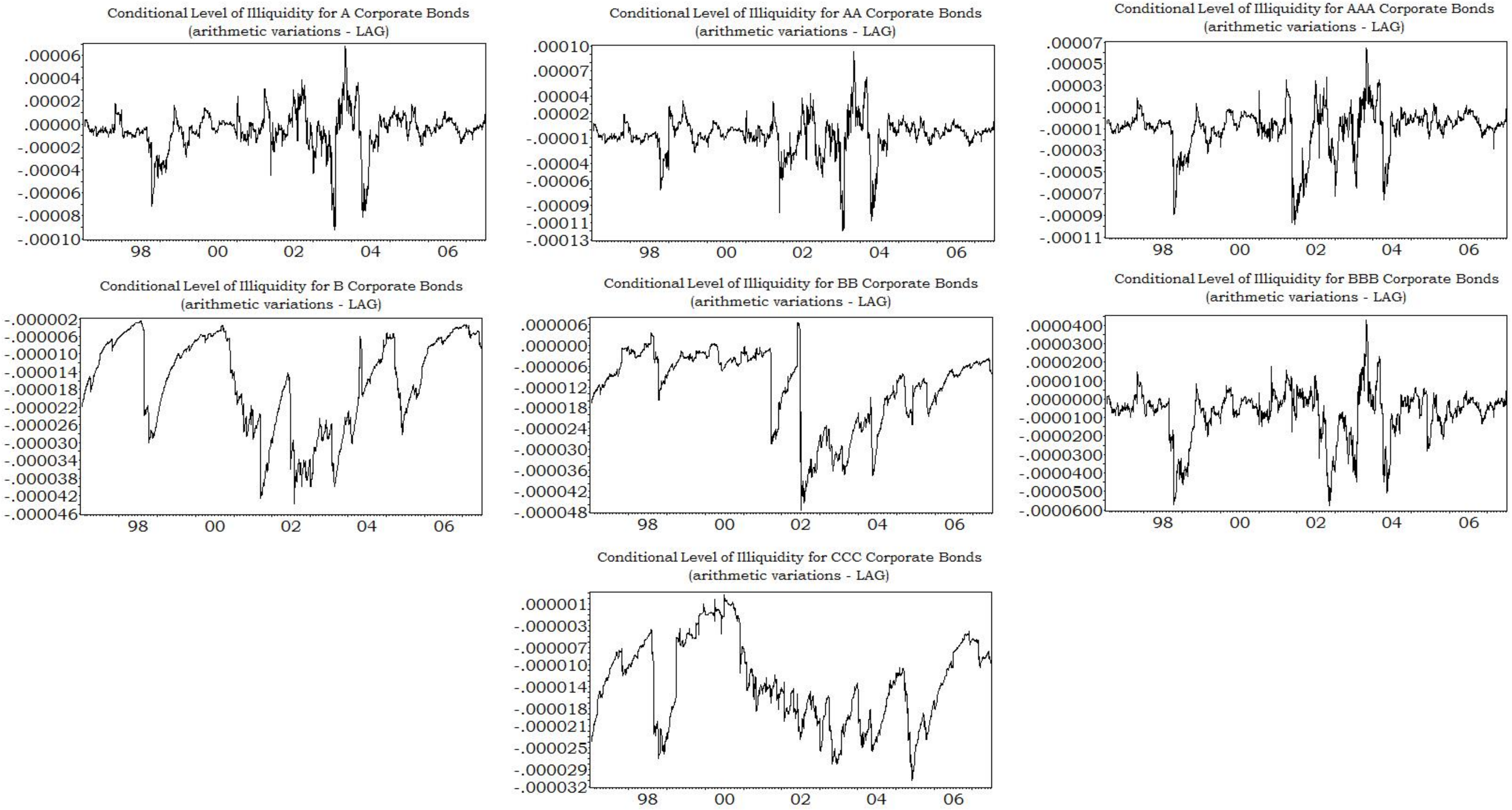


Figure 4.2: The conditional level of illiquidity, based on *ARITHMETIC* variations (DURING THE FINANCIAL CRISIS)

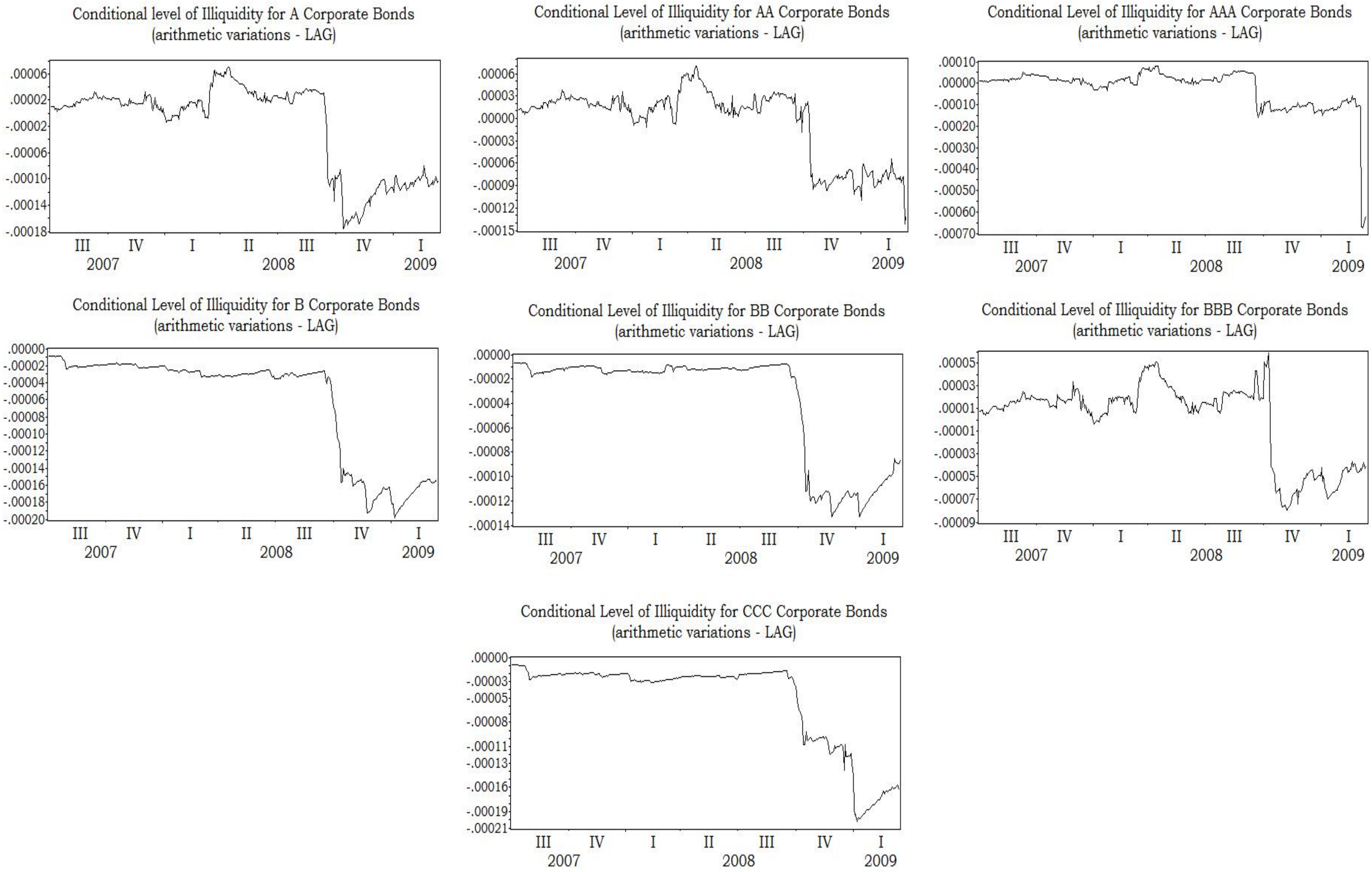


Figure 4.3: The conditional level of illiquidity, based on *ARITHMETIC* variations (AFTER THE FINANCIAL CRISIS)

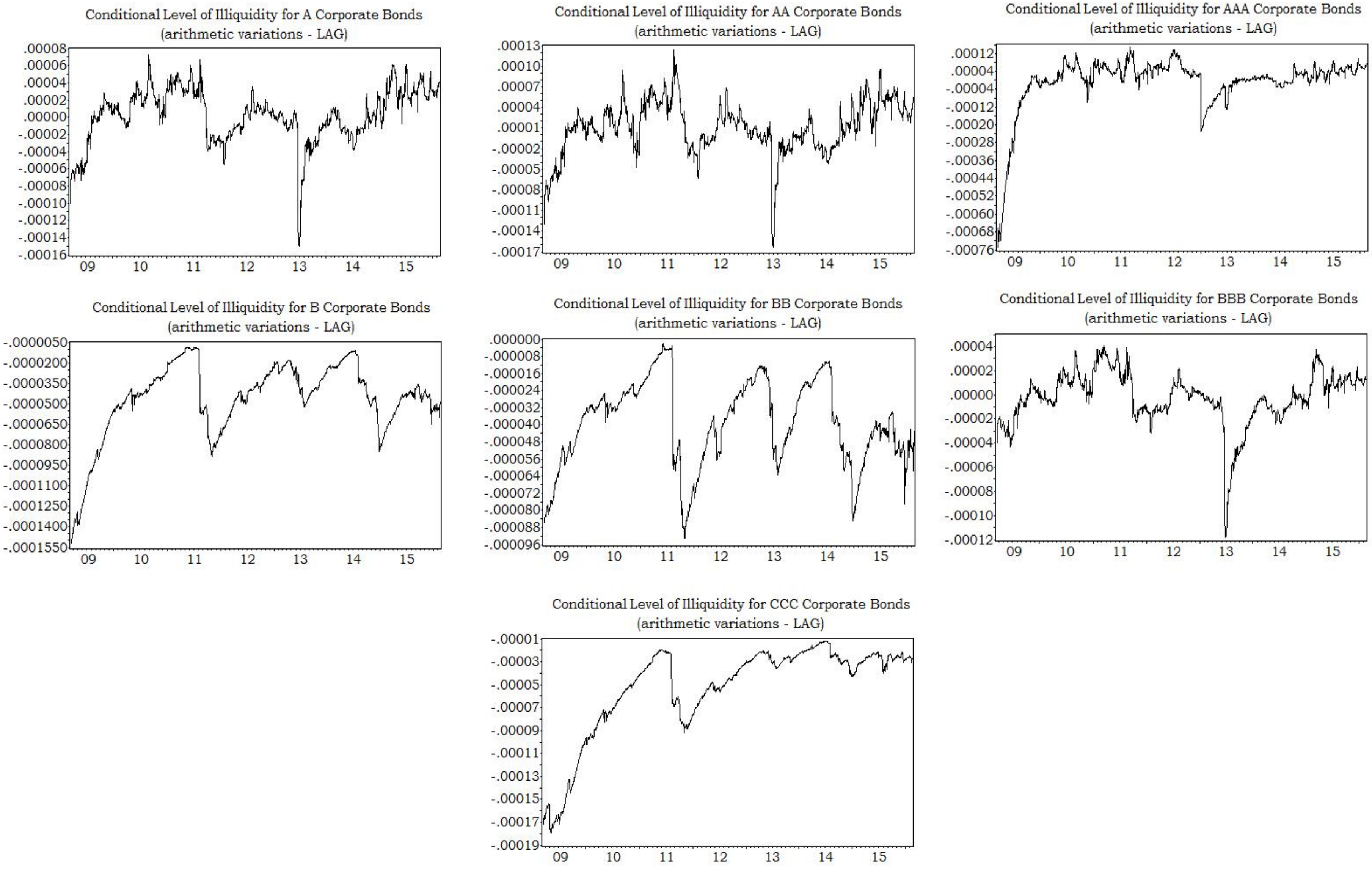


Figure 5.

Conditional level of Illiquidity for U.S. Corporate Bond Yields (logarithmic variations)

The figure shows the dynamics of the conditional levels of illiquidity for the following categories of U.S. corporate bond yields: **A, AA, AAA, B, BB, BBB** as well as **CCC or below**. **Figure 5.1** shows the conditional level of illiquidity for U.S. corporate bond yields, BEFORE THE FINANCIAL CRISIS (**BC**). **Figure 5.2** shows the conditional level of illiquidity for U.S. corporate bond yields, DURING THE FINANCIAL CRISIS (**FC**). **Figure 5.3** shows the conditional level of illiquidity for U.S. corporate bond yields, AFTER THE FINANCIAL CRISIS (**AFC**). The data are downloaded from *The Bank of America Merrill Lynch* database and the figures report the estimated values from **01/01/1997** to **02/19/2016**.

Figure 5.1: The conditional level of illiquidity, based on *LOGARITHMIC* variations (BEFORE THE FINANCIAL CRISIS)

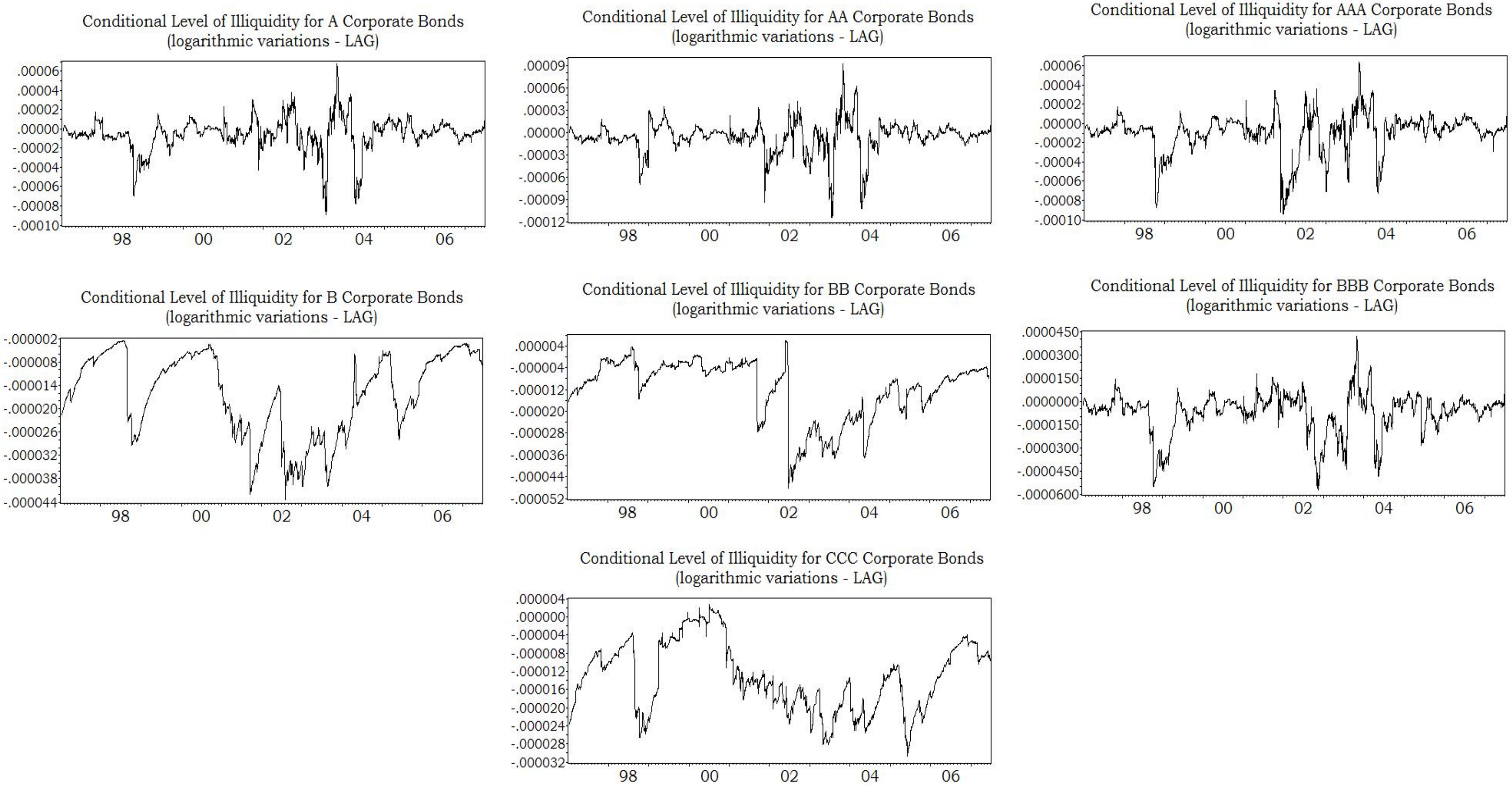
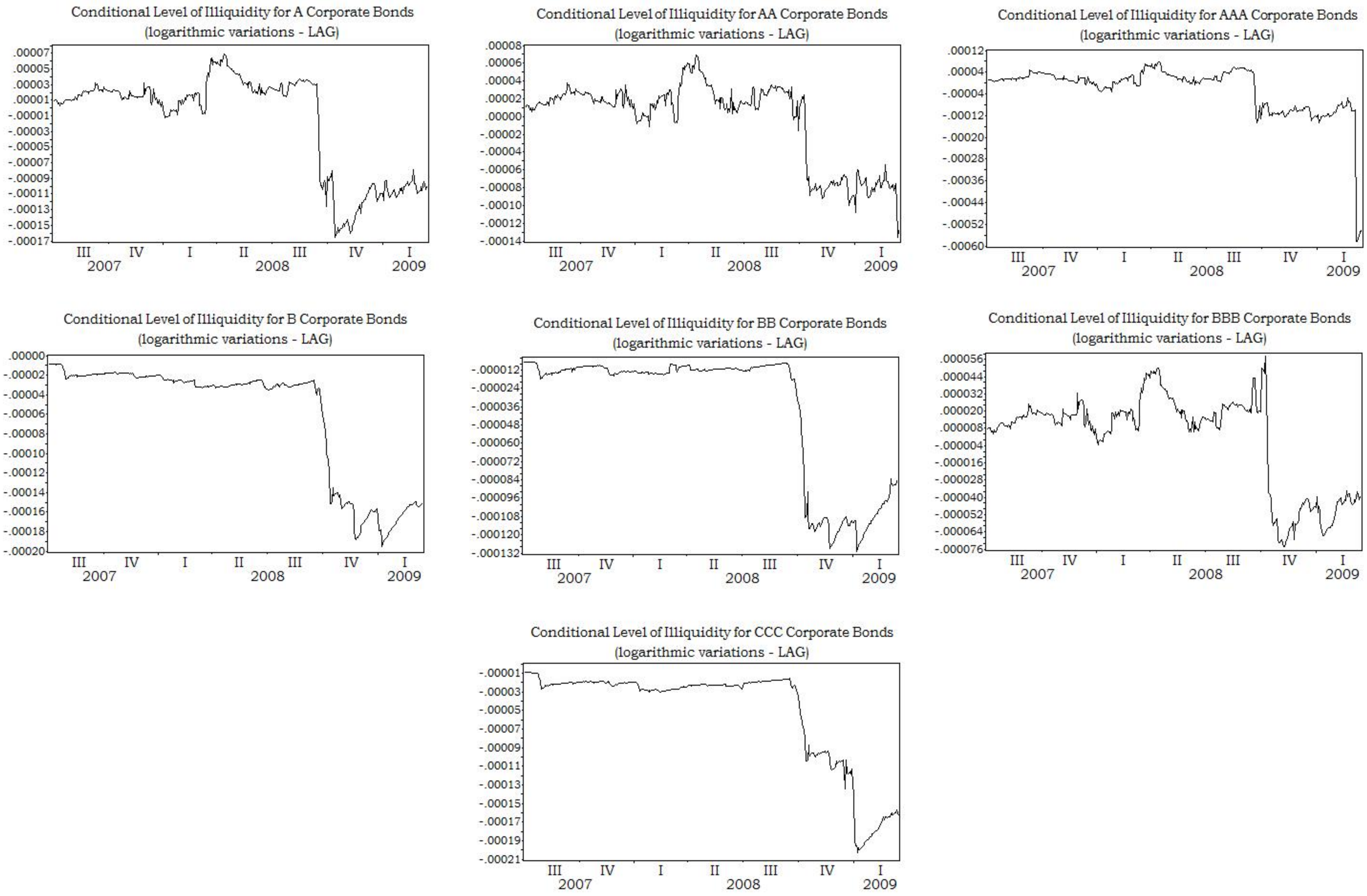


Figure 5.2: The conditional level of illiquidity, based on *LOGARITHMIC* variations (DURING THE FINANCIAL CRISIS)



Conditional Level of Illiquidity for A Corporate Bonds (logarithmic variations - LAG)

Conditional Level of Illiquidity for AA Corporate Bonds (logarithmic variations - LAG)

Conditional Level of Illiquidity for AAA Corporate Bonds (logarithmic variations - LAG)

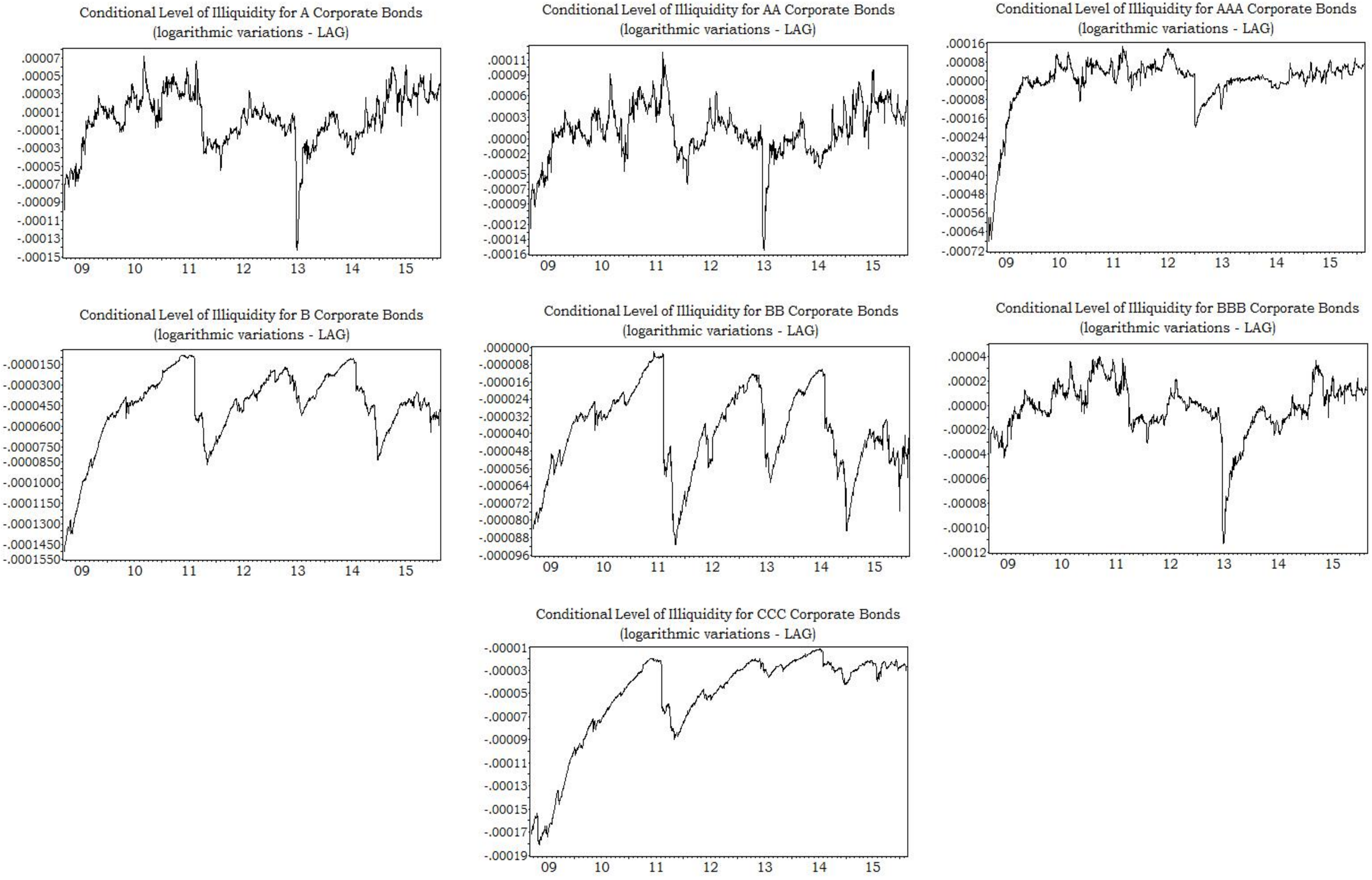
Conditional Level of Illiquidity for B Corporate Bonds (logarithmic variations - LAG)

Conditional Level of Illiquidity for BB Corporate Bonds (logarithmic variations - LAG)

Conditional Level of Illiquidity for BBB Corporate Bonds (logarithmic variations - LAG)

Conditional Level of Illiquidity for CCC Corporate Bonds (logarithmic variations - LAG)

Figure 5.3: The conditional level of illiquidity, based on *LOGARITHMIC* variations (AFTER THE FINANCIAL CRISIS)



Conditional Level of Illiquidity for A Corporate Bonds (logarithmic variations - LAG)

Conditional Level of Illiquidity for AA Corporate Bonds (logarithmic variations - LAG)

Conditional Level of Illiquidity for AAA Corporate Bonds (logarithmic variations - LAG)

Conditional Level of Illiquidity for B Corporate Bonds (logarithmic variations - LAG)

Conditional Level of Illiquidity for BB Corporate Bonds (logarithmic variations - LAG)

Conditional Level of Illiquidity for BBB Corporate Bonds (logarithmic variations - LAG)

Conditional Level of Illiquidity for CCC Corporate Bonds (logarithmic variations - LAG)