“The Prayer”: The 10 Steps of Advanced Risk and Portfolio Management

THE QUANT CLASSROOM by ATTILIO MEUCCI

The first of a two-part article on the path from data analysis to optimal execution across all asset classes and investment styles.

In this article, we present Steps 1–4 of “The Prayer,” a blueprint of 10 sequential steps for quants across the board to achieve their common goal (see Figure 1, above). Steps 5–10 will be discussed in the next Quant Classroom column, which will be published in the June issue of Risk Professional. By following the “Prayer,” quants can avoid common pitfalls and ensure that they are not missing important points in their models. Furthermore, quants are directed to areas of advanced research that extends beyond the traditional quant literature. We use the letter “Y” to signify the true probability space of the buy-side P&L, which stands in contrast to the risk-neutral probability space “Q” used on the sell-side to price derivatives (see Meucci [2011b]).

Each step of the “Prayer” is concisely encapsulated into a definition with the required rigorous notation. Then a simple case study with a portfolio of only stocks and call options illustrates the steps with analytical solutions.

Within each step, we prepare the ground for, and point to, advanced research that fine-tunes the models, or enhances the models’ flexibility, or captures more realistic and nuanced empirical features. Each of these steps is deceptively simple at first glance. Hence, we highlight a few common pitfalls to further clarify the conceptual framework.

1: Quest for Invariance

The “quest for invariance” is the first step of the “Prayer,” and the foundation of risk modeling. The quest for invariance is necessary for the practitioners to learn about the future by observing the past in a stochastic environment.

Key concept. The quest for invariance is the process of extracting from the available market data the “invariants” i.e., those patterns that repeat themselves identically and independently (i.i.d.) across time. The quest for invariance consists of two sub-steps: identification of the risk drivers and extraction of the invariants from the risk drivers.

The first step of the quest for invariance is to identify for each security the risk drivers among the market variables.

Key concept. The risk drivers of a given security are a set of random variables, Yt ≡ (Yt,1, ..., Yt,p)

that satisfy the following two properties: (a) the risk drivers Zt, together with the security terms and conditions, completely specify the security price at any given time t; and (b) the risk drivers Zt, although not i.i.d., follow a stochastic process that is homogeneous across time, in that it is impossible to ascertain the sequential order of the realizations of the risk drivers from the study of the risk drivers past time periods [P(Yt) ↦ P(Yt−1) ↦ ... ↦ P(Y1)]

The risk drivers are variables that fully determine the price of a security, but in general they are not the price, because the price can be non-homogeneous across time: think for instance of a zero-coupon bond. Homogeneity ensures that we can apply statistical techniques to the observed time series of the risk drivers [Zt], t ∈ (1, ... , T) and project future distributions.

Note that we use the standard convention where lower-case letters, such as xt, denote realized variables, whereas upper-case letters, such as Zt, denote random variables.

Illustration. Consider first the asset class of stocks. Denote by S the random price of one stock at the generic time t. The log-price of the stock S = ln S is a random process, which can be steered by one or more factors. The factors can be identified by variance-covariance analysis.

In the stock case, the factors are the realized log-returns of the stock, which are zero-mean random processes, independent of each other and of the stock price (or of the stock log-price).

The next step – i.e., at time t = 1 – is to identify the risk drivers behind the stock price.

Next, let’s consider a second asset class, namely stock options. Denote by C(t,e) the random price of a European call option on the stock, where t is a given strike and e is the given expiry date, or time of expiry. The call price, or its log-price, is not a risk driver, because the presence of the expiry date breaks the time homogeneity in the statistical behavior of the call option price.

In order to identify the risk drivers behind the call option, we transform the price into an equivalent, but homogeneous, variable – namely, the implied volatility at a given time to expiry. More precisely, consider the Black-Scholes pricing formula

C(t,e) = C(t,
e) (ln S_t − ln K, Σ_t, ν_t)

where ν = t − the time to expiry, K is the yet to be defined implied volatility for that time to expiry, and C(t,
e) is the Black-Scholes formula

C(t,
e) (m, ν, σ) = \frac{m e^{-r ν}}{\sqrt{2 \pi σ^2 ν}} \left[ e^{-\frac{(m−r ν)^2}{2σ^2 ν}} + \Phi\left(\frac{ln \frac{m}{e^{-r ν}} + (r + \frac{1}{2}σ^2) ν}{σ\sqrt{ν}}\right)\right]

with Φ the standard normal cdf. At each time t, the price C(t,
e) is observable, and so are S_t and ν. Therefore, the option formula (2) defines a value for Σ_t, which for this reason is called implied volatility.

The implied volatility for a given time to expiry, or better, the logarithm of the implied volatility ln Σ_t displays a homogeneous behavior through time and thus it is a good candidate for an invariant. For more on this and related tests see Meucci (2005).

Since the time to expiry is deterministic, the call option requires two risk drivers to determine its price fully:

\[ Y_{e,t} = \ln S_t \]

The second step of the quest for invariance is the extraction of the invariants – i.e., the repeated patterns – from the homogeneous series of the risk drivers.

Key concept. The invariants are shocks that steer the stochastic process of the risk drivers Y_t over a given time step \( r^t + 1 \):

\[ (\ln S_t)= C(t_{r+1}) + C(t_r) \] 

The invariants satisfy the following two properties: (a) they are identically and independently distributed (i.i.d.) across different time steps; and (b) they become known at the end of the step – i.e., at time \( t+1 \).
Note that each of the Biisk drivers \( \epsilon \) can be steered by one or more drivers. If the latter is the case, then \( Q_{i,j} \neq 0 \).

To determine whether a variable is i.i.d. across time, the easiest test is to scatter-plot the series of the variable versus its own lag. If the scatter-plot is not better, its location-distribution ellipse is a circle, then the variable is a good candidate for an invariant. For more on this and related tests, see Meucci (2005).

Steering to identify the invariants that steer the dynamics of the risk drivers is of crucial importance because it allows us to project the market randomness to the desired investment horizon. Often, practitioners make the mistake of projecting variables they have on hand, most notably returns, instead of the invariants. This, of course, leads to incorrect measurement of risk at the horizon, and thus to suboptimal trading decisions.

The stochastic process for the risk drivers \( \epsilon \) is steered by the randomness of the invariants \( \epsilon_{t-1} \). The most basic dynamics, yet the most statistically robust, which connects the invariants \( \epsilon_{t-1} \) with the risk drivers \( \epsilon_{t} \), is the random walk \( \epsilon_{t} = \epsilon_{t-1} + \epsilon_{t} \).

More advanced processes for the risk drivers account for such features as autocorrelations, stochastic volatility and long memory. We refer to Meucci (2009a) for a review of these more general processes and how they related to random walk and invariants both in discrete and in continuous time, with theory, case study, and code. We also refer to Meucci (2009b) for the multivariate case and how it relates to cointegration and statistical arbitrage.

Illustration. Consider our first asset class example, the stock. As discussed, the only risk driver is the log-price \( \epsilon = \ln S_t \). The aforementioned scatter-plot generally indicates that the compounded return \( \ln S_t / \ln S_{t-1} \) are approximately invariants \( \epsilon_{t-1} = \ln S_{t-1} - \ln S_t \).

Therefore, the risk driver \( \epsilon = \ln S_t \) follows a random walk, as in (6).

Now, consider our second asset class, the call option example. The empirical scatter-plot shows that the changes of the log-impvol volatility are approximately i.i.d. across time. Furthermore, our analysis of the stock example (7) implies that the changes of the log-price are invariants. Therefore, using notation similar to (7), we obtain

\[
\begin{align*}
\epsilon_{t-1} - \epsilon_{t-2} = & \ln S_{t-1} - \ln S_t - (\ln S_{t-1} - \ln S_t), \\
\epsilon_{t-1} - \epsilon_{t-2} = & \ln S_{t-1} - \ln S_t.
\end{align*}
\]

This is also a random walk, as in (6). Notice that this is a multivariate random walk.

The outcome of the quest for invariance — i.e., the set of risk drivers and their corresponding invariants — depends on the asset class and on the time scale of our analysis. For instance, in the case of risk for interest rates, a simple random walk assumption \( \epsilon \) can be viable for time steps of one day, but for time steps of the order of one year, mean-reversion becomes important.

Similarly, for stock returns, the order of the past observations in the series \( \{\epsilon_t\}_{t=1}, \ldots, T \) of the historical scenarios. Alternatively, for the distribution of the invariants, one can make parametric assumptions such as multivariate normal, elliptical, etc.

Illustration. To illustrate the parametric approach, we consider our example (8), where the invariants \( \epsilon \) are changes in moneyness and changes in log-impvol from \( t \) to \( t+1 \).

We can assume that the distribution \( f_{\epsilon} \) is bivariate normal with 2x1 expectation vector \( \mu = [\mu_1, \mu_2] \) and 2x2 covariance matrix, \( \sigma \) as below:

\[
\begin{align*}
\epsilon_{t-1} - \epsilon_{t-2} = & \ln S_{t-1} - \ln S_t \\
\epsilon_{t-1} - \epsilon_{t-2} = & \ln S_{t-1} - \ln S_t.
\end{align*}
\]

We can assume that the distribution \( f_{\epsilon} \) is bivariate normal with 2x1 expectation vector \( \mu = [\mu_1, \mu_2] \) and 2x2 covariance matrix, \( \sigma \) as below:

\[
\begin{align*}
\epsilon_{t-1} - \epsilon_{t-2} = & \ln S_{t-1} - \ln S_t \\
\epsilon_{t-1} - \epsilon_{t-2} = & \ln S_{t-1} - \ln S_t.
\end{align*}
\]

The expectation can be estimated with the sample mean \( \mu = [\mu_1, \mu_2] \) and the sample covariance matrix \( \sigma^2 = \frac{1}{T-2} \mathbf{e}^\top (\epsilon - \mu)(\epsilon - \mu)^\top \), where \( \mathbf{e} \) denotes the transpose.

In large multivariate markets, it is important to impose structure on the correlations of the distribution of the invariants \( \epsilon \). This is often achieved by means of linear factor models. Linear factor models are an essential tool of risk and portfolio management, as they play a key role in the estimation of the invariants distribution. This, of course, leads to incorrect measurement of risk at the horizon, and thus to suboptimal trading decisions.

Available at the current time \( T \), the risk driver \( \epsilon \) can be steered by one or more invariants; therefore, \( Q_{i,j} \neq 0 \).

The simplest of all estimators for the invariants distribution is the nonparametric empirical distribution, justified by the law of large numbers — i.e., “i.i.d. history repeats itself.”

This, of course, leads to incorrect measurement of risk at the horizon, and thus to suboptimal trading decisions. For instance, for stock returns, the order of the past observations in the series \( \{\epsilon_t\}_{t=1}, \ldots, T \) of the historical scenarios. Alternatively, for the distribution of the invariants, one can make parametric assumptions such as multivariate normal, elliptical, etc.

Illustration. To illustrate the parametric approach, we consider our example (8), where the invariants \( \epsilon \) are changes in moneyness and changes in log-impvol from \( t \) to \( t+1 \).

We can assume that the distribution \( f_{\epsilon} \) is bivariate normal with 2x1 expectation vector \( \mu = [\mu_1, \mu_2] \) and 2x2 covariance matrix, \( \sigma \) as below:

\[
\begin{align*}
\epsilon_{t-1} - \epsilon_{t-2} = & \ln S_{t-1} - \ln S_t \\
\epsilon_{t-1} - \epsilon_{t-2} = & \ln S_{t-1} - \ln S_t.
\end{align*}
\]

The expectation can be estimated with the sample mean \( \mu = [\mu_1, \mu_2] \) and the sample covariance matrix \( \sigma^2 = \frac{1}{T-2} \mathbf{e}^\top (\epsilon - \mu)(\epsilon - \mu)^\top \), where \( \mathbf{e} \) denotes the transpose.

In large multivariate markets, it is important to impose structure on the correlations of the distribution of the invariants \( \epsilon \). This is often achieved by means of linear factor models. Linear factor models are an essential tool of risk and portfolio management, as they play a key role in the estimation of the invariants distribution. This, of course, leads to incorrect measurement of risk at the horizon, and thus to suboptimal trading decisions.

The simplest of all estimators for the invariants distribution is the nonparametric empirical distribution, justified by the law of large numbers — i.e., “i.i.d. history repeats itself.”

This, of course, leads to incorrect measurement of risk at the horizon, and thus to suboptimal trading decisions. For instance, for stock returns, the order of the past observations in the series \( \{\epsilon_t\}_{t=1}, \ldots, T \) of the historical scenarios. Alternatively, for the distribution of the invariants, one can make parametric assumptions such as multivariate normal, elliptical, etc.

Illustration. To illustrate the parametric approach, we consider our example (8), where the invariants \( \epsilon \) are changes in moneyness and changes in log-impvol from \( t \) to \( t+1 \).

We can assume that the distribution \( f_{\epsilon} \) is bivariate normal with 2x1 expectation vector \( \mu = [\mu_1, \mu_2] \) and 2x2 covariance matrix, \( \sigma \) as below:

\[
\begin{align*}
\epsilon_{t-1} - \epsilon_{t-2} = & \ln S_{t-1} - \ln S_t \\
\epsilon_{t-1} - \epsilon_{t-2} = & \ln S_{t-1} - \ln S_t.
\end{align*}
\]

The expectation can be estimated with the sample mean \( \mu = [\mu_1, \mu_2] \) and the sample covariance matrix \( \sigma^2 = \frac{1}{T-2} \mathbf{e}^\top (\epsilon - \mu)(\epsilon - \mu)^\top \), where \( \mathbf{e} \) denotes the transpose.

In large multivariate markets, it is important to impose structure on the correlations of the distribution of the invariants \( \epsilon \). This is often achieved by means of linear factor models. Linear factor models are an essential tool of risk and portfolio management, as they play a key role in the estimation of the invariants distribution. This, of course, leads to incorrect measurement of risk at the horizon, and thus to suboptimal trading decisions.

The simplest of all estimators for the invariants distribution is the nonparametric empirical distribution, justified by the law of large numbers — i.e., “i.i.d. history repeats itself.”

This, of course, leads to incorrect measurement of risk at the horizon, and thus to suboptimal trading decisions. For instance, for stock returns, the order of the past observations in the series \( \{\epsilon_t\}_{t=1}, \ldots, T \) of the historical scenarios. Alternatively, for the distribution of the invariants, one can make parametric assumptions such as multivariate normal, elliptical, etc.

Illustration. To illustrate the parametric approach, we consider our example (8), where the invariants \( \epsilon \) are changes in moneyness and changes in log-impvol from \( t \) to \( t+1 \).

We can assume that the distribution \( f_{\epsilon} \) is bivariate normal with 2x1 expectation vector \( \mu = [\mu_1, \mu_2] \) and 2x2 covariance matrix, \( \sigma \) as below:

\[
\begin{align*}
\epsilon_{t-1} - \epsilon_{t-2} = & \ln S_{t-1} - \ln S_t \\
\epsilon_{t-1} - \epsilon_{t-2} = & \ln S_{t-1} - \ln S_t.
\end{align*}
\]
Key concept. The Projection Step is the process of obtaining the distribution at the investment horizon $T+\tau$, of the relevant risk drivers $Y$ from the distribution of the invariants and additional information $\delta$, available at the current time $T$, $f_Y \Rightarrow f_{Y|\tau \leq T}$. 

In order to project the risk drivers, we must go back to their connection with the invariants analyzed in the Quest for Invariance Step $P$. 

If the drives evolve as a random walk $\delta$, then by recursion of the random walk definition $Y_{t+1} = Y_t + \varepsilon_{t+1}$, or $Y_t = Y_0 + \sum_{i=1}^{t} \varepsilon_i$ we obtain that the risk drivers at the horizon $H_\tau$ are the current observable value $\mu$ plus the sum of all the intermediate invariants $Y_{\tau \leq t} \equiv Y_t \equiv \sum_{i=1}^{\tau} \varepsilon_i$. 

Using the independence of the invariants, (12) yields for the variance

$$\text{Var}(Y_{\tau \leq t}) = \text{Var}(Y_0) + \sum_{i=1}^{\tau} \text{Var}(\varepsilon_i).$$

Since all the $\varepsilon_i$s in (12) are i.i.d., all the variances in (13) are equal, and thus we obtain the well-known “square-root rule” for the projection of the standard deviation $\sigma_\tau = \sqrt{\tau} \sigma_0$, or $\sigma_{\tau \leq t} = \sqrt{\tau} \sigma_0$. 

Note that we did not make any distributional assumption such as normality to derive the square-root rule. 

Simple results to project other moments under the random walk assumption such as normality to derive the square-root rule.

Pitfall. “...To project the market I assume normality and therefore I multiply the standard deviation by the square root of the horizon $\tau$. The square root rule is true for random walks with finite-variance invariants, regardless of their distribution.

However, the square-root rule only applies to the standard deviation and does not allow to determine all the other moments of the distribution, unless the distribution is normal.

F3: Pricing. Now that we have the distribution of the risk drivers at the horizon $H_\tau$ from the Projection Step $P$, we are ready to compute the distribution of the security prices in our book. Recall that the value of the security at the horizon $H_\tau$ is determined by $f_{Y|\tau \leq T}$, by design, is fully determined by (a) risk drivers at the horizon $H_\tau$, and (b) non-random information $\delta$ known at the current time, such as terms and conditions $\Pi_{\tau \leq t}$, or $\Pi_{\tau \leq t} \equiv f_{Y|\tau \leq t}$. 

Then, given the security price at the horizon $H_\tau$, the security P&L from the current date to horizon $H_\tau$ is the difference between the horizon value (15), which is a random variable, and the current value, which is observable and thus part of the available information set $\delta$. Consequently, the horizon profit function reads

$$\Pi_{\tau \leq t} \equiv f_{Y|\tau \leq t} - f_Y.$$

While the investment horizon is much shorter than the time to expiry of the option — i.e., $\tau < T$ — the following first-order Taylor approximation suffices to price the option $p(y)$:

$$p(y) \approx p(y_0, T) \equiv p(y_0, \ln s, T; \tau) \approx \frac{\partial p(y_0, T)}{\partial \ln s} \bigg|_{y=y_0}$$

At times it is convenient to approximate the pricing function (15) by its Taylor expansion

$$p(y_0, T) \approx p(y_0, T) + \frac{\partial p(y_0, T)}{\partial \ln s} \bigg|_{y=y_0} (\ln s - \ln s_0)$$

for the log-changes $\ln s - \ln s_0$.

We conclude the pricing step by highlighting two problems. First, a data availability concern: the available pricing functions in all terms and conditions.

Second, we have the problem of liquidity risk. The pricing step assumes the existence of one single price, which is fully determined by the risk drivers $f(P\&L|\delta)$ and, in (15), by any additional cash flow term in (16).

By following the Prayer, quants can avoid common pitfalls and ensure that they are not missing important points in their models.

Effective value of the risk drivers, often the current value $p\delta = p\delta(s, T; \tau)$, for the vector of the first derivatives, and $\delta^2 p\delta^2$ denotes the matrix of the second cross-derivatives.

Depending on its end users, the coefficients in the Taylor approximation (18) are known under different names. In the derivatives world, they are called the “Greeks” — delta, gamma, vega, etc.

In the fixed-income world, the coefficients are called carry, duration and convexity.

Illustration. In our stock example, the single risk driver is the log-price $W$, i.e., $W = \ln s$. Therefore, the horizon pricing function (15) reads $p(y=\ln s)$. Its Taylor approximation reads $p(y) \approx p(y_0) + \frac{\partial p(y_0)}{\partial y} (y - y_0)$, then by recursion of the random walk assumption $\delta_{\tau \leq t}$, or by an additional cash flow term $\Pi_{\tau \leq t}$.

Then, given the security price at the horizon $H_\tau$, the security P&L from the current date to horizon $H_\tau$ is the difference between the horizon value (15), which is a random variable, and the current value, which is observable and thus part of the available information set $\delta$. Consequently, the horizon profit function reads

$$\Pi_{\tau \leq t} \equiv f_{Y|\tau \leq t} - f_Y$$

At times it is convenient to approximate the pricing function (15) by its Taylor expansion

$$p(y_0, T) \approx p(y_0, T) + \frac{\partial p(y_0, T)}{\partial \ln s} \bigg|_{y=y_0} (\ln s - \ln s_0)$$

for the log-changes $\ln s - \ln s_0$.

We stated in the distribution of the risk drivers (14) that the log-changes in (22) are jointly normal. Therefore, the distribution of the P&L is normal, because the linear combination of jointly normal variables is normal, as follows:

$$\Pi_{\tau \leq t} \Rightarrow N(\mu_{\tau \leq t}, \sigma_{\tau \leq t}^2)$$

where $\mu_{\tau \leq t}$ is a significant value of the risk drivers, often the current value $\delta_{\tau \leq t}$, $\sigma_{\tau \leq t}^2$ denotes the vector of the first derivatives, and $\delta^2 p\delta^2$ denotes the matrix of the second cross-derivatives.

Using the independence of the invariants, (12) yields for the variance

$$\text{Var}(Y_{\tau \leq t}) = \text{Var}(Y_0) + \sum_{i=1}^{\tau} \text{Var}(\varepsilon_i).$$

Since all the $\varepsilon_i$s in (12) are i.i.d., all the variances in (13) are equal, and thus we obtain the well-known “square-root rule” for the projection of the standard deviation $\sigma_\tau = \sqrt{\tau} \sigma_0$, or $\sigma_{\tau \leq t} = \sqrt{\tau} \sigma_0$. 

Note that we did not make any distributional assumption such as normality to derive the square-root rule.

Simple results to project other moments under the random walk assumption such as normality to derive the square-root rule.

Pitfall. “...To project the market I assume normality and therefore I multiply the standard deviation by the square root of the horizon $\tau$. The square root rule is true for random walks with finite-variance invariants, regardless of their distribution.

However, the square-root rule only applies to the standard deviation and does not allow to determine all the other moments of the distribution, unless the distribution is normal.

F3: Pricing. Now that we have the distribution of the risk drivers at the horizon $H_\tau$ from the Projection Step $P$, we are ready to compute the distribution of the security prices in our book. Recall that the value of the security at the horizon $H_\tau$ is determined by $f_{Y|\tau \leq T}$, by design, is fully determined by (a) risk drivers at the horizon $H_\tau$, and (b) non-random information $\delta$ known at the current time, such as terms and conditions $\Pi_{\tau \leq t}$, or $\Pi_{\tau \leq t} \equiv f_{Y|\tau \leq t}$. 

Then, given the security price at the horizon $H_\tau$, the security P&L from the current date to horizon $H_\tau$ is the difference between the horizon value (15), which is a random variable, and the current value, which is observable and thus part of the available information set $\delta$. Consequently, the horizon profit function reads

$$\Pi_{\tau \leq t} \equiv f_{Y|\tau \leq t} - f_Y.$$ 

Note that the P&L must be adjusted for coupons and dividends, either by reinvesting them in the pricing function (15), or by an additional cash flow term in (16).

Key concept. The Pricing Step is the process of obtaining the distribution of the securities P&Ls over the investment horizon from the distribution of the risk drivers at the horizon and current information such as terms and conditions, by means of the pricing function

$$f_Y \Rightarrow f_{Y|\tau \leq T}$$

At times it is convenient to approximate the pricing function (15) by its Taylor expansion

$$p(y_0, T) \approx p(y_0, T) + \frac{\partial p(y_0, T)}{\partial \ln s} \bigg|_{y=y_0} (\ln s - \ln s_0)$$

for the log-changes $\ln s - \ln s_0$. 

We concluded the pricing step by highlighting two problems. First, a data availability concern: the available pricing functions in all terms and conditions.

Second, we have the problem of liquidity risk. The pricing step assumes the existence of one single price, which is fully determined by the risk drivers $f(P\&L|\delta)$, and, in (15), by any additional cash flow term in (16).

By following the Prayer, quants can avoid common pitfalls and ensure that they are not missing important points in their models.

It is worth noticing that pricing becomes a surprisingly easy task when the distribution of the risk drivers is represented in terms of scenarios, as (16) is simply repeated, scenario by scenario, by inputting discrete realized risk drivers values.

We conclude the pricing step by highlighting two problems. First, a data and analytics problem: in many financial companies, there might not be readily available pricing functions with all terms and conditions.

Second, we have the problem of liquidity risk. The pricing step assumes the existence of one single price, which is fully determined by the risk drivers $f(P\&L|\delta)$, and, in (15), by any additional cash flow term in (16).

By following the Prayer, quants can avoid common pitfalls and ensure that they are not missing important points in their models.
Often, practitioners make the mistake of projecting variables they have on hand, most notably returns, instead of the invariants. This, of course, leads to incorrect measurement of risk at the horizon, and thus to suboptimal trading decisions.

**Pitfall.** “... The delta approximation gives rise to parametric risk models that assume normality...” The Taylor approximation of the pricing function can be paired with any distributional assumption, not necessarily normal, on the risk drivers. “... The goodness of the Taylor approximation depends on the specific security...” The goodness of the Taylor approximation depends on the security and on the investment horizon: due to the square-root propagation of the standard deviation (13), the longer the horizon, the wider the distribution of the risk drivers. Therefore the approximation worsens with longer horizons.

*'...To be continued in the next “classroom.”'*

Attilio Meucci is the chief risk officer at Kepos Capital LP. He runs the 6-day “Advanced Risk and Portfolio Management Bootcamp,” see symmys.com. The author is grateful to Gavri Beibi, Luca Spampinato and an anonymous referee.

**REFERENCES**


