How to Value Guarantees

What are financial guarantees? What are their risk benefits, and how can risk control practices be used to help value guarantees? **Gordon E. Goodman** outlines multiple methods for valuing guarantees and explores the expansion of the risk manager's role in financial reporting.

The expanded financial use of fair value measurements (traditionally called "mark to market" within the risk community) has resulted in the need for relatively complex calculations to be captured in standard financial performance reports. Consequently, while the risk management profession maintains its historical role as an integral part of the trading floor, financial reporting is becoming an increasingly important component of the expanded responsibilities of the contemporary risk manager.

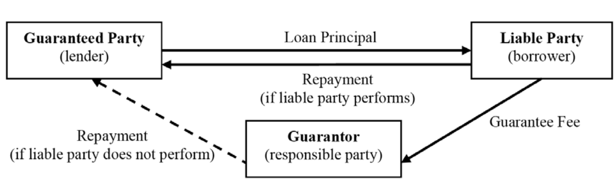
An example of this increasing complexity is evident in FASB Interpretation No. 45 (FIN 45), a financial accounting rule that requires that the fair value of certain financial guarantees be disclosed by the guarantor. FAS 157 sets out the general provisions for measuring all fair values. This article provides background information on guarantees and outlines specific procedures for guarantee valuation. Additionally, to illustrate better the valuation process, several hypothetical guarantees will be valued.

It is important to note that though FASB has required the application of fair value measurement to guarantees, it has provided little practical guidance on how to measure the value of guarantees. This article attempts to use best-in-class risk control practices to assist practitioners in preparing these valuations.

Definition of a Guarantee

A guarantee is the assumption of responsibility for payment of a debt or performance of an obligation if the liable party fails to perform to expectations. Below is an illustration of a guarantee that supports a loan.

Diagram: How a Guarantee Supports a Loan



A guarantee reduces the risk to the guaranteed party and creates a contingent liability for the guarantor. The guarantee's value from the guaranteed party's standpoint is usually higher than the value (actual cost) from the guarantor's standpoint. This difference in values, however, diminishes as the creditworthiness of the guarantor increases. In the valuation procedures described a bit later in this article, the guarantor's perspective is the one being considered (please note that this is also the perspective required for valuation under FIN 45).

Guarantees can be broadly classified as one of two types: those in which a single event (such as a default) can trigger payouts and those in which more than one event can trigger payouts. The more common types are those with single-event triggers. The event is typically non-performance (i.e., default) by the liable party with regard to the obligation. Examples of single-event guarantees include guarantees of loan repayments and guarantees of payments to contractors.

In the "multiple triggers" guarantee type, the triggers might be mutually exclusive, independent or not independent. Events might trigger the same payout amount or varying payout amounts. Examples include guarantees of project milestones and performance guarantees.

The Current Approach

The goal of this article is to develop a guarantee valuation framework that would ensure a consistent approach to valuation, be broad enough to encompass almost any type of guarantee and conform to the guidelines set forth in FIN 45 and FAS 157.

The current approach is based on two underlying principles in guarantee valuation. First, the value of a risk-free transaction is equal to the value of a risky transaction plus the value of the guarantee. This relationship, which combines the risky transaction with the guarantee results in a synthetic risk-free transaction, can be stated as

(1) Value of Guarantee = Value of Risk-Free Transaction - Value of Risky Transaction

The second basic valuation principle is that the value of any contingent liability, including guarantees, equals its expected present value. As defined by FASB, the expected present value is "the sum of the probability-weighted present values in a range of estimated cash flows, all discounted using the same interest rate convention." Therefore, according to this principle,

(2) Value of Guarantee = Present Value of the Probability-Weighted Estimated Cash Flows

Our approach also employs the FASB's so-called fair value hierarchy in the selection of the valuation method and the inputs used to calculate the fair value. This hierarchy is described in FAS 157. (Please note, however, that FAS 157 does not specifically address guarantees.) There are three levels in this hierarchy:

- **Level 1**: Models and values based on external, quoted prices in active markets for identical assets/liabilities.
- **Level 2:** Models and values based on external, quoted prices for similar assets/liabilities (with adjustments).
- **Level 3:** Models and values based on internal inputs.

Three Valuation Methods

Based on the principles described earlier, three methods of valuing guarantees have been developed.

Table 1: Guarantee Valuation Methods

Method	Guarantee Value Equals	Key Inputs/Assumptions	Applies To
Market Value	• fee received … or difference in market value of guaranteed debt and non-guaranteed debt.	• guarantee fees … or market value of debt with and without guarantees.	• sovereign debt. • bank deposit insurance.
Credit Spread	• difference in present value of the transaction's cash flows when discounted at the guaranteed and risky rates.	• risky (i.e., non-guaranteed) discount rate • risk-free (i.e., guaranteed) discount rate.	• default-triggered guarantees with simple payout structures.
Contingent Claims	• present value of the probability-weighted estimated cash flows.	• all payout scenarios identified. • probability of each scenario.	• default-triggered with atypical payout structures.
		• all payout scenarios identified. • probability of each scenario. • correlations (if any) among scenarios.	• multiple-trigger guarantees.

If a guarantee has the characteristics that allow it to be valued with all three methods, then each method should produce a similar value. The three methods are described in more detail in the subsequent sections.

A Pair of Methods: Market Value and Credit Spread

The market value method is the simplest to apply, but the required inputs are seldom available. It is consistent with Level 1 of the fair value hierarchy. Generally, it can be applied in two cases.

In the first case, the comparable risk-free (i.e., guaranteed) and risky (non-guaranteed) instruments exist with the liable party, the market values of these instruments are known and the value of the guarantee is simply the difference in the value of the risky and risk-free instruments. This could be applied to a guarantee on an entity that has both typical debt and guaranteed debt (e.g., backed by the federal government).

In the second case, a fee is received for providing the guarantee and the guarantee's value is equal to the fee.

The credit spread method, in contrast to the market value method, is consistent with Level 2 of the fair value hierarchy. It is based on the first valuation principle (i.e., Value of Guarantee = Value of Risk-Free Transaction - Value of Risky Transaction) outlined earlier in this article.

The value of the guarantee calculated this way is valid only when the guarantor's probability of default is zero. Nevertheless, we could approximate a guarantee's value when the guarantor is not default-free by

Value of Guarantee = Value of Guaranteed Transaction - Value of Risky Transaction

The credit spread method can be used if (1) the guarantee covers an obligation that is structured like a loan/bond; (2) the credit spread of the liable party can be estimated; or (3) it can be assumed that the loss-given default (LGD) of the guarantee is the same as the LGD of the instruments used to imply the credit spread.

The credit spread is the difference in the risky rate (i.e., non-guaranteed rate) and the rate with a guarantee. In many real-world circumstances, this is the most useful and also the most widely used valuation technique for guarantees.

The value of the guaranteed obligation/loan is calculated by discounting the expected cash flows (principal and coupon payments under the risky rate) at the guaranteed rate, while the value of the non-guaranteed loan is discounted at the risky rate.

The difference between the guaranteed and non-guaranteed values of the loan is the value of the guarantee. In general, discounting a risky loan at the risky rate for that loan should equal the initial amount lent — i.e., the value of the risky (non-guaranteed loan) is equal to the principal. Thus, in reality, the discounted cash flows at the guaranteed rate are being compared with the amount lent.

In most cases (the standard approach), the true/market discount rate of the guaranteed transaction is not known. In such instances, one can assume that the discount rate of the guaranteed transaction is the risk-free rate. This is a conservative assumption that will overstate the guarantee's value. The higher the creditworthiness of the guarantor, the lower the deviation from the true value of the guarantee in the future.

"Guarantee contracts represent contingent claims into the future. Consequently, the methodology for pricing contingent claims could be applied to estimating the value of guarantees. This valuation approach can be used to value almost any type of guarantee."

Alternatively, one can assume that the discount rate of the guaranteed transaction is the same as the discount rate of the guarantor. In effect, this says that the guaranteed transaction's "risk" is equal to the risk that the guarantor will not perform. In reality, the guaranteed transaction is slightly less risky than this, because its "risk" actually occurs only when *both* the liable party and the guarantor fail to perform. Consequently, this approach will also tend to overstate the value, albeit slightly.

A second alternative is a theoretically correct method that accounts for the joint probability that both the liable party and the guarantor fail to perform. This method is the most accurate, but can be more complicated than the other methods.

If the *standard* approach is applied, the value of a particular guarantee will be the same regardless of the creditworthiness of the guarantor. If the first or second alternative approaches are used, the value (i.e., liability recognized) of a particular guarantee will be increased (decreased) as the credit worthiness of the guarantor increases (decreases).

The risky rate can be obtained or estimated in a number of ways, including a review of the known cost of debt (or borrowing rate), the applicable corporate bond yields and the cost of debt of entities with comparable credit ratings (or from comparable project financing rates).

Occasionally, the risky rates obtained by these methods may need to be adjusted, given that they incorporate both the probability of default (PD) and the LGD. The PD is the same for all guarantees on a given entity; however, the LGD may depend on the type of guarantee and its place in the capital structure.

LGD is the percentage of the guaranteed amount that is expected to be lost if default occurs. In many cases, it might be reasonable and/or practical to assume that the LGD is similar for all the liabilities (bonds, guarantees, etc.) of a given entity. In these cases, the credit spread method can be used. However, in cases where the LGD is expected to be significantly different from that of the other liabilities, then the contingent claim methods should rather be used.

Contingent Claims Valuation Methods

Guarantee contracts represent contingent claims into the future. Consequently, the methodology for pricing contingent claims could be applied to estimating the value of guarantees. This valuation approach can be used to value almost any type of guarantee, including those that can be valued with the first two (e.g., market value and credit spread) methods. But given the inherent complexity of this methodology, it should only be used when the first two methods are either not possible or not available.

The contingent claims method is consistent with Level 3 of the fair value hierarchy, and it is based on the second valuation principle described earlier:

Value of Guarantee = Present Value of Expected Future Guarantee Payments.

Depending on how expectations are calculated — i.e., what probabilities are assigned to different events — different discount rates should be used.

Binomial Tree with the Actual Probabilities of Default

This method can be summarized as follows: adjust the cash flows for the (actual) probability of default and then discount the probability-weighted cash flows (CFs) at the risk-free rate + a margin to reflect the *systematic risk of default only*. In order to implement this method of valuation, the following information is needed: (1) the cash flows under all possible scenarios; (2) estimates of the actual probabilities of default; and (3) an estimate of the margin required for bearing systematic risk.

Arguably, identifying the CFs should be the easiest. *Actual default probabilities* can be estimated from historical data. Most rating agencies produce tables similar to Table 2 (next page). It shows, for example, that for a bond that starts with an A rating, within one year there is a chance of only 0.04% that it goes into default, a 91.76% chance to remain an A bond, a 5.19% chance to be downgraded to BBB, and so on. The probability of the same A-rated bond being in default in two years, which is calculated by multiplying row A with the entire matrix (table), is 0.01%.

Table 2: One-Year Ratings Migration Probabilities (Average, 1981-2000)

original rating	probability of migrating to rating by year end (%)							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	93.66	5.83	0.40	0.08	0.03	0.00	0.00	0.00
AA	0.66	91.72	6.94	0.49	0.06	0.09	0.02	0.01
A	0.07	2.25	91.76	5.19	0.49	0.20	0.01	0.04
BBB	0.03	0.25	4.83	89.26	4.44	0.81	0.16	0.22
BB	0.03	0.07	0.44	6.67	83.31	7.47	1.05	0.98
B	0.00	0.10	0.33	0.46	5.77	84.19	3.87	5.30
CCC	0.16	0.00	0.31	0.93	2.00	10.74	63.96	21.94
Default	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Source: Standard & Poor's

Estimating the margin to be added to the risk-free rate for discounting is by far the most subjective part in this valuation method. Most businesses prefer relying on the capital asset pricing model (CAPM) when calculating this margin, as the model is well known and widely accepted. To calculate the risk-adjusted discount rate, you need estimates of the market return premium over the risk-free rate (MRP) and the β of a similar class bond (Bloomberg is a good source for a bond's β).

Risk-adjusted rate = $r^f + \beta$*MRP

Option pricing is another alternative, and highly useful, method for valuing complex guarantees. The general idea behind option pricing methods is the no-arbitrage condition — i.e., assets with the same payoffs must have the same price. The option pricing approach entails replicating the payoff of the loan guarantee by a portfolio made up of risk-free bonds and assets of the borrowing firm; thus, the value of the loan guarantee can be inferred from the cost of the replicating portfolio. Option pricing techniques are also known as risk-neutral valuation — i.e., any contingent claim could be priced as the sum of the expected cash flows discounted at the risk-free rate, the expectation being taken under what is known as the risk-neutral probability measure, *Q*.

Calculating the Value of a Loan Guarantee Explicitly as a Put Option

As shown by Merton (1977), a loan guarantee for a single, homogeneous term discount debt is equivalent to a European put option written on the assets of the borrower, with an exercise price equal to the maturity value of the debt obligation, maturity corresponding to that of the loan and the value of the firm's assets as the underlying asset. To understand this explanation, observe that at any point of time there are two possible outcomes: the liable party is either solvent or bankrupt.

In the first case, the guarantor is not called upon, because the firm has sufficient funds to honor its commitments. In the second case, the value of debt (D_t) is higher than the value of the firm (V_t), and the guarantor has to cover the difference (D_t-V_t). Thus, the payoff of the guarantee is either 0 (when $V_t \geq D_t$; i.e., the firm is solvent), or D_t-V_t (when $V_t < D_t$):

Guarantee Payoff = max{0, D_t-V_t}

The above put option is a standard one, and the Black-Scholes option pricing formula can be applied, giving the value of the guarantee (G) as

$$G = -V_0 \cdot N(-d_1) + D \cdot e^{-rT} \cdot N(-d_2)$$

where

$$d_1 = \frac{\ln(V_0 / D) + (r + \sigma_V^2 / 2)T}{\sigma_V \sqrt{T}}$$

$$d_2 = d_1 - \sigma_V \sqrt{T}$$

N(.) is the cumulative standard normal density function; σ_V is the volatility of the returns on the borrower's assets; D is the amount of debt interest and principal due to be repaid at time T; and V_0 is the value of the borrower's assets today. Notice that N(-d_2) is just the risk-neutral probability of default.

The above solution for the value of the guarantee requires estimates of both the market value of the borrower's assets, *V*, and the volatility of their returns, σ_V . Both of these variables cannot be observed. However, if the liable party is a publicly traded company, we can observe the company's equity value today, E_0, and its volatility, σ_E. Black and Scholes (1973) demonstrated that a firm's equity at maturity of the debt can be interpreted as the value of a call option on its own assets, i.e.:

$$E_T = \max\{0, V_T - D\}$$

Thus, using the Black-Scholes call option formula gives us the value of the equity today:

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$

where N(.), d_1 and d_2 are as before.

By applying Ito's lemma to $dE(V,t)$, we can get the following relationship:

$$\sigma_E = \frac{N(d_1)\sigma_V V_0}{E_0}$$

Now, we have two equations that have to be solved for the two unknowns, V_0 and σ_E. Together with the other known variables, D and T, they can be inserted in the previously described formula for the loan guarantee (G) and thus obtain the value of the guarantee.

Binomial Tree with Given Risk-Neutral PD

If we knew the risk-neutral probability of default beforehand, then the value of a loan guarantee could be calculated as

$$G = \frac{E^Q[CF]}{1+r^f} = \frac{(1-q)*\$0 + q*\$1bn}{1.05} = \frac{0.444*\$1bn}{1.05} = \$0.42bn$$

There are several ways to obtain estimates of the risk-neutral probabilities of default, including: credit-default swap (CDS) spreads (CDS spreads); the yield spread between the risk-free and the risky bonds; an adjustment of the actual probabilities (see Jarrow, et al., 1997); and the use of Merton's (1974) model (the value of $N(-d_2)$).

If we are willing to assume that the higher yields on corporate bonds are entirely compensation for possible losses from default,[1] then, given a constant recovery rate (RR), the cumulative probability of default through time *t* is

$$Q(t) = \frac{1-\exp[-t\cdot(y(t)-y^f(t))]}{1-RR}$$

where y(t) is the yield on a t-year corporate zero-coupon bond, and y^f - is the yield on the t-year risk-free bond. For example, if the spread is 175 basis points for the 5-year, over the risk-free rate as well as for loans with shorter maturities, and the RR=0%, then

Table 3: Probability of Default

year	cumulative probability of default	default probability for a given year
1	1.73%	1.73%
2	3.44%	1.70%
3	5.11%	1.68%
4	6.76%	1.65%
5	8.38%	1.62%

Monte Carlo Simulation Method

From standard option pricing, we know that the price of any contingent claim is equal to the sum of its expected cash flows under the risk-neutral probability measure, *Q*, and is discounted to today at the risk-free rate:

$$G = e^{-rT} E^Q[CashFlows]$$

The Monte Carlo simulation method is based on the idea of approximating the above expectation by simulating sufficiently many paths of the underlying asset, *V*, and averaging the resultant cash flows from each path. The advantage of the Monte Carlo simulation method is that it can be used in pricing highly complex guarantees, such as when the payoff of the guarantee depends on the path followed by the underlying assets (or just on their final value[s]).

"The Monte Carlo simulation method can be used with any stochastic process for the underlying assets; it easily handles multiple trigger events guarantees, even when the default events are correlated."

This simulation method can be used with any stochastic process for the underlying assets; it easily handles multiple trigger events guarantees, even when the default events are correlated. A drawback of the method is that it is computationally very time consuming.

It is clear that the valuation of guarantees has become an increasingly important part of the financial framework, as various instruments — including many publicly traded corporate and government bonds — are marketed with attached guarantees (or insurance) that form an integral part of the total value received by investors. Clarifying the guarantee valuation process itself should improve investors' and creditors' understanding of the true value of their holdings, and it should also help guarantors understand the contingent liabilities that they have created on their books. If risk managers and financial analysts can agree upon and adopt a standardized methodology for the valuation of guarantees and other contingent liabilities (such as the framework we have outlined in this article), there should be increased confidence in the complex valuations now regularly performed by the risk profession. These valuations are increasingly required for the preparation of modern financial statements. ■

FOOTNOTES:

1. Please note that other factors — such as differential tax treatment, transaction costs and liquidity — also contribute to the spread over the risk-free rate.

✎ **GORDON GOODMAN** is the Trading Control Officer for Occidental Petroleum. In this capacity, he serves as Occidental's chief risk officer for energy commodities. He is a current member of the FASB's Valuation Resource Group and he is a past chairman and member of the American Petroleum Institute's (API's) Risk Control Committee. He can be reached at Gordon_Goodman@oxy.com

GEORGI VASSILEV, a PhD candidate in economics at the University of Southern California, has been working for the past two years as a quantitative consultant for Occidental Petroleum. He can be reached at Georgi_Vassilev@oxy.com

The authors wish to thank William C. Hamilton and Tushar Patel, who worked on earlier drafts of this article.