Measuring Default Risk for a Portfolio of Equities

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December 4, 2017

Abstract

This work evaluates some changes proposed by the Basel Committee (BCBS) Regulation for capital allocation due to a company default in an equities portfolio. In the last decade, some measures were designed to account for the risk of default of an enterprise that generally would not be caught by a 10-day Value at Risk measure. The first and more conservative measure was the Incremental Risk Charge. The second and less severe is the Incremental Default Charge. To designed this measures we use a Merton Model to compute the Probability of Default. This probability is compared with simulated asset returns so we can compute a one year value at risk. The idea is capture the risk of a company defaulting. The results shown are based in a portfolio of Ibovespa companies and a portfolio of S&P500 companies. Additionally, we propose a manner to account for the correlation in the companies. And we compare the effects of the standard method of allocation to the models developed in this study.

Keywords:FRTB; Default Risk; Equities; Multi-factor Model.

1 Introduction

The main goal of this work is to evaluate some of the changes of regulation proposed by the Basel Committee in the last decade. Right after the crisis of 2008, the Incremental Risk Charge measure of risk was proposed to account for the possibility of some company running into a default. This was proposed since the existing metrics where designed to capture short term risk.

The measure Incremental Risk Charge proposed in 2008 is based on a VaR calculation using a one-year time horizon and calibrated to a 99.9th percentile confidence level. A few years later this measure was substituted by a simpler measure which intend to capture only the jump to default possibility. This was done in the Fundamental Revision Trading Book, which after received a standardized version.

In this paper we explore these new risk measures, comparing the older version with the younger one and with the standardized one. All the results obtained were focusing in two equities portfolios, one based in the Ibovespa Index and one based in a S&P 500 Index.

In the first part of the paper we will go through a brief historic of regulatory changes where we can see in a broader context what happened in the past years and how this affects the capital requirements. Moreover we go through a bibliographic review about themes related to the subject. In the second part we formulate the problem as proposed by regulation as a Internal Model Approach and as a Standardized Approach. In the last part discuss what will be simulated and the results obtained in these simulations.

1.1 The Historic of Regulatory Changes

The first document about this subject was released in 2008 by the BCBS, which is the Guidelines for Computing Capital for Incremental Risk in the Trading Book, see [1]. In this document there are the rules for construction a measure to compute the effect of credit risk in the Trading Book. This measure is the Incremental Risk Charge (IRC), which is basically a Value at Risk with a 99.9% confidence interval and a one year time horizon. The IRC must consider the correlation effect between obligors, must the liquidity horizon of the traded position and a the constant level of risk over one-year capital horizon for the portfolio among other things.

In 2014, the BCBS issued the second document about the subject, which is the Fundamental review of the trading book: A revised market risk framework, see [2]. This document changed the idea of IRC which was designed to capture the effects of price change and default to only consider the default. This happened because the BCBS understood that a double counting was being done since the price effect was already contemplated in other measures. An important observation is that we are talking about the Internal Model Approach (IMA) permitted by the BCBS. This new version of IRC was now called the Incremental Default Risk.

In 2016, the BCBS issued the third document, which is Minimum capital requirements for market risk, see [3]. This document completed the previous one and gave the standardized version as a complementary possibility for the IMA one. This means that instead of implementing a model that captures the effect of default following the rules which the regulation demands, we may be calculate the measure by a standardized approach following some parameters. In the end of the process we may select the measure which best capture the risk, given that we follow some criteria selection. Additionally, this new document excluded the constant level of risk and called this final version of measure Default Risk Charge.

1.2 The Current Regulation

The Default Risk Charge (DRC) presented in [3] will be the focus of this article. The DRC is a Credit Value at Risk model and it must be set considering the following conditions:

- One year time horizon with one tail Value at Risk;
- The confidence interval must be adjusted to the 99.9% level;
- The modeled returns must consider a systematic and idiosyncratic framework, where the systematic must have a global component and a industry one.
- Default correlations must be based on credit spreads or on listed equity prices;
- The liquidity horizon must vary between 60 days (very liquid equities) and one year;
- it must the greater of: the average of the default risk charge model measures over the previous 12 weeks or the most recent default risk charge model measure;
- the Probability of Default (PD) of each obligor must be subject to a floor of 0.03%;
- Risk neutral PDs should not be used as estimates of observed (historical) PDs.

There are some more specific conditions to be met. In the list above there are the ones important to build the equities model. For more details see [3] and [4].

1.3 Brief Review about this subject

Right after the document [1] was released many authors published some interesting works about the subject. We will briefly describe the most important of these works.

The first work about the IRC theme was see [5]. In this work the author explores IRC for bonds and equities. The work explains how to build the model and what the demands from the regulation meant for the modeling perspective. At the end the author evaluate some results, but those lack some more profound exploration.

A second work about IRC is [6]. The author begins by describing the CreditMetrics model and the need to modify it to attend the Basel Committees modeling requirements. He divided the study in three parts: first, the requirements and principles of the model were discussed so a complete model could be derived in terms of a simulation model and a correlation structure; second, the estimation of the required inputs in the derived model; third, the assessment of the model and its inputs and assumptions. The conclusion is that the copula assumption, the assumed lengths of the liquidity horizons of the assessed positions, the applied conditionality in credit migration matrix and the average level of issuer asset correlations in the model are crucial inputs in the estimation of the required risk measure in any IRC model.

A third paper about the IRC subject is [7]. The paper introduces the common multi-factor model for portfolio credit risk by first giving an overview of the foundation univariate and multivariate Merton (1974) model and then proceed to discuss the multi-factor model version. They calculated an IRC and in particular analyzed the effect of the liquidity horizon. They demonstrated that the market and regulatory rationale for assigning short liquidity horizons for investment grade credits and longer liquidity horizons for non-investment grade is aligned with banks incentives of how to allocate the liquidity horizons across different credit grades by minimizing the IRC add-on.

A forth paper about IRC is [8]. The focus of this paper is about the methodology for building a transition probability matrix (TPM), since to model IRC is necessary modeling default and migration with a period shorter than one year.The author concludes that given the importance of TPMs and their PDs in the IRC, financial institutions will need to make discretionary choices regarding their preferred methodology while ensuring that uncertainties are well understood, managed and communicated properly to local regulators.

These last works all focused on the IRC as a measure of risk for price and default. The first work, to analyze default in separate was [9]. Additionally, in [9] we see in this work a application to the Brazilian equity market. The conclusion of the work is that the amount of capital charge generated by the IRC formulation, considering only the default would be comparable to a one year Value at Risk for price. This result occurs mainly because of the constant level of risk condition in the IRC measure. In this paper we continue to work in [9], but following the new modifications seen in [3].

More recently, the work [14] explores the DRC in its new format. But the paper explores in more detail european companies and it did note use a structural model for modeling the PD. In addition of using the Brazilian companies, we also compare the obtained DRC results with a standard Value at Risk for market risk.

2 Problem Formulation

2.1 FRTB Internal Model

In this section we see how to model the DRC measure. We will do this in a few steps: first we need to compute the probability of default for each obligor;

second we discuss how to build a multi-factor model; and the third part how to compute the Portfolio Loss, this means the DRC.

2.1.1 The Merton Model for PD

Since our goal is to compute the DRC for an equities portfolio. We are going to compute the Probability of Default for each obligor using the Merton Model. Before explaining how to calculate the probability of default its important to bring to light the fact that the regulation requires that banks use PDs which are not risk neutral. Usually banks have processes that use observed default data that permits to compute PD which are not risk neutral. In this article we simple use a structural model, i.e Merton Model, as a way to compute a Probability of Default for the exercise of understanding the DRC measure. To get out of the risk neutral measure, we may use an adaptation which comes from bond data, see [13]. More over, this is a simple model, that consider returns as log-normal distributed. This is typically not true since credit events are heavy tailed. We intend to address these two points in future works.

The model analyses the Balance Sheet of a firm which is composed by assets, which are equal to liabilities plus owner's equity, see table 1. By using the Merton structural bond pricing model [11], we are able to price the value of the assets of a firm by considering a single class of homogeneous debt and a residual claim on the equity.

<table>
<tr><th>Asset</th><th>Liabilities</th></tr>
<tr><td rowspan="2">Firm Value: A_t</td><td>Debt: $L(t, A_t)$</td></tr>
<tr><td>Equity: $E(t, A_t)$</td></tr>
<tr><td colspan="2">Total: $A_t = L(t, A_t) + E(t, A_t)$</td></tr>
</table>

Table 1: An illustration of the balance sheet structure.

In the Merton Model, the shareholders get the residual claim on the equity. If $E(t, A_t) \leq 0$ (this is the same as $A_t \leq L(t, A_t)$), the shareholders have the option to give the company away to the debt holders without any additional charge. The Debt $L(t, A_t)$ is a zero coupon bond with face value L and maturity in T. Thus, the Probability of Default is $P(A_t \leq L(t, A_t))$.

First we derive the PD by using the Black and Scholes formula and second we associate the asset's volatility to the owners equity volatility, see [10] and [11].

Supposing that the value of the assets of the firm, A_t, follows a log-normal distribution, meaning that $G = lnA_t$ follows a normal. By Itô Lemma's (for the following demonstration we will consider $t_0 = 0$ and $t_{maturity} = T$):

$$dG = \left(\mu_A A \frac{\partial G}{\partial A} + \frac{\partial G}{\partial t} + \frac{\sigma_A^2 A^2}{2} \frac{\partial^2 G}{\partial A^2}\right) dt + \sigma_A A \frac{\partial G}{\partial A} dW_t \tag{1}$$

In eq. (1) the brownian $W_t \sim \mathcal{N}(0, t)$. Additionally the eqs. (2) follow directly from the definition of $G = lnA_t$:

$$\frac{\partial G}{\partial A} = \frac{1}{A} \; ; \; \frac{\partial G}{\partial t} = 0 \; ; \; \frac{\partial^2 G}{\partial A^2} = -\frac{1}{A^2} \tag{2}$$

Thus, by substituting eqs. (2) in eq. (1), we arrive at the eq. (3):

$$d(lnA_t) = \left(\mu_A - \frac{\sigma_A^2}{2}\right)dt + \sigma_A dW_t \tag{3}$$

From eq. (3) follows eq. (4):

$$lnA_t - lnA_0 = \left(\mu_A - \frac{\sigma_A^2}{2}\right)t + \sigma_A W_t \tag{4}$$

From eq. (4) is easy to see that $lnA_t \sim \mathcal{N}\Big(lnA_0 + \Big(\mu_A - \frac{\sigma_A^2}{2}\Big)t,\, \sigma_A\sqrt{t}\Big)$. The PD will be given by $P(A_t \leq L(t, A_t))$, since the logarithm function is monotonic, we may consider the PD given by $P(lnA_t \leq ln(L(t, A_t)))$. By using the arguments of the previous expressions, we have the Probability of Default at T:

$$PD = N\left(\frac{lnL - lnA_0 - (\mu_A - \frac{\sigma_A^2}{2})T}{\sigma_A\sqrt{T}}\right) \tag{5}$$

To compute the probability of default in eq. (5), we need to approximate the value of σ_A by σ_E. This is necessary, since there is no liquidity in the asset, the only information is the volatility of the equity. Under Mertons assumption, equity is a call option on the value of the firms assets, and it follows the stochastic differential equation eq. (6):

$$dE = \mu_E E dt + \sigma_E E dB_t \tag{6}$$

By the Black and Scholes, see [10], equation to the option pricing model, we get the equation:

$$E_t = A_t N(d_1) - L\exp(-rT)N(d_2) = f(t, A_t) \tag{7}$$

Where, the stochastic process to A_t follows a geometric Brownian motion:

$$A_t - A_0 = \mu_A \int_0^t A_s ds + \sigma_A \int_0^t A_s dB_s \tag{8}$$

And, the stochastic process to E_t follows a geometric Brownian motion:

$$E_t - E_0 = \mu_E \int_0^t E_s ds + \sigma_E \int_0^t E_s dB_s \tag{9}$$

By Itô's Lemma:

$$f(t, A_t) = C_t(A_t, \sigma_A, L, T, r) \tag{10}$$

$$df = \left(\mu_A A \frac{\partial f}{\partial A} + \frac{\partial f}{\partial t} + \frac{\sigma_A^2 A^2}{2} \frac{\partial^2 f}{\partial A^2} \right) dt + \frac{\partial f}{\partial A} \sigma_A A dB_t \tag{11}$$

Comparing diffusion terms in eqs. (6) and (11), we can retrieve the relationship in eq. (12):

$$\sigma_E E_t dB_t = f_A(t, A_t) \sigma_A A_t dB_t \tag{12}$$

$$\sigma_A = \frac{\sigma_E E_t}{f_A(t, A_t) A_t} \tag{13}$$

Where it follows from eq.(7) that $f_A(t, A_t) = N(d_1)$, for a more detailed deduction see [12]. Thus,

$$\sigma_A = \frac{\sigma_E E_t}{N(d_1) A_t} \tag{14}$$

The eq. (14) relates asset volatility to equity volatility. The eqs. (14) and (7) form a system of nonlinear equations with two variables: the asset volatility and the asset value. By solving this system we obtain the elements to be used in the eq. (5) to calculate the PD of an issuer.

2.1.2 The Multi-factor Model for Asset Returns

The second step is to decompose the asset returns into factors. According to Basel Committee, see [4], the decomposition will be done in two types of factors, the systematic risk and the idiosyncratic risk. Moreover, the systematic risk needs to be divided into industry risk and region risk, as reference see [12]. Based on that, we will consider the asset return of each issuer i as in eq. (15). For a portfolio of N different companies, we have:

$$r_i = \sum_{j=1}^{M} \beta_{i,j}^{reg} Y_j^{reg} + \sum_{j=1}^{N} \beta_{i,j}^{ind} Y_j^{ind} + \gamma_i \epsilon_i \ ; \ i = 1, 2, \ldots, N \tag{15}$$

By assuming normal distribution such that the asset return $r_i \sim \mathcal{N}(0, 1)$, the region component $Y_i^{reg} \sim \mathcal{N}(0, 1)$, the industry component $Y_i^{reg} \sim \mathcal{N}(0, 1)$ and the idiosyncratic component $\epsilon_i \sim \mathcal{N}(0, 1)$, and that they are independent and identically distributed (i.i.d.) we get:

$$\mathbf{V}(r_i) = \sum_{j=1}^{M} (\beta_{i,j}^{reg})^2 \mathbf{E}((Y_j^{reg})^2) + \sum_{j=1}^{N} (\beta_{i,j}^{ind})^2 \mathbf{E}((Y_j^{ind})^2) + \gamma_i^2 \mathbf{E}(\epsilon_i^2) = 1 \tag{16}$$

$$\gamma_i = \sqrt{1 - \sum_{j=1}^{M} (\beta_{i,j}^{reg})^2 - \sum_{j=1}^{N} (\beta_{i,j}^{ind})^2} \tag{17}$$

For our exercise, we will use both parts of the systematic risk with one component. So we will have:

$$r_i = \beta_i^{reg} Y^{reg} + \beta_i^{ind} Y^{ind} + \gamma_i \epsilon_i \ ; \ i = 1, 2, \ldots, N \tag{18}$$

$$\gamma_i = \sqrt{1 - (\beta_i^{reg})^2 - (\beta_i^{ind})^2} \tag{19}$$

This means that the region part will measure the correlation with the respective index that the stock belongs and the industry part the correlation with the respective stock industry. It is important to emphasize that the industry and region components only vary when the index change or the company industry changes.

We will compare the results from this form of obtaining the correlation with the IRB approach from Basel II, see [5]. This approach is a closed formula, obtained from a copula model called Assymtotic Single Risk Factor model (ASRF). The correlation is obtained by using eq. (21).

$$r_i = \beta_i Y + \gamma_i \epsilon_i \ ; \ i = 1, 2, \ldots, N \tag{20}$$

Where,

$$\beta_i = 0.12\lambda_i + 0.24(1 - \lambda_i) \ ; \ i = 1, 2, \ldots, N \tag{21}$$

And,

$$\lambda_i = \frac{1 - \exp(-50PD_i)}{1 - \exp(-50)} \ ; \ i = 1, 2, \ldots, N \tag{22}$$

2.1.3 The Portfolio Loss

In order to compute the portfolio loss, meaning the DRC. We need to verify if a company defaults. If so, we need to evaluate the amount lost. Since we are studying equity portfolio, when a company goes into default it is not possible recover anything. This means that its Loss Given Default (LGD) is 100% and its Exposure at Default is equal to the amount traded in the portfolio.

The portfolio loss is obtained by simulating the region, industry and idiosyncratic terms as normally distributed. The main idea is to simulate the asset returns and compare them to the inverse normal of the probability of default of each issuer. If the value of asset return simulated is smaller than the value of the inverse of the PD, this issuer is considered to be in default.

The simulation of the region and industry returns are associated to scenarios, which are common to the comparable issuers. In each scenario we will simulate each component and then multiply it by its load. Additionally, each issuer has an idiosyncratic term, which is particular to them, see table (2). In that example, we simulated K scenarios, for N companies in a single country and in M different industry, like the Ibovespa Index companies. If for a particular scenario, the position is in default, then we compute the Loss. The Loss for an issuer is the product of the LGD by the EAD. See table (3) for a picture.

The portfolio loss with 99,9% confidence level, i.e the DRC will be given by the eq. (23) by:

$$P\left(\left(\sum_{i=1}^{n} LGD_i EAD_i \mathbb{1}_{[r_i<N^{-1}(PD_i)]}\right) > DRC\right) = 99.9\% \tag{23}$$

Scenario	Region	Industry	Idiosyncratic
1	Y_1^{Reg}	$Y_1^{Ind_1}$	$\epsilon_{1,1}$
	Y_1^{Reg}	$Y_1^{Ind_1}$	$\epsilon_{1,2}$
	Y_1^{Reg}	$Y_1^{Ind_2}$	$\epsilon_{1,3}$
	Y_1^{Reg}	$Y_1^{Ind_2}$	$\epsilon_{1,4}$
	$\vdots$	$\vdots$	$\vdots$
	Y_1^{Reg}	$Y_1^{Ind_M}$	$\epsilon_{1,N-1}$
	Y_1^{Reg}	$Y_1^{Ind_M}$	$\epsilon_{1,N}$
2	Y_2^{Reg}	$Y_2^{Ind_1}$	$\epsilon_{2,1}$
	Y_2^{Reg}	$Y_2^{Ind_1}$	$\epsilon_{2,2}$
	Y_2^{Reg}	$Y_2^{Ind_2}$	$\epsilon_{2,3}$
	Y_2^{Reg}	$Y_2^{Ind_2}$	$\epsilon_{2,4}$
	$\vdots$	$\vdots$	$\vdots$
	Y_2^{Reg}	$Y_2^{Ind_M}$	$\epsilon_{2,N-1}$
	Y_2^{Reg}	$Y_2^{Ind_M}$	$\epsilon_{2,N}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
K	Y_K^{Reg}	$Y_K^{Ind_1}$	$\epsilon_{K,1}$
	Y_K^{Reg}	$Y_K^{Ind_1}$	$\epsilon_{K,2}$
	Y_K^{Reg}	$Y_K^{Ind_2}$	$\epsilon_{K,3}$
	Y_K^{Reg}	$Y_K^{Ind_2}$	$\epsilon_{K,4}$
	$\vdots$	$\vdots$	$\vdots$
	Y_K^{Reg}	$Y_K^{Ind_M}$	$\epsilon_{K,N-1}$
	Y_K^{Reg}	$Y_K^{Ind_M}$	$\epsilon_{K,N}$

Table 2: Monte Carlo simulation of asset values to generate correlated loss.

Scenario	Loss
1	$\sum_{i=1}^{n_1} LGD_i EAD_i \mathbb{1}_{[r_i<N^{-1}(PD_i)]}$
2	$\sum_{i=1}^{n_2} LGD_i EAD_i \mathbb{1}_{[r_i<N^{-1}(PD_i)]}$
$\vdots$	$\vdots$
K	$\sum_{i=1}^{n_K} LGD_i EAD_i \mathbb{1}_{[r_i<N^{-1}(PD_i)]}$

Table 3: Monte Carlo simulation of asset values to generate correlated loss.

2.2 Minimum capital requirements model

In this paper we will compare the standardized value DRC that regulation sets with the one modeled in the last section, see [3]. For the equities case it is a very simple formula, see eq. (24).

$$DRC_s = \max\left[\Big(\sum_{i\in long} RM_i JTD_i\Big) - WtS\Big(\sum_{i\in short} RM_i JTD_i\Big); 0\right] \tag{24}$$

Where,

$$JTD_i = \max\left(LGD_i EAD_i + P\&L_i, 0\right) \tag{25}$$

$$WtS = \frac{\sum JTD_{long}}{\sum JTD_{long} + \sum |JTD_{short}|} \tag{26}$$

For the equities case, the $LGD_i = 100\%$, $EAD_i = notial$ and $P\&L_i = 0$. We are not going to simulate portfolios with short positions, so $WtS = 1$ and the short exposure is zero.

The RM_i according to [3] follows a table that is based in the issuer agency rating, which gives the PD. In our exercise we will use the PD obtained for the Merton Model, i.e. we will not consider the correlation like the modeled developed here.

The number obtained in this section will be compared with the one modeled. If the Internal model is bigger the 80% of the value of the standardized one, then the number used in capital allocation is the Internal model one. Otherwise we set 80% of the standardized value as the capital for allocation.

3 Numerical Simulation

We are going to show some results obtained for two kind of portfolios. The first one is base in a portfolio of Ibovespa Stocks. The second one is base in a portfolio of S&P500 Stocks. These portfolios will be simulated in a 10 year window so we can see the effects. Moreover, we will compare the standardized methodology with the internal model one. To obtain the PD it is necessary to solve, simultaneously, the two nonlinear eqs. (7) and (14) to work out the value and volatility of the firm's assets. Afterwards, we start to solve the Probability of Default eq. (5), the parameters to compute the PD are described in the following itens:

- The volatility of equity: the volatility of the equity is calculated by the historical equity return data. In eq. (27) we have the volatility of the equity and the log return, which will be calculated for 252 trading days.

$$\sigma_E = \sqrt{\frac{n}{n-1}\sum_{i=1}^{n}\mu_i^2 - \frac{1}{n-1}(\sum_{i=1}^{n}\mu_i)^2} \quad ; \quad \mu_i = \ln\frac{S_i}{S_{i-1}} \tag{27}$$

- The market value of equity: this is calculated as the number of shares times the equity price. The number of shares was obtained from the quarterly balance sheet.

- The drift component: will be substituted by the Risk-free interest rate. In the case of the Ibovespa stocks, the first future of DI was used as the risk-free interest rate. In the case of the S&P500 stocks the Libor rate was used as the risk-free interest rate.

- Time: the company maturity was set as one year to calculate the Probability of Default. Naturally an adaption had to be made, since we need to consider the liquidity horizon. For liquid stocks this period may be considered as three months, see [3]. After computing the one year PD, we convert to the three month PD by the eq. (28):

$$(1 - PD_{3month})^4 = 1 - PD_{1year} \Rightarrow PD_{1month} = 1 - \sqrt[4]{1 - PD_{1year}} \tag{28}$$

- Liability of the Firm: obtained from the quarterly balance sheet. The liabilities are equal to the short-term plus one half of the long-term one.

- The value an volatility of the firm's asset: the two nonlinear eqs. (7) and (14) use as input the five parameters above. To solve this system of equations they are modified to the equations eqs. (29). To solve this system we will use a Newton-Raphson method for a system of non-linear equations, just as described before.

$$f(E_t) = A_t N(d_1) - L\exp(-rT)N(d_2) - E_t = 0 \tag{29}$$

$$f(\sigma_E) = N(d_1)A_t\sigma_A - \sigma_E E_t = 0 \tag{30}$$

- Probability of Default: lastly, we apply the result obtained in the previous items to calculate the PD using eq. (5).

For the simulation of the portfolio, we still need to make a couple of calculations:

- Asset return sensibility to the region: this beta is simply the equity correlation of a certain stock to its index. This approximation is permitted by the Basel Committee, see [3].
- Asset return sensibility to the industry: this beta is the equity correlation of a certain stock to a index composed of all the stocks from a specific industry belonging to a certain stock index. This composed index as built with a market cap setting.
- Asset return time adaptation: since the PD used will be for a three month period, it is necessary to adapt the asset return, by multiplying by the square root of time.

4 Main Results

In this section, we go through the results obtained by applying the developed model. In figs. (1), (2) and (3) we see the results when applied to a portfolio which was built equal to the Ibovespa Index. In figs. (4), (5) and (6) we see the results when applied to a portfolio which was built equal to the S&P500 Index. In both cases, we took from off the companies belonging to the Financial Sector, and maintained the proportionality of each member to the total.

In fig. (1) we show the PD calculated for each member of the Ibovespa Index separately. In this first graph, time is months, we see that during the crisis of 2008 the PD increases considerably. This happens mainly due to an increase of volatility of the companies, we may see this in fig. (2). The PD of these companies also increased in 2016, but for a different reason. Brazil lost its investment grade in the last quarter of 2015, which made the Brazilian currency depreciate. Since the debt of many Brazilian companies are in US dollars, with the currency depreciation the company debt increased doing the same to the PD, see fig. (2). We can also see in fig. (1) a pick by the end of 2013. This was due to OGX and MMX both companies filled for bankruptcy by the end of that year.

In fig. (3) we see the results of the three methodologies proposed in this work for the Ibovespa companies. The lower curve in the first figure is the DRC calculated as standard model. Since in this form we lack the effect of the correlation, the results underestimate the risk. The curve in the middle, DRC IRB, is the ASRF model which seems to underestimate the results as well. The higher curve, uses the correlation as calculated from equity returns.

Finally, we compare in the second graph of fig. (3) the standard 10-day VaR with the DRC modeled with correlation from the equity returns. We see in the graph that, in average, that the DRC is twice the value of the 10-day VaR. This demonstrates the importance of considering the default risk from listed companies.

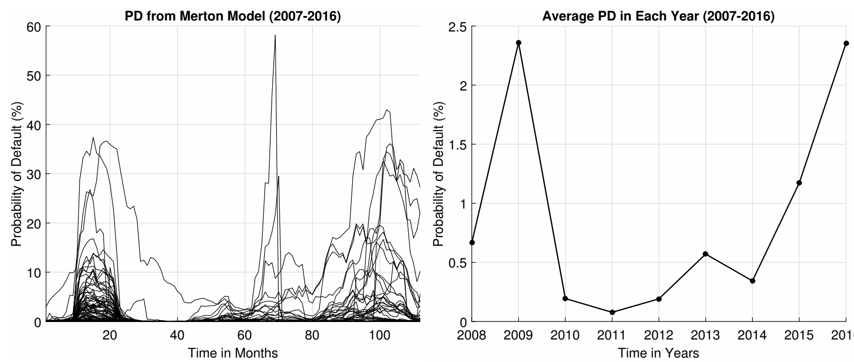

Figure 1: Probability of Default for Ibovespa Members

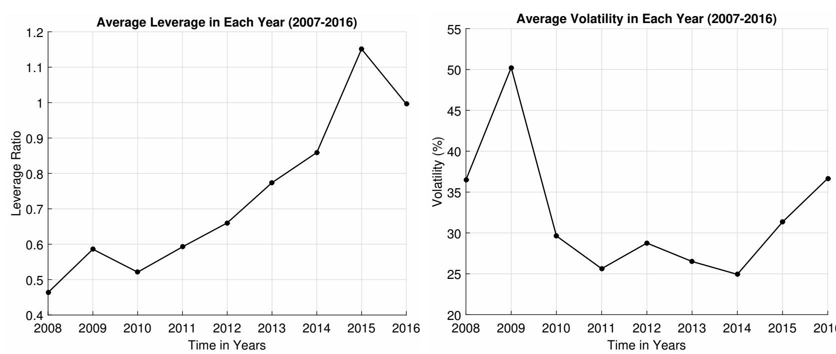

Figure 2: Average Leverage and Average Volatility for Ibovespa Members

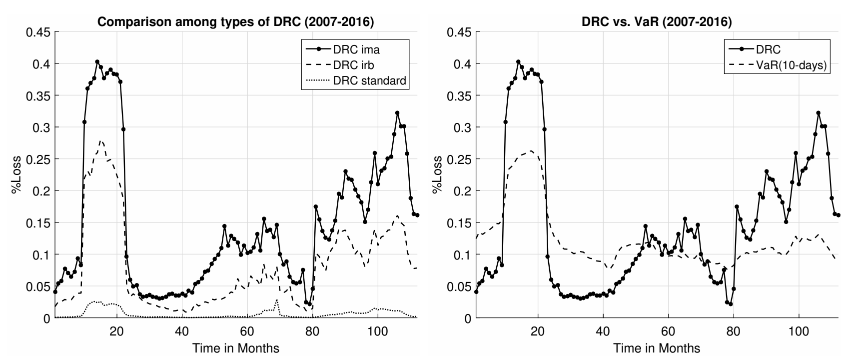

Figure 3: Comparison of Default Risk Charge Methodologies and Value at Risk for Ibovespa

In fig. (4) we show the PD calculated for each member of the S&P500 Index separately. In this first graph, time is months, we see that during the crisis of

2008 the PD increases considerably. This happens mainly due to an increase of volatility of the companies, we may see this in fig. (5). The PD of these companies also increased in 2016, for the same reason.

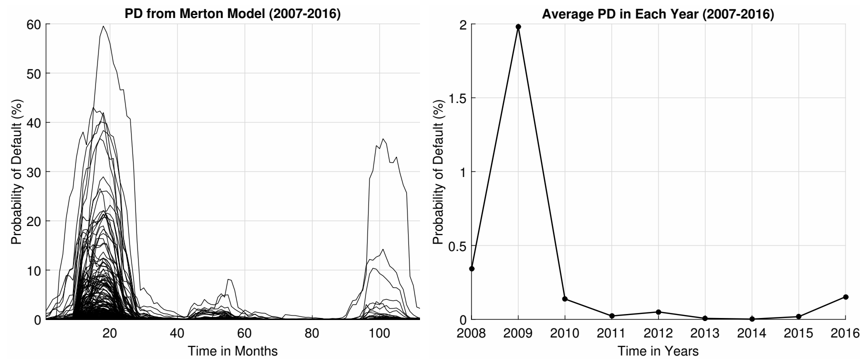

Figure 4: Probability of Default for S&P500 Members

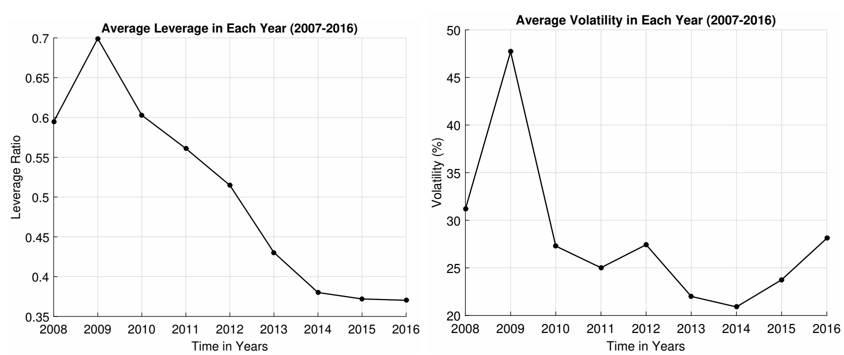

Figure 5: Average Leverage and Volatility for S&P500 Members

In fig. (6) we see the results of the three methodologies proposed in this work for the S&P500 companies. The results show the same conclusion as for Ibovespa companies. Finally, we compare in the second graph of fig. (6) the standard 10-day VaR with the DRC modeled with correlation from the equity returns. We see in the graph that, in average, that the DRC is half the value of the 10-day VaR. Again this demonstrates the importance of considering the default risk from listed companies.

5 Conclusion

This work discussed the importance of allocation the risk of default in the trading book for equities. We analyzed results from two case studies, Ibovespa and

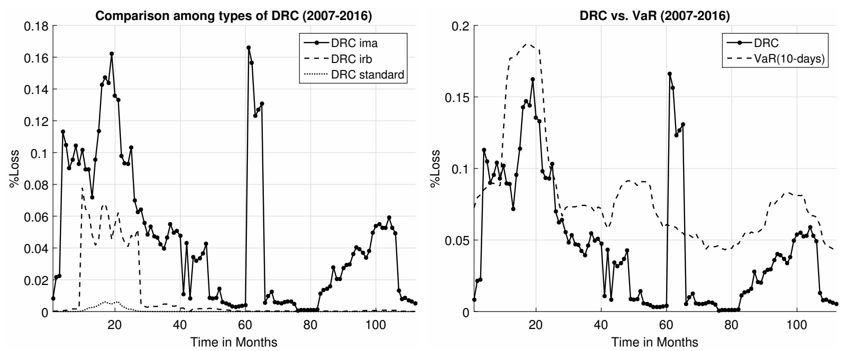

Figure 6: Comparison of Default Risk Charge Methodologies and Value at Risk for S&P500

S&P500 companies. Which were simulated as long portfolios without companies from the Financial Sector. Both cases demonstrate that the DRC is comparable with a 10-day VaR. We demonstrated how to build the process of calculating the DRC measure as an internal model and discussed the necessary changes required to make the process acceptable in the eyes of regulator. Since the PD needs to be calculated from observed default cases.

In this work the PD was modeled using a structural approach from the classical Merton Model. This gives results which are underestimated since default do not follow log-normal processes. In a future work, we intend to simulate these case studies with more adequate models, to better measure the default risk from the companies.

Finally, we saw that using correlation from equity returns in a multi-factor model, we obtained results witch reflect in a better way the risk companies suffer when in systemic events.

References

[1] Basel Committee (2009) *Guidelines for computing capital for incremental risk in the trading book.* Consultative Document.

[2] Basel Committee (2014) *Fundamental review of the trading book: A revised market risk framework.* Consultative Document.

[3] Basel Committee (2016) *Minimum capital requirements for market risk.* Consultative Document.

[4] Basel Committee (2017) *Frequently asked questions on market risk capital requirements.* Consultative Document.

[5] T. Xiao (2009) *Incremental Risk Charge Methodology.* Available at SSRN: https://ssrn.com/abstract=2426836.

[6] Y. Stel (2010) *The Incremental Risk Model.* Master Dissertation, Department of Finance, Erasmus School of Economics.

[7] J. Skoglund & W. Chen (2010) *Calculating incremental risk charges: The effect of the liquidity horizon.* Munich Personal RePEc Archive.

[8] T. Yavin, H. Zhang, E. Wang & M. Clayton (2011) *Transition Probability Matrix Methodology for Incremental Risk Charge.* Munich Personal RePEc Archive.

[9] M. Rodrigues (2015) *he effect of default risk on trading book capital requirements for public equities: an irc application for the Brazilian market.* Master Dissertation, Department of Economics, Sao Paulo School of Economics.

[10] F. Black & M. Scholes. (1973) The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, v. 81, n. 3, p. 637-654.

[11] R. Merton. (1974) On the pricing of corporate debt: The risk structure of interest rates. *The Journal of Finance*, v. 29, n. 2, p. 449-470.

[12] C. Bluhm, L. Overbeck & C. Wagner. (2016) *Introduction to Credit Risk Modeling.*, Crc Press.

[13] J. Hull, M. Predescu & A. White (2005). *Bond prices, default probabilities and risk premiums.* Available at SSRN: https://ssrn.com/abstract=2173148.

[14] J. Laurent, M. Sestier & S. Thomas (2016). *Journal of Banking & Finance.* Trading book and credit risk: How fundamental is the Basel review?, v. 73, p. 211-223.