

# Wrong-way-risk in tails

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## Abstract

With new regulations like the credit valuation adjustment (CVA) the assessment of wrong-way-risk (WWR) is of utter importance. We analyze the effect of a counterparty's credit risk and its influence on other asset classes (equity, currency, commodity and interest rate) in the event of extreme market movements like the counterparty's default. With an extreme value approach we model the tail of the joint distribution of different asset returns belonging to the above asset classes and counterparty credit risk indicated by changes in CDS spreads and calculate the effect on the expected shortfall when conditioning on counterparty credit risk. We find the conditional expected shortfall to be 2% to 440% higher than the unconditional expected shortfall depending on the asset class. Our results give insights both for risk management as well as for setting an initial margin for non-centrally cleared derivatives which becomes mandatory in the European Market Infrastructure Regulation (EMIR).

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# 1 Introduction

In this paper we study the dependence of different asset classes in extreme events and increasing stress in the banking sector. Using extreme value theory we model the tail of the joint distribution of asset returns and counterparty credit risk and analyse the impact on the risk measure expected shortfall when conditioning on mayor stress in the banking sector for a period from December 2005 until January 2016.

We find increasing correlation between assets belonging to either equity, currency, commodity or interest rates on government bonds and mayor stress in the European and American banking sector. Applying our framework to the calculation of the on financial stress conditioned expected shortfall leads to considerably higher values compared to the unconditioned expected shortfall although periods like the financial crisis are already considered in the calculation. For instance, the weekly conditioned expected shortfall on a 97.5 percent level is about 2 to 440 percent higher depending on the asset than the unconditioned. These results are relevant for margin requirements for non-centrally cleared derivatives, a framework released by the Basel Committee on Banking Supervision (BCBS) and the International Organization of Securities Commissions (IOSCO) in 2013 with revision in 2015 (Basle Committee on Banking Supervision, 2015).

The margin is split into the variation margin which compensates for the daily profit and loss and the initial margin which covers the loss between the default of the counterparty and the close-out of the position. By this definition the variation margin solely depends on the changes of the underlying assets whereas the initial margin should include counterparty credit risk and possible wrong-way-risk.

Wrong-way-risk is especially subject of discussions for the calculation of the credit valuation adjustment (CVA) which is the difference of the value of a derivative or portfolio without counterparty credit risk and a derivative or portfolio exposed to counterparty credit risk. For the calculation of CVA including wrong-way-risk the whole joint distribution of asset returns and counterparty credit risk is modelled by correlating the marginal distributions by some dependence structure. For instance, Hull and White (2012) proposed a valuation method how to include counterparty credit risk in the calculation of CVA by linking the exposure and the

probability of default by a function. Including wrong-way-risk into the calculation of CVA can also be achieved by correlating the exposure and the probability of default by copulas (see Rosen and Saunders, 2012; Hofer, 2016). A recent approach is to look at the worst case as firstly considered by Glasserman and Yang (2015) and adopted by Kenyon and Green (2016). In contrast to the approach in context of CVA calculations, we study wrong-way-risk in tail events with focus on a possible non linear dependence structure of asset returns and counterparty credit risk.

The model and data is presented in the following section. In Section 3 we present the results for the dependence between the different asset classes and mayor financial stress in the banking sector. Our model shows a significant correlation in case of almost all asset classes components. For some like the equity indices, some currency pairs and interest rates on government bonds the correlation actually increases significantly. As an indicator for stress in the banking sector we take CDS spreads of the iTraxx senior-financial 5y for European banks and an CDS index for American banks. We take the Euro Stoxx50 Index and the S&P 500 Index, the mayor currency pairs, the Goldman Sachs Commodity Index and gold as well as interest rates on government bonds with different maturities. In section 4 we use our results to do a sensitivity analysis on the expected shortfall conditioned on mayor financial stress addressing the wrong-way-risk topic when computing initial margins for non-centrally cleared derivatives becoming mandatory as part of the European Market Infrastructure Regulation (EMIR). Section 5 concludes.

## 2 Model and data

The focus is to model the dependency between stress in the banking sector and extreme returns of different asset classes. We apply a two-dimensional distribution function  $F_R^\theta$  which consists of two generalized Pareto distributions  $G_{R_i}^{\theta_i}$ ,  $i = 1, 2$ , to model the extremes over predefined thresholds  $\theta_1$  and  $\theta_2$  (see Longin and Solnik, 2001). The dependence structure, in case both thresholds are exceeded, is modelled by a Gumbel–Hougaard copula  $C(u_1, u_2) = \exp\left(-\left((- \log u_1)^{1/\alpha} + (- \log u_2)^{1/\alpha}\right)^\alpha\right)$  which is also known as logistic copula (see Gumbel, 1961). Since the Pearson’s correlation coefficient  $\rho$  can be calculated by  $\rho = 1 - \alpha^2$ , we focus on linear dependence (see Tiago de Oliveira, 1973). With  $\theta = (\theta_1, \theta_2)$  the two-dimensional

distribution function is defined by

$$F_R^\theta(x_1, x_2) = C(F_{R_1}^{\theta_1}(x_1), F_{R_2}^{\theta_2}(x_2)) \quad (1)$$

with marginal distributions

$$F_{R_i}^{\theta_i}(x_i) = (1 - p_i) + p_i \times G_{R_i}^{\theta_i} = 1 - p_i(1 + \xi_i(x_i - \theta_i)/\sigma_i)_+^{-1/\xi_i} \quad (2)$$

where  $p_i$  is the probability of exceeding the threshold  $\theta_i$  and  $\xi_i$  and  $\sigma_i$  being the shape and scale parameters of the generalized Pareto distribution  $G_{R_i}^{\theta_i}$ .

An illustration of the distribution and its behaviour is given in [Figure 1](#). On the left hand side 1,000 realizations of a bivariate normal distribution with correlation  $\rho = 0.6$  are plotted together with thresholds  $\theta_1$  and  $\theta_2$ . The plot on the right hand side illustrates the theoretical development of the correlation coefficient  $\rho = 1 - \alpha^2$  when increasing the thresholds simultaneously to the quantiles of  $R_1$  and  $R_2$  where  $\alpha$  is estimated via the maximum likelihood method applied to  $F_R^\theta$ .

< Figure 1 here >

At the 50 percent quantile the correlation is 0.6 and decreases when moving the thresholds to the extremes resulting asymptotically in  $\rho = 0$  although the correlation is constant at 0.6 for the whole sample. Therefore, we say that the correlation increases (decreases) for extreme events if  $\rho$  is significantly above (below) its theoretical value implied by the model assuming a constant correlation for the whole sample. If  $\rho$  is neither above nor below, we say that the correlation stays constant for extreme events.

### *2.1 Test for increasing/decreasing correlation in the extremes*

Longin and Solnik (2001) develop an approach to test whether the correlation increases (decreases) in the extremes by sampling from a bivariate normal distribution with correlation coefficient equal to the estimated correlation of their empirical data. The corresponding sampled points are used to estimate the correlation coefficient in the extremes compared to the estimated correlation with the likelihood ratio test.

In order to test for increasing (decreasing) correlation without assuming the marginal distributions being the normal distribution, we proceed as follows: Let  $(R_1, R_2)$  be a set of realizations of some unknown two-dimensional distribution function  $H$  with marginal distribution functions  $F_1$  and  $F_2$ , respectively, with some unknown dependence structure. From these realizations we compute the corresponding marginal empirical distribution functions  $\hat{F}_1$  and  $\hat{F}_2$  and the Pearson correlation coefficient  $\hat{\rho}$ . Using a Gaussian-Copula with correlation coefficient  $\hat{\rho}$  of dimension two, we generate  $N$  two-dimensional points to sample from  $\hat{F}_1$  and  $\hat{F}_2$ . By this construction the resulting sample of  $N$  simulated points  $(R_1^{sim}, R_2^{sim})$  has the bivariate distribution function  $H_{sim}$  with marginal distributions  $\hat{F}_1$  and  $\hat{F}_2$  and a linear dependence structure with correlation coefficient  $\hat{\rho}$ . Then, the dependence coefficient  $\alpha$  of the distribution function (1) is estimated on the one hand from the simulated points  $(R_1^{sim}, R_2^{sim})$ , referred to as  $\check{\alpha}$ , as well as from  $(R_1, R_2)$ , referred to as  $\hat{\alpha}$ . Finally,  $\check{\alpha}$  and  $\hat{\alpha}$  are compared via the likelihood ratio test. If  $\hat{\alpha}$  is significantly greater (smaller) than  $\check{\alpha}$ , the correlation increases (decreases) in the extremes.

## 2.2 Data

To test the setting empirically we analyse the behaviour of four different asset classes in extreme events when there is significant stress in the banking sector. Representing the asset class equity, we use the Euro Stoxx50 Index and the S&P 500 Index. In the asset class currency we take the four major currency pairs for EUR and for USD to each other and GBP, CHF and JPY. The third asset class includes commodities covered by the Goldman Sachs Commodity Index (GSCI) and gold as a hybrid between currency and commodity. Finally, we analyse interest rates of government bonds of countries in the Euro area with long-term (10-years), mid-term (5-years) and short-term (6-months) maturity provided by the ECB<sup>1</sup> and the corresponding U.S. Treasury securities provided by the FED<sup>2</sup>.

As proxy for stress in the banking sector, we choose the CDS Index iTraxx senior-financials from Bloomberg with a 5-year maturity (on-the-run quote) for

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<sup>1</sup>Yield curve spot rate, 10-year maturity - Government bond, nominal, all issuers all ratings included - Euro area (changing composition)

<sup>2</sup>Market yield on U.S. Treasury securities at constant maturity, quoted on investment basis

the European banks which is composed of the 25 most liquid credit default swaps of financial institutions. Similarly, we create a CDS Index US Financials 5y for the American banks including 5-year CDS from Datastream of JPMorgan Chase & Co., Citigroup, Bank of America, Wells Fargo & Company, Goldman Sachs, Lehman Brothers and Bear Stearns. Each dataset consists of weekly changes from 1. December 2005 until 29. January 2016 corresponding to 578 observations.

< Table 1 here >

**Table 1** shows a summary of the (log)-returns and contains the mean, standard deviation (SD), maximum and minimum of the weekly returns. Additionally, the correlation with the CDS indices is included. Except for gold and the European government bonds all correlations with the iTraxx senior-financials are negative, i.e. prices tend to decrease when stress in the banking sector increases and vice versa. The correlations with the CDS Index US Financials 5y has the same sign for equity and commodities but the correlation with the US Treasury securities is negative for all maturities and except for USD/JPY the correlation with the currencies is positive. The correlations are all significantly different from zero on the 1 percent level except for EUR/GBP, USD/CHF and gold in USD.

### 3 Wrong-way-risk in tails: empirical findings

To illustrate the method proposed we focus on the behaviour of four asset classes, namely equity, currency, commodities and interest rates, in their extremes under high stress in the banking sector. In a first step we calculate the correlation of the weekly changes of the different assets and the iTraxx and the CDS Index US Financials 5y respectively for the full sample period. The sign of the correlation determines if the negative extremes or the positive extremes of the assets are considered. For example, we estimate  $\alpha$  of the distribution function (1) from the positive extreme returns for gold and the negative extreme returns for the equity indices like the Stoxx50 via the maximum likelihood method. Therefore, when we say the correlation for the Stoxx50 increases in its extreme, we mean the correlation increases in case of extreme negative returns. The thresholds for the weekly CDS indices changes start at zero and then increase gradually in 5 basis

point (bp) steps to 25 bp. A weekly change of 25 basis points of the iTraxx can be considered as significant stress in the European banking sector. There are only 15 observations which are larger than 25 basis points, of which five are connected to the financial crisis in 2007-2009 and 10 associated with the European debt crisis 2010-2013. Similarly the CDS Index US Financials 5y exceeds 25 bp six times connected with the financial crisis in 2008 and 2009 and four times during the European debt crisis 2010 and 2011. Accordingly, the thresholds for the assets' (log)-returns start at zero and gradually increase from the value corresponding to the 59 percent quantile to the value corresponding to the 95 percent quantile by a step size of 9 percentage points. The thresholds for the CDS indices and the assets are increased simultaneously, i.e. they start with (0,0) and then go to (5, 59 percent quantile) until (25, 95 percent quantile). The quantile values for all assets can be found in [Table 2](#).

< Table 2 here >

The main results are shown in [Figure 2](#) and [Figure 3](#) for the iTraxx and the CDS Index US Financials 5y, respectively.

< Figure 2 and 3 here >

*Equity (Figure 2 and Figure 3 top left)*

The co-movement between negative returns in stock markets and the iTraxx is very strong as expected since a CDS insures against the default of a company. The correlation significantly increases for the Stoxx50 on a 5 percent level in contrast to the S&P 500 where the correlation stays constant approaching the extremes. European banks are traditionally more closely connected to European companies than US companies in terms of debt and equity. Furthermore, banks are listed in those equity indices and a bank crisis leads to simultaneously dropping stock prices and spiking CDS spreads for banks.

Similar results obtain for the CDS Index US Financials 5y, but here for both indices the correlation increase significantly on a 5 percent level.

*Currency (Figure 2 and Figure 3 top right)*

The results for currencies in times of financial turmoil is mixed. While the correlation for EUR/USD and EUR/CHF increases significantly, the correlation for EUR/JPY stays constant in their extremes. The correlation for EUR/GBP does not change significantly from zero. These results clearly support the assumption of wrong-way-risk for non-European banks and investors holding positions in derivatives on especially EUR/USD and EUR/CHF but also EUR/JPY with an European bank as counterparty. This kind of counterparty credit risk is quantified in the next section.

In case of the CDS Index US Financials 5y all correlations stay constant in the extremes except for USD/CHF. This is especially interesting since the correlation for USD/CHF is not significantly different from zero when using the full sample. Since all correlations are positive except for USD/JPY, the direction of the wrong-way-risk is switched. While EUR devalues when European banks are under financial stress, the USD appreciates in times of financial turmoil in the American banking sector. This can be explained by the fact the USD is the primary reserve currency and financial turmoil in the American banking sector spreads to other countries and is no isolated event.

*Interest rates (Figure 2 and Figure 3 bottom left)*

The interest rates of European government bonds show a clear ranking of the different maturities. The 10-year maturity shows overall the lowest correlation followed by the 5-years maturity and then the 6-months. Furthermore, only the 5-year and 6-months maturities increase significantly in their extremes while 10-years stay constant. Since the correlation is positive, it means interest rates hike in times of financial stress in the European banking sector. One explanation for this dependence between interest rates of European government bonds and stress in the banking sector could be the bank bailouts during the financial crisis in 2008. As Alter and Schuler (2012) show a shock of the financial sector affects sovereign CDS spreads more after bailouts where this effect is significant in the short term and insignificant in the long-term which coincides with our results.

For the CDS Index US Financials 5y and the US Treasury securities the results are opposite. The correlation is negative, meaning interest rates drop in times of

financial stress. The 10-years maturity shows overall the highest correlation followed by the 5-years maturity and then the 6-months. Thus in times of financial stress, investors looking for investments with lower risks in the USA buy US Treasury bonds. For the long term maturity this correlation increases significantly on a 5 percent level in the extremes.

*Commodities (Figure 2 and Figure 3 bottom right)*

The GSCI shows a significantly increasing correlation for the CDS Index US Financials 5y in its extremes whereas gold stays constant for both CDS indices. The increasing correlation for the GSCI can be a sign of the involvement of banks in the commodity sector during the past decade. Surprisingly, gold shows no increasing correlation in the extremes although gold is often referred to as hedge against financial turmoil (see Baur and McDermott (2010)).

## 4 Sensitivity analysis of the expected shortfall

In this section we perform a sensitivity analysis for the expected shortfall in case of counterparty credit risk motivated by the empirical findings of section 3. The expected shortfall without any conditions is compared to the expected shortfall conditioned on default of the counterparty. The analysis focuses on wrong-way-risk, which means the disadvantageous correlation between an increasing value of a contract and a worsening probability of default of the counterparty. It adds insight to the topic of initial margin which is about to become a standard in the OTC market as it is proposed by the Basel Committee on Banking Supervision and the International Organization of Securities Commissions.

Since defaults of large banks are rarely observed, we approximate the expected shortfall conditioned on default by the expected shortfall conditioned on the CDS spread change in basis points exceeding a predefined threshold. We use the iTraxx financials and the CDS Index US Financials 5y as a common bank risk factor for expected shortfall regarding counterparty credit risk for European and American banks, respectively. Denoting the value at risk with level  $\beta$  by  $VaR_{1-\beta}$  and  $\Delta CDS$  being the absolute CDS Spread change in basis points, we compare the un-

conditioned expected shortfall defined by equation (3) to the conditioned expected shortfall defined by equation (4):

$$ES_{1-\beta}(R_1) = E[R_1 | R_1 \geq VaR_{1-\beta}(R_1)] \quad (3)$$

$$ES_{1-\beta}^{CR}(R_1) = E[\tilde{R}_1 | \tilde{R}_1 \geq VaR_{1-\beta}(\tilde{R}_1)] \quad (4)$$

with  $\tilde{R}_1 = R_1 | R_2 \geq \theta_{CDS}$ . We compute the expected shortfalls in equations (3) and (4) by fitting the distribution function  $F_R^\theta$  from equation (1) to the pair  $R = (R_1, R_2)$  where  $R_1$  is the assets return and  $R_2$  the CDS spread change  $\Delta CDS$  from 1. December 2005 until 29. January 2016 as done in section 3. The conditioned expected shortfall in equation (4) is calculated by

$$ES_{1-\beta}^{CR}(R_1) = \frac{1}{1-\beta} \int_{1-\beta}^{\infty} VaR_\gamma(\tilde{R}) \gamma$$

with  $VaR_{1-\beta}(\tilde{R}) = F_{\tilde{R}}^{-1}(1-\beta)$  being the inverse of the probability function having the density function

$$f_{\tilde{R}}(x_1) = \frac{\int_{\theta_{CDS}}^{\infty} f_R^\theta(x_1, x_2) / f_{R_2}^{\theta_2}(x_2) dx_2}{\int_{\theta_{CDS}}^{\infty} f_{R_2}^{\theta_2}(x_2) dx_2}.$$

The probability density functions  $f_R^\theta(x_1, x_2)$  and  $f_{R_2}^{\theta_2}$  are defined as the derivatives of the probability functions defined in (1) and (2), respectively. The unconditioned expected shortfall in equation (3) is calculated with  $R_1$  having the probability function defined in (2), which is a generalized Pareto probability function when exceeding the threshold. Using extreme value theory like generalized Pareto distributions for the calculation of value at risk or expected shortfall has been proposed by, eg. McNeil and Frey (2000) and Yamai and Yoshihara (2005), however, when we are interested in the ratio of the conditioned and unconditioned expected shortfall rather than the levels of the conditioned expected shortfall, we calculate the unconditioned expected shortfall for a portfolio exposed to counterparty credit risk using any convenient method and multiply it with the ratio  $ES_{1-\beta}^{CR}/ES_{1-\beta}$  calculated as described above.

Using weekly returns and setting  $\beta = 0.025$ , we look at the weekly expected shortfall on a 97.5 percent level. We emphasize that the calculation is based on the entire period from December 2005 until January 2016 including periods of mayor stress like the financial crisis. The threshold  $\theta_1$  is set to be the 97.5 quantile value. The threshold  $\theta_2 = \theta_{CDS}$  for the changes of the CDS indices ranges from zero to 25 basis points showing the development of the relative difference when stress increases.

< Figure 4 and 5 here >

The results illustrated in [Figure 4](#) and [Figure 5](#) show significant differences of the expected shortfall unconditioned and conditioned on stress in the European and American banking sector. Except for EUR/GBP and USD/JPY the conditioned expected shortfall is considerably greater than the unconditioned when increasing the threshold  $\theta_{CDS}$ . The values for  $\theta_{CDS} = 25$  are given in [table 3](#).

< Table 3 here >

The conditioned expected shortfall on stress in the European banking sector (American banking sector) is 235 (235) percent higher for the Stoxx50 Index and 190 (212) percent higher for the S&P 500 Index. Except for the EUR/GBP currency pair which is not significantly correlated to stress in the banking sector at all and USD/JPY, the other currency pairs show a similar behaviour. For example, the EUR/USD (USD/EUR) conditioned expected shortfall is 98 (46) percent and the EUR/CHF (USD/CHF) 357 (226) percent higher. Also for commodities the conditioned expected shortfall is distinctly higher with 75 (83) percent for gold and 153 (236) percent for the GSCI. The conditioned expected shortfalls for interest rates for European government bonds are also significantly higher ranging from 51 percent for long-term, 109 percent for mid-term and 253 percent for short-term rates. Considering the US Treasury securities, the 10 years maturity shows the highest relative difference with 120 percent, followed by 6 month with 107 percent and 5 years with 51 percent. These results for the European and the American banking sector show that counterparty credit risk can have a considerably effect on risk measures like the expected shortfall. Furthermore, this has implications

on the topic of initial margin since an initial margin shall cover the loss between default of the counterparty and the close-out of the position. Assuming a short forward position on the Stoxx50 with a nominal of 100 Mio. EUR, the one week expected shortfall on a 97.5 percent level is 9.5 Mio. EUR while the conditioned expected shortfall is 31.8 Mio. EUR. Supposing the initial margin is computed by the one week expected shortfall without considering counterparty credit risk, one faces an expected gap of 22.3 Mio. EUR until the close-out of the position.

## 5 Conclusion

In this paper we analyze the dependence of four major asset classes (equity, currency, commodities and interest rates) and stress in the European and American banking sector separately. We take the iTraxx senior-financials 5y as an indicator for the European and an corresponding artificial CDS Index US Financials 5y for the American banks. Using a bivariate extreme value distribution we model the tails by a generalized Pareto distribution and the dependence by a Gumbel-Hougaard copula and find significant correlation for all four asset classes. Except for gold and European government bonds interest rates, the correlation is always negative with changes in the iTraxx senior-financials 5y, meaning prices tend to decrease in times of major financial stress for European banks. In some cases, like for the Stoxx50, EUR/USD, EUR/CHF or mid and short term interest rates of European government bonds, this correlation even increases in the extremes showing that wrong-way-risk becomes even more important in tail events. In case of stress in the American banking sector, gold and all currencies except USD/JPY are positively correlated and all other assets are negatively correlated with this stress. For the Stoxx50, S&P500, USD/CHF, long term interest rates on US Treasuries and the Goldman Sachs Commodity Index (GSCI) the correlation significantly increases in extreme events. This shows that a crisis in the banking sector has a significant impact on equity prices, the domestic currency and sovereign interest rates. Based on these findings we do a sensitivity analysis of the expected shortfall in the case of mayor stress in the banking sector. When using a 97.5 percent level, the weekly expected shortfall under counterparty credit risk is considerably higher compared to an expected shortfall without counterparty credit risk although pe-

riods like the financial crisis are included in the calculation. For instance, the expected shortfall conditioned on counterparty credit risk of European banks is higher by about 235 percent for the Stoxx50 Index, 98 percent for EUR/USD, 153 percent for the GSCI and 253 percent for short term interest rates of European government bonds compared to the unconditioned expected shortfall. In case of conditioning on counterparty credit risk of American banks, the expected shortfall is 212 percent higher for the S&P500, 226 percent for USD/CHF, 236 percent for the GSCI and 107 percent for short-term interest rates of US Treasury securities. This quantification of the so called wrong-way-risk gives some important insight on the calculation of the initial margin which is about to become a standard for the OTC market as part of the European Market Infrastructure Regulation (EMIR). Calculating the initial margin without considering the dependence of asset prices and financial stress of banks leads to significant gaps if the counterparty defaults resulting in considerable losses until the close-out of the position. Future research could extend the setting to the case of a multi-asset-class portfolio under certain assumptions for the intra-asset-class correlation and thus mimicking the practical situation of a typical portfolio.

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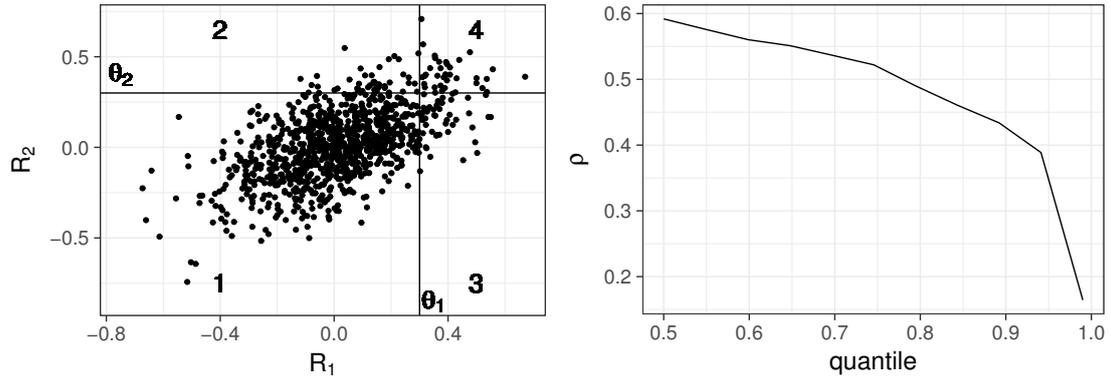


Figure 1: Theoretical results for the correlation when approaching the extremes

The figure shows an illustration of the bivariate distribution and its behaviour when increasing the thresholds. On the left hand side 1,000 realizations of a bivariate normal distribution with correlation  $\rho = 0.6$  are plotted together with thresholds  $\theta_1$  and  $\theta_2$ . The points in area two and four are fitted to a generalized Pareto distribution associated to  $R_2$  and the points in area three and four are fitted to a generalized Pareto distribution associated to  $R_1$ . Additionally, the points in area four are used to estimate the dependence parameter  $\alpha$  of the Gaussian copula. The right graphic illustrates the theoretical development of the correlation coefficient  $\rho = 1 - \alpha^2$  when increasing the thresholds simultaneously to the quantiles of  $R_1$  and  $R_2$ .

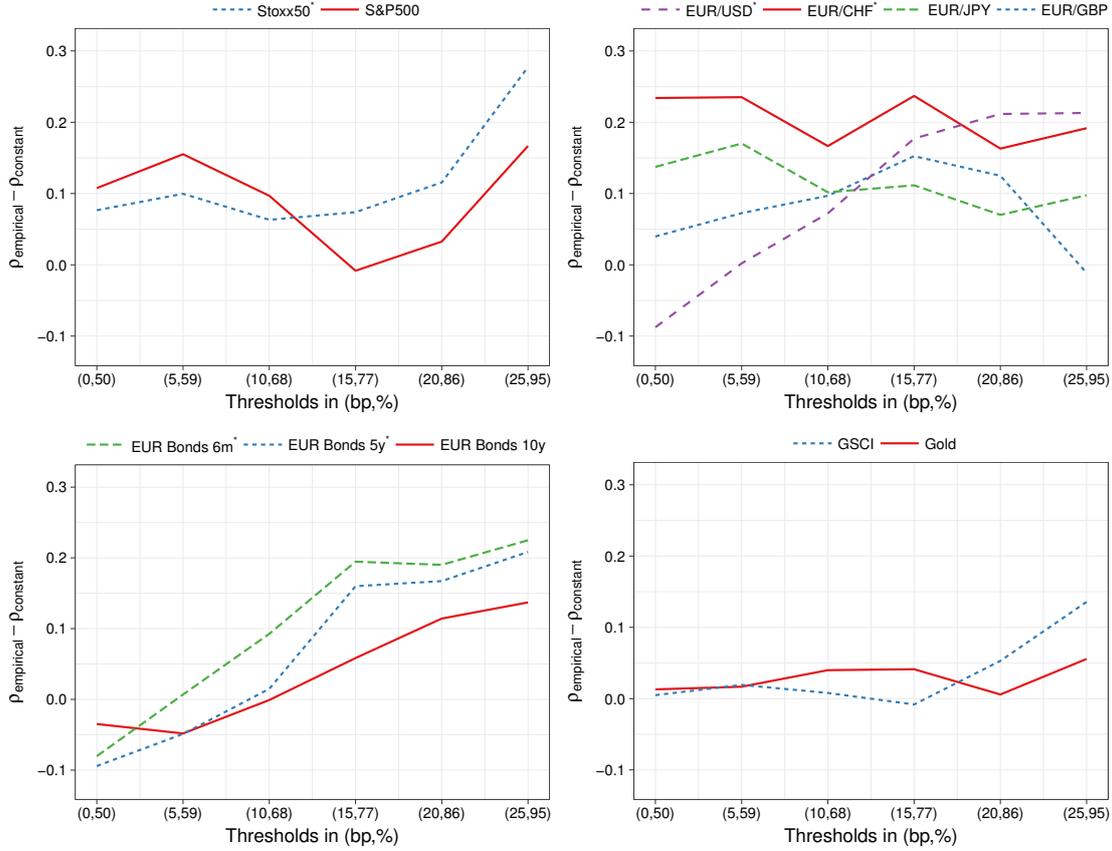


Figure 2: Empirical correlations minus their theoretical value, assuming correlation to be linear and constant, for the four asset classes and stress in the European banking sector when approaching the extremes

This figure shows the development of the empirical correlation  $\rho_{empirical}$  minus its theoretical value  $\rho_{constant}$ , assuming the correlation to be linear and constant according to [section 2](#), for the four different asset classes when increasing the thresholds for the assets as well as the iTraxx senior-financials 5y. The thresholds for the assets start at zero, which is almost the 50 percent quantile. Then, it is increased to the 95 percent quantile by a step size of 9 percentage points. The threshold for the iTraxx is accordingly set to start from zero and then increased by 5 basis points. Since the correlation is negative for all assets except for gold and interest rates on European government bonds, we look at the negative extremes for all assets and positive for gold and interest rates on European government bonds. The likelihood ratio test shows on a 5 percent confidence level, indicated by \*, that the correlation increases for Stoxx50, EUR/USD, EUR/CHF and the mid and short term interest rates on government bonds. For all other assets the correlation stays constant in the extremes.

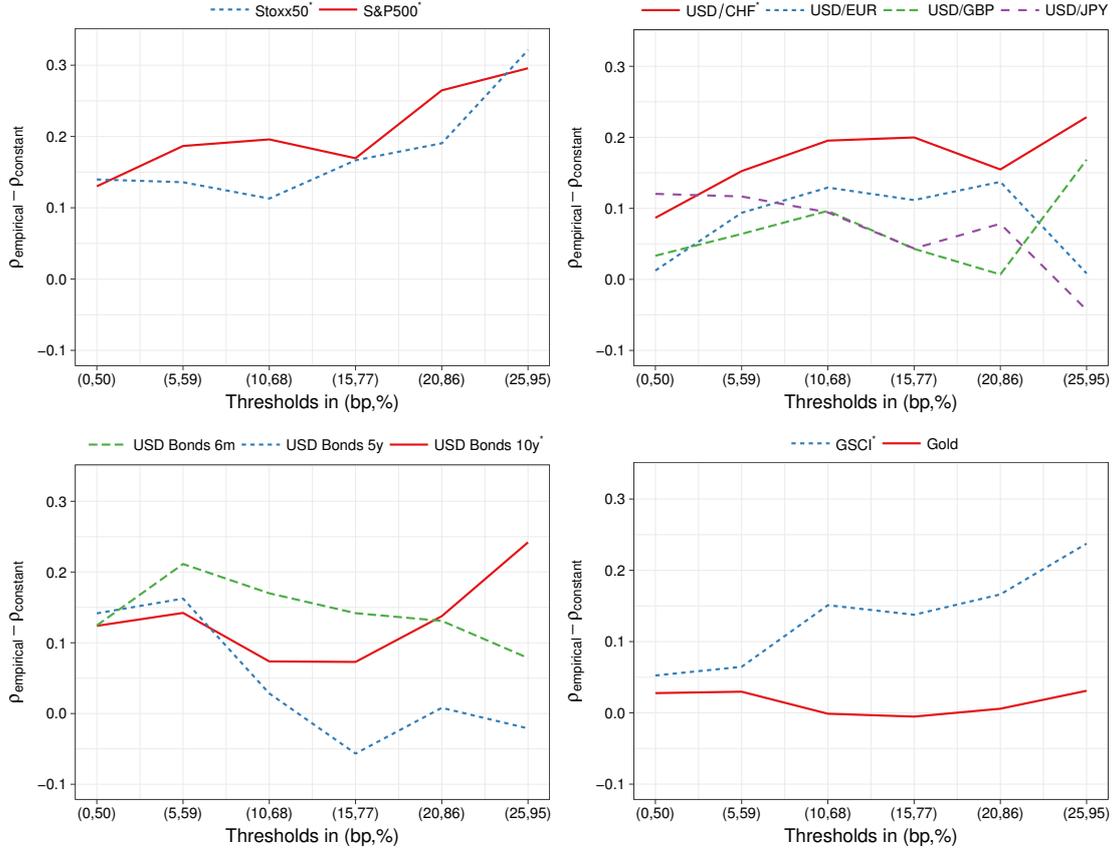


Figure 3: Empirical correlations minus their theoretical value, assuming correlation to be linear and constant, for the four asset classes and stress in the American banking sector when approaching the extremes

This figure shows the development of the empirical correlation  $\rho_{empirical}$  minus its theoretical value  $\rho_{constant}$ , assuming the correlation to be linear and constant according to [section 2](#), for the four different asset classes when increasing the thresholds for the assets as well as the CDS Index US Financials 5y. The thresholds for the assets start at zero, which is almost the 50 percent quantile. Then, it is increased to the 95 percent quantile by a step size of 9 percentage points. The threshold for the CDS Index US Financials 5y is accordingly set to start from zero and then increased by 5 basis points. Since the correlation is negative for all assets except for gold, USD/EUR, USD/GBP and USD/CHF, we look at the negative extremes for all assets and positive for gold, USD/EUR, USD/GBP and USD/CHF. The likelihood ratio test shows on a 5 percent confidence level, indicated by \*, that the correlation increases for Stox50, S&P500, USD/CHF, GSCI and the interest rates on government bonds with 10 years maturity. For all other assets the correlation stays constant in the extremes.

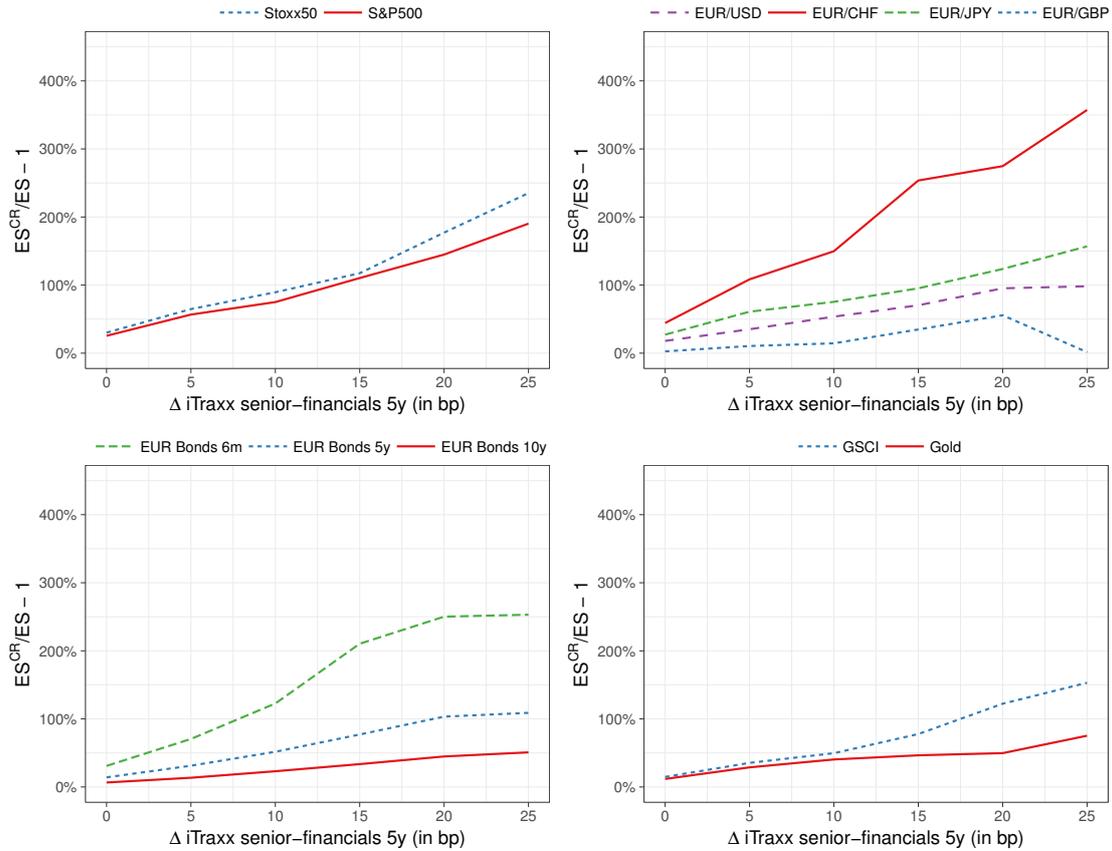


Figure 4: Relative difference of the expected shortfall conditioned on stress in the European banking sector and unconditioned when increasing the level of stress

This figure shows the relative difference of the expected shortfall under counterparty credit risk and without counterparty credit risk on a 97.5 percent level for the period of one week. The threshold for the weekly changes of the iTraxx in basis points (x-axis) starts at zero and rises until 25 basis points. The y-axis shows the relative difference of the expected shortfall. For instance, for the Stoxx50 index in the upper left corner the expected shortfall conditioned on counterparty credit risk is 235 percent higher than without this condition.

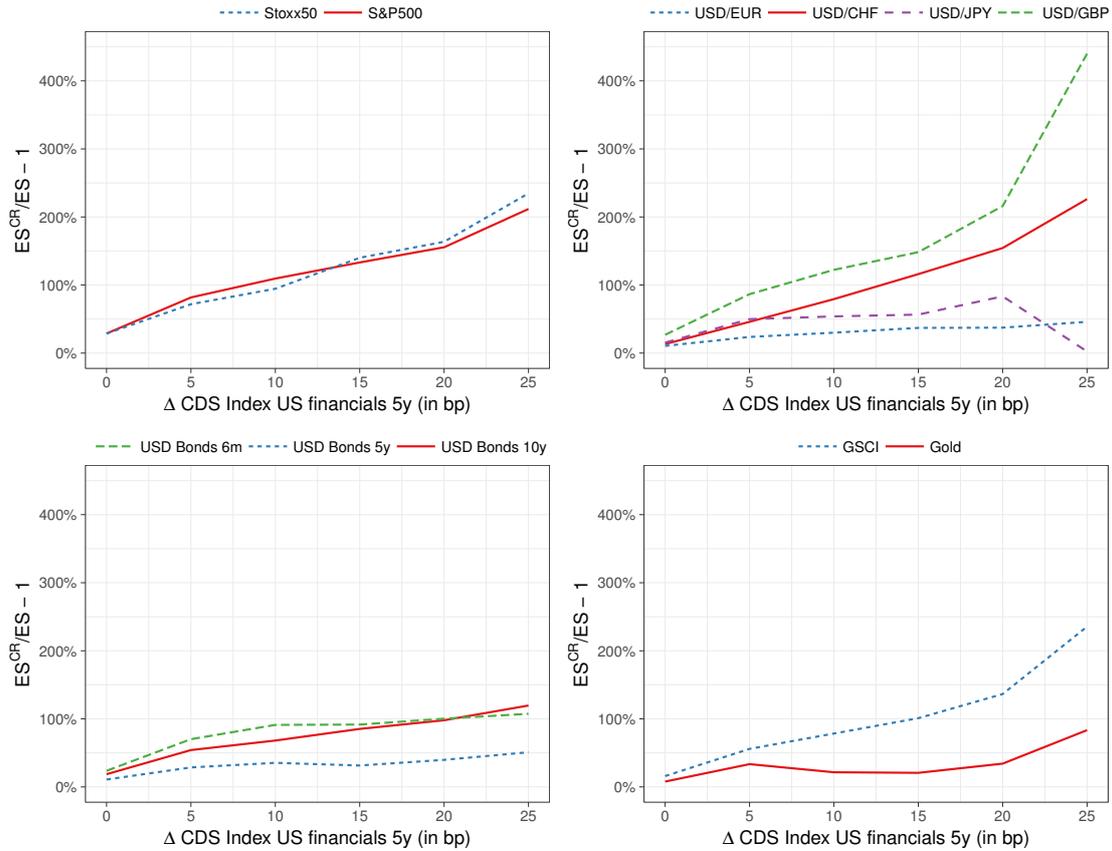


Figure 5: Relative difference of the expected shortfall conditioned on stress in the American banking sector and unconditioned when increasing the level of stress

This figure shows the relative difference of the expected shortfall under counterparty credit risk and without counterparty credit risk on a 97.5 percent level for the period of one week. The threshold for the weekly changes of the CDS Index US in basis points (x-axis) starts at zero and rises until 25 basis points. The y-axis shows the relative difference of the expected shortfall. For instance, for the Stoxx50 index in the upper left corner the expected shortfall conditioned on counterparty credit risk is 235 percent higher than without this condition.

Table 1: Descriptive statistics and correlation with the CDS indices

	Mean	SD	Max	Min	CDS Index US Financials 5y	Correlation with iTraxx senior-financials 5y
Stoxx50 <sup>†</sup>	0.01	3.12	11.52	-25.13	-0.50***	-0.61***
S&P500 <sup>†</sup>	0.08	2.50	11.36	-20.08	-0.49***	-0.50***
USD/EUR <sup>†</sup>	0.04	1.37	6.28	-5.21	0.20***	-
USD/CHF <sup>†</sup>	-0.02	1.65	11.73	-16.51	0.06	-
USD/JPY <sup>†</sup>	0.03	1.38	4.65	-7.77	-0.20***	-
USD/GBP <sup>†</sup>	0.05	1.36	9.48	-5.14	0.17***	-
Gold <sup>†</sup> (in USD)	0.16	2.83	14.69	-13.79	0.06	-
GSCI <sup>†</sup>	-0.16	3.33	12.13	-21.13	-0.19***	-0.21***
USD Bonds 10y <sup>††</sup>	-0.39	12.21	38.00	-52.00	-0.28***	-
USD Bonds 5y <sup>††</sup>	-0.38	12.12	51.00	-39.00	-0.25***	-
USD Bonds 6m <sup>††</sup>	-0.20	5.33	21.00	-48.00	-0.20***	-
CDS Index US Financials 5y <sup>††</sup>	0.08	10.41	91.11	-67.92	1.00***	0.54***
iTraxx senior-financials 5y <sup>††</sup>	0.13	11.97	66.73	-74.40	0.54***	1.00***
EUR/USD <sup>†</sup>	-0.04	1.34	5.19	-6.22	-	-0.29***
EUR/CHF <sup>†</sup>	-0.06	1.21	8.87	-17.04	-	-0.11***
EUR/JPY <sup>†</sup>	-0.01	1.73	6.25	-14.39	-	-0.38***
EUR/GBP <sup>†</sup>	0.01	1.16	6.16	-7.14	-	-0.05
Gold <sup>†</sup> (in EUR)	0.20	2.61	13.07	-13.39	-	0.12***
EUR Bonds 10y <sup>††</sup>	-0.48	8.95	37.51	-47.89	-	0.12***
EUR Bonds 5y <sup>††</sup>	-0.51	9.70	45.27	-52.03	-	0.18***
EUR Bonds 6m <sup>††</sup>	-0.41	8.77	54.94	-66.95	-	0.26***

<sup>†</sup> Weekly log-returns in percentage

<sup>††</sup> Weekly differences in basis points

\*\*\* Statistically significantly different from 0 on the 1% levels

Table 2: Used quantile values for the thresholds in [section 3](#) and [section 4](#)

Quantile	59%	68%	77%	86%	95%
Stoxx50 <sup>†</sup>	-0.29	-1.06	-1.89	-2.92	-4.93
S&P500 <sup>†</sup>	-0.15	-0.55	-1.15	-2.13	-4.00
USD/EUR <sup>†</sup>	0.29	0.52	0.92	1.27	2.44
USD/CHF <sup>†</sup>	0.37	0.70	1.03	1.50	2.12
USD/JPY <sup>†</sup>	-0.21	-0.48	-0.82	-1.30	-2.34
USD/GBP <sup>†</sup>	0.29	0.52	0.82	1.28	2.12
Gold <sup>†</sup> (in USD)	0.83	1.45	2.05	2.85	4.42
GSCI <sup>†</sup>	-0.54	-1.33	-2.01	-3.23	-6.12
USD Bonds 10y <sup>††</sup>	-4.00	-6.00	-9.00	-13.00	-18.00
USD Bonds 5y <sup>††</sup>	-3.00	-6.00	-9.00	-13.00	-19.00
USD Bonds 6m <sup>††</sup>	0	-1.00	-2.00	-3.00	-9.15
EUR/USD <sup>†</sup>	-0.32	-0.61	-0.91	-1.3	-2.32
EUR/CHF <sup>†</sup>	-0.11	-0.23	-0.41	-0.67	-1.36
EUR/JPY <sup>†</sup>	-0.23	-0.6	-1.04	-1.64	-2.58
EUR/GBP <sup>†</sup>	-0.2	-0.53	-0.76	-1.06	-1.61
Gold <sup>†</sup> (in EUR)	0.68	1.22	1.77	2.67	4.04
EUR Bonds 10y <sup>††</sup>	0.82	2.96	5.28	8.1	14.11
EUR Bonds 5y <sup>††</sup>	1.18	3.06	5.39	9.00	14.30
EUR Bonds 6m <sup>††</sup>	1.09	1.88	2.82	4.68	8.86

<sup>†</sup> Weekly log-returns in percentage

<sup>††</sup> Weekly differences in basis points

Table 3:  $ES^{CR}/ES - 1$  when  $\Delta CDS \geq 25$

	CDS Index US Financials 5y	iTraxx senior-financials 5y
Stoxx50	235%	235%
S&P500	212%	190%
USD/EUR	46%	-
USD/CHF	226%	-
USD/JPY	2%	-
USD/GBP	440%	-
Gold (in USD)	83%	-
GSCI	236%	153%
USD Bonds 10y	120%	-
USD Bonds 5y	51%	-
USD Bonds 6m	107%	-
EUR/USD	-	98%
EUR/CHF	-	357%
EUR/JPY	-	157%
EUR/GBP	-	2%
Gold (in EUR)	-	75%
EUR Bonds 10y	-	51%
EUR Bonds 5y	-	109%
EUR Bonds 6m	-	253%

This table shows the relative difference of  $ES^{CR}$  to  $ES$  when  $\Delta CDS \geq 25$ . For instance, the expected shortfall conditioned on  $\Delta CDS \geq 25$  for the CDS Index US Financials 5y is 212% higher for the S&P500 than without conditioning.