The Role of the Dynamic Conditional Quartic Beta and the Capital Markets*

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Abstract

The paper proposes an alternative methodology able to measure the sensitivity of a risky asset with respect to the market portfolio, such as the Dynamic Conditional Quartic Beta (DCQB) able to incorporate the dynamics for the fourth conditional centered co-moments. The empirical analysis, developed for the 49 Fama-French U.S. industry portfolios, discusses the following results: (i) the divergence between the DCQB and the Black Dynamic Conditional Beta (The Black DCB), with the aim to measure the departure from a benchmark indicator and compute the potential gain in terms of acquired information; (ii) the statistical relationship between the percentage variation of the fourth conditional centered co-moments, the *"asymmetry effect"* and the *"leverage effect"* concerned about the distributions of returns; (iii) a multivariate bootstrapping procedure with the aim to simulate the dynamics of the proposed methodology, at several days before some event dates; (iv) the asymptoticity and the accuracy of the bootstrapped indicator.

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1. Introduction

The theoretical framework proposed by Black (1972) as well as Ross (1973, 1976) relies on some CAPM assumptions (Sharpe 1964, Lintner 1965, Mossin 1966) questioned in several papers and proposals for testing the adequacy of this framework have been the main goal for scholars involved in asset pricing studies (Fama and Macbeth 1973; Jagannathan and Wang 1996; Roll 1997; Fama and French 2004; Gibbons and Ferson 1985; Lettau and Ludvigson 2001; Bali and Engle 2010). The assumptions behind the CAPM framework are mainly concerned about: 1) security markets that are perfectly competitive and there are many small price taker investors; (2) the behavior of investors, that in a hypothetical economy, live for only one period (myopic investors) and choose their efficient allocation of resources (portfolios), on the basis of their mean and variance; (3) the condition regarding the absence of taxes or transaction costs; (4) all investors have on average homogeneous believes regarding the joint probability of returns for the financial assets; (5) there is a riskless asset; (6) the activities for borrowing and lending are riskless and allowed[1].

In a real world, investors live for many periods and might be concerned about extreme fluctuations that drive the dynamics of a risky asset as well as the market portfolio. These concerns might create a different allocation of resources able to replicate the time-varying evolution of a risky asset. This is the logic to provide a framework in which the market beta for the assets varies over time and incorporates the dynamics of the fourth conditional centered co-moments. The changes of the market beta are related to variations of the relative risk of a firm's cash flows, linked to changes of the business cycle, motivated by the flow of the available information regarding each individual asset and the entire market portfolio (Jagannathan and Wang 1996). This flow of information, also available to market participants, impacts the dynamics regarding the higher moments, particularly the fourth conditional centered (co)-moments that represent the conditional (co)-spikes that are possible to observe along the time.

This paper proposes the *Dynamic Conditional Quartic Beta* (DCQB), as an alternative indicator able to depict the sensitivity of a risky asset with respect to the market portfolio and incorporate the dynamics for the fourth conditional centered (co)-moments that create extreme fluctuations for a risky asset, the market portfolio as well as the joint evolution between the excess rates of return for a risky asset and the market portfolio, where the excess rates of return are computed with respect to a benchmark interest rate.

[1]Many other studies in finance (Black et al. 1972; Fama and Miller 1972) provide similar results with the aim to test the flat relationship, between the beta and average returns. This statistical relationship is also consistent for financial time series (Friend and Blume (1970) as well as Stambaugh (1982)).

Under the assumptions that *there is not a riskless asset* and *the activities for borrowing and lending are allowed and risky*, the relationship between risk and return can be described with a dynamic framework able to incorporate the fourth conditional centered (co)-moments. Therefore, the main difference between the framework proposed by Bollerslev et al. (1988), Jagannathan and Wang (1996) as well as Corvasce (2015) and the proposed framework relies on the investor decisions to allocate resources also based on the concerns for the extreme fluctuations experienced by individual securities as well as the market portfolio that might create dramatic changes for the risk-adjusted returns. This means that the premium per unit of beta risk is equal to the difference between the expected market return and the expected return of the asset uncorrelated with the market portfolio, where, the beta risk is adjusted for the dynamics of the fourth conditional (co)-moments.

The empirical results also report some metrics of accuracy able to describe the divergences between the *Dynamic Conditional Quartic Beta* and The Black Dynamic Conditional Beta (Corvasce 2015) that relies on the theoretical framework proposed by Black (1972) and Ross (1973, 1976) as well as on the validity of the "separation theorem" (Tobin 1958), where, in case of absence of a risk-free asset, the price of an asset is simply a linear combination of two risky assets.

The results are discussed for the 49 Fama-French industry portfolios, with a particular emphasis for agriculture (AGRIC), food (FOOD), gold (GOLD) and oil (OIL) industries. The analysis also discusses the evolution of the fourth conditional co-moments, valuating the frequency of the conditional (co)-spikes that are possible to observe, during the period from January 1982 to December 2014. In particular, the recent financial crisis (July 2007 to March 2009) shows a raise of the conditional volatility for industry portfolios that reflects an increase of the magnitude for the fourth conditional centered (co)-moments, the market portfolio as well as the conditional dependence between the risky portfolio and the market portfolio. The upward movement for the centered conditional co-spikes corresponds to an increase of the risk aversion as well as leverage risk (Jacobs and Levy 2012, 2013, 2014) caused by concerns of the investors for allocating capital, based on the mean and variance, particularly during the period that spans from July 2007 to March 2009. The leverage risk represents a crucial component that an asset manager would like to control and it is usually sensitive to the dynamics of the fourth conditional centered co-moments.

In particular, there is a positive and statistically significant relationship between the coefficient (ϱ) that accounts for the asymmetry effect able to describe the dynamics of the conditional volatility for U.S. industry portfolios and the percentage variation for the fourth conditional centered co-moments; whereas, the coefficient (Φ), able to depict the statistical effect of the squared residuals on the dynamics of the volatility, is also positive and significant, implying that an increase of this component creates a slight raise for the fourth conditional centered co-moments.

The unknown parameters are estimated via the Ordinary Least Square (OLS) procedure with a Newey-West covariance matrix and compared with the MM estimation technique (Yohai 1987) that relies on the S estimation step with a breakdown value and a tuning value that are respectively equal to 0.5 and 1.5477. The M estimation step is based on a Tukey's bisquare weighted function with a breakdown value that is equal to 4.684 and robust standard errors (Huber 1973, 1981) able to estimate the covariance matrix. The *pseudo random number generator* of the seeds relies on the technique proposed by Knuth (1998).

There is a positive and statistically significant relationship, between the asymmetry effect (ϱ) and the percentage variation concerned about the fourth conditional centered co-moments. This statistical result is consistent for the time period between January 1st, 1982 and October 19th, 1987 as well as for the period between January 1st, 1982 and September 15th, 2008. The relationship is associated with the so called "*leverage effect*" (Black 1976, Christie 1982, French et al. 1987, Campbell and Hentschel 1992, Ait-Sahalia et al. 2013), where, the fall of asset prices is linked to the increase of the riskiness, that is usually measured in terms of conditional volatility for a company that compensates the higher level of leverage, with the relative value of the debt that tends to increase with the equity value. Therefore, a dramatic variation of the fourth conditional co-moments is also linked to the *"asymmetry effect"* (Nelson 1991, Engle and Ng 1993), where, drastic declines in stock prices are accompanied by larger increases in volatility than declines in volatility that are related to rising stock prices.

An increase of the fourth conditional centered (co)-moments is also evident for almost all Fama-French U.S. industry portfolios, five trading days before October 19th, 1987 (The Black Monday) and September 15th, 2008 (The Chapter 11 for Lehman Brothers), showing the reliability of the methodology able to depict the centered conditional (co)-spikes, before the dates of a dramatic decline of the market portfolio. A raise of the conditional volatility for the industry and the market portfolios creates an increase of the uncertainty and so a higher level of the conditional centered (co)-spikes. This phenomenon is usually caused by an increase of the leverage risk and so large stock price declines.

The analysis also reports the pseudo out-of-sample results with the aim to depict the forecasting power of the proposed methodology before some event dates, in which the market portfolio drastically declines. The procedure relies on a multivariate bootstrapping technique (Corvasce 2015, Giannopoulos and Tunaru

2005) and further depicts the divergence, in terms of some metrics of accuracy, such as the root mean square divergence (RMSD) and the symmetric mean absolute percentage divergence (SMAPD), between the *estimated* and the *simulated* values of the *Dynamic Conditional Quartic Beta*, computed at several trading days before the drawdown of the market portfolio. The analysis also reports the simulated values computed with n. 50000 and n. 100000 trials, showing the adequacy and the asymptoticity (Andrews and Buchinsky 1997) of this alternative indicator, at several trading days before some event dates.

The paper is organized in the following way: section 2 derives the *Dynamic Conditional Quartic Beta*; section 3 discusses the econometric methodology; in section 4, the framework is applied for the 49 Fama-French U.S. industry portfolios, where, summary and descriptive statistics are provided; in section 5, the empirical results are discussed, with a particular emphasis for the commodities; section 6 provides the conclusions.

2. Derivation of the Model

The framework considers a hypothetical economy with no taxes or transaction costs, without a riskless asset and where the activities for borrowing and lending are based on a *benchmark* interest rate. In this economy, investors live for more than one period, choosing the efficient allocation of resources (portfolios) based on the mean, variance and fourth conditional (co)-moments concerned about the distributions of the excess returns, computed with respect to a *benchmark* interest rate that is time-varying. Therefore, on average, investors are also concerned about extreme events that might create dramatic shocks to the evolution of the risky asset as well as to the market portfolio and have on average homogeneous believes regarding the joint probability of the excess rates of return for the assets.

The relationship between the rate of return for a risky asset i at time $t+1$, $(R_{i,t+1})$ and the rate of return for the market portfolio $(P_{i,t+1})$, with $t \geqslant 0$, can be written in the canonical way[2]:

$$R_{i,t+1} - z_{t+1} = \beta_{t+1} \cdot [P_{i,t+1} - z_{t+1}] + \eta_{i,t+1} \tag{1}$$

[2]The relationship between risk and return is developed by Sharpe (1964), Lintner (1965), Mossin (1966), Black (1972), Ross (1973, 1976), Jagannathan and Zhenyu (1996), Bollerslev et al. (1988), Lettau and Ludvigson (2001), Corvasce (2015).

$$\tilde{R}_{i,t+1} = \beta_{t+1} \cdot \tilde{P}_{i,t+1} + \eta_{i,t+1} \tag{2}$$

where, $\tilde{R}_{i,t+1}$ and $\tilde{P}_{i,t+1}$ are respectively the excess rates of return for a risky asset as well as for the market portfolio, computed with respect to a benchmark interest rate z, at time $t+1$. The quantities β_{t+1} and $\eta_{i,t+1}$ respectively represent the sensitivity of the risky asset with respect to the market portfolio and an error component, computed at time $t+1$ and based on the information set F, at time t.

The quantity $\eta_{i,t+1}$ can be written in the following way:

$$\eta_{i,t+1} = \tilde{R}_{i,t+1} - \beta_{t+1} \cdot \tilde{P}_{i,t+1} \tag{3}$$

The quartic loss function $L(\beta_{t+1})$, that depends on the quantity β_{t+1}, is able to incorporate the influence that the fourth conditional (co)-moments might create to the efficient allocation of resources. Therefore, it can be written in the following way:

$$L(\beta_{t+1}) = \sum_{i=1}^{n} (\eta_{i,t+1})^4 \tag{4}$$

where,

$$L(\beta_{t+1}) = \sum_{i=1}^{n} \left(\tilde{R}_{i,t+1} - \beta_{t+1} \cdot \tilde{P}_{i,t+1}\right)^4. \tag{5}$$

The previous quantity can be developed in the following way:

$$L(\beta_{t+1}) = \sum_{i=1}^{n} \left(\tilde{R}^4_{i,t+1} + 6 \cdot \beta^2_{t+1} \cdot \tilde{R}^2_{i,t+1} \cdot \tilde{P}^2_{i,t+1} - 4 \cdot \beta_{t+1} \cdot \tilde{R}^3_{i,t+1} \cdot \tilde{P}_{i,t+1} - 4 \cdot \beta^3_{t+1} \cdot \tilde{R}_{i,t+1} \cdot \tilde{P}^3_{i,t+1} + \beta^4_{t+1} \cdot \tilde{P}^4_{i,t+1}\right), \tag{6}$$

or,

$$L(\beta_{t+1}) = \tilde{\mu}_{04,t+1} + 6 \cdot \beta^2_{t+1} \cdot \tilde{\mu}_{22,t+1} - 4 \cdot \beta_{t+1} \cdot \tilde{\mu}_{13,t+1} - 4 \cdot \beta^3_{t+1} \cdot \tilde{\mu}_{31,t+1} + \beta^4_{t+1} \cdot \tilde{\mu}_{40,t+1} \tag{7}$$

where, $\tilde{\mu}_{04,t+1} = \sum_{i=1}^{n} \tilde{R}^4_{i,t+1}$ represents the contribution to the quartic loss function $L(\beta_{t+1})$, concerned about the fourth conditional moment and related to the evolution of the excess rates of return for the risky asset; $\tilde{\mu}_{22,t+1} = \sum_{i=1}^{n} \tilde{R}^2_{i,t+1} \cdot \tilde{P}^2_{i,t+1}$ as well as $\tilde{\mu}_{31,t+1} = \sum_{i=1}^{n} \tilde{R}^3_{i,t+1} \cdot \tilde{P}_{i,t+1}$ and $\tilde{\mu}_{13,t+1} = \sum_{i=1}^{n} \tilde{R}_{i,t+1} \cdot \tilde{P}^3_{i,t+1}$ indicate the contributions to the quartic loss function concerned about the fourth conditional co-moments, between the excess rates of return for the risky asset and the market portfolio; whereas, $\tilde{\mu}_{40,t+1} = \sum_{i=1}^{n} \tilde{P}^4_{i,t+1}$ represents the contribution to the quartic loss function concerned about the fourth conditional moment and related to the excess rates of return for the market portfolio.

The conditional beta is based on the dynamics regarding the contributions to the fourth conditional centered (co)-moments. It is derived, imposing the first derivatives of the quartic loss function $L(\beta_{t+1})$ equals to 0. Therefore,

$$\frac{\partial L}{\partial \beta_{t+1}}(\beta_{t+1}) = 0. \tag{8}$$

The previous equation can be written in the following way:

$$12 \cdot \beta_{t+1} \cdot \mu_{22,t+1} - 4 \cdot \mu_{13,t+1} - 12 \cdot \beta_{t+1}^2 \cdot \mu_{31,t+1} + 4 \cdot \beta_{t+1}^3 \cdot \mu_{40,t+1} = 0 \tag{9}$$

or simply,

$$4 \cdot \left(3 \cdot \beta_{t+1} \cdot \mu_{22,t+1} - \mu_{13,t+1} - 3 \cdot \beta_{t+1}^2 \cdot \mu_{31,t+1} + \beta_{t+1}^3 \cdot \mu_{40,t+1}\right) = 0. \tag{10}$$

This is a polynomial equation with three unknown solutions[3]. It is *time dependent* and has one real solution and two imaginary ones. Without the loss of generality, this section reports the real solution of the polynomial equation[4], since the empirical framework is concerned about the dynamics of the estimated conditional beta that belongs to the set of real numbers. As such, the quantities A_{t+1}, B_{t+1} and C_{t+1} are respectively defined, in the following way:

$$A_{t+1} = \left(\frac{\mu_{22,t+1}}{\mu_{40,t+1}} - \frac{\mu_{31,t+1}^2}{\mu_{40,t+1}^2}\right) \tag{11}$$

$$B_{t+1} = (A_{t+1})^3 + \left(\frac{\mu_{13,t+1}}{(2 \cdot \mu_{40,t+1})} + \frac{\mu_{31,t+1}^3}{\mu_{40,t+1}^3} - \frac{(3 \cdot \mu_{22,t+1} \cdot \mu_{31,t+1})}{(2 \cdot \mu_{40,t+1}^2)}\right)^2 \tag{12}$$

$$C_{t+1} = \left(\frac{\mu_{13,t+1}}{(2 \cdot \mu_{40,t+1})} + \frac{\mu_{31,t+1}^3}{\mu_{40,t+1}^3} - \frac{(3 \cdot \mu_{22,t+1} \cdot \mu_{31,t+1})}{(2 \cdot \mu_{40,t+1}^2)}\right). \tag{13}$$

Therefore, the real solution $(\beta_{1,t+1})$ of the polynomial equation n. 10 is computed as follows:

$$\beta_{1,t+1} = \frac{\mu_{31,t+1}}{\mu_{40,t+1}} - \frac{A_{t+1}}{\left(B_{t+1}^{\frac{1}{2}} + C_{t+1}\right)^{\frac{1}{3}}} + \left(B_{t+1}^{\frac{1}{2}} + C_{t+1}\right)^{\frac{1}{3}}. \tag{14}$$

For the purpose of the analysis, this manuscript considers the assumption, where the joint distribution of the excess rates of return for the risky asset as well as the market portfolio is a multivariate normal.

[3] For the purpose of the analysis, the quantities $\mu_{04,t+1}$ and $\mu_{40,t+1}$ respectively represent the fourth conditional moments, for the risky asset and the market portfolio; whereas, the quantities $\mu_{22,t+1}$, $\mu_{13,t+1}$ and $\mu_{31,t+1}$ represent the fourth mixed conditional moments, between the risky asset and the market portfolio, centered around the level of 0.

[4] Appendix A derives the imaginary solutions of the polynomial equation n. 10, that are a function of the unit imaginary quantity j.

Therefore, the fourth conditional moments concerned about the dynamics of the risky asset ($\mu_{04,t+1}$) and the dynamics of the market portfolio ($\mu_{40,t+1}$) are respectively equal to $3\cdot\sigma^4_{\tilde{R}_i,t+1}$ and $3\cdot\sigma^4_{\tilde{P}_i,t+1}$. These quantities depict the time-varying evolution of the spikes, assuming the expected values for the rates of return around the level of 0.

Conversely, the fourth conditional centered co-moments with more weight to the risky asset $(\mu_{13,t+1})$; with more weight to the market portfolio ($\mu_{31,t+1}$) and equal weights to the risky asset and the market portfolio ($\mu_{22,t+1}$) are respectively computed, in the following quantities: $3\cdot\rho_{\tilde{R}_i\tilde{P}_i,t+1}\cdot\sigma_{\tilde{P}_i,t+1}\cdot\sigma^3_{\tilde{R}_i,t+1}$, $3\cdot\rho_{\tilde{R}_i\tilde{P}_i,t+1}\cdot\sigma^3_{\tilde{P}_i,t+1}\cdot\sigma_{\tilde{R}_i,t+1}$ and $\sigma^2_{\tilde{P}_i,t+1}\cdot\sigma^2_{\tilde{R}_i,t+1}\cdot\left(1+2\cdot\rho^2_{\tilde{R}_i\tilde{P}_i,t+1}\right)$. The fourth conditional centered co-moments depict the dynamics of the joint co-movements between the risky asset and the market portfolio.

The conditional correlation between the excess rates of return for the risky asset and the market portfolio is equal to $\rho_{\tilde{R}_i\tilde{P}_i,t+1}$ and represents the time-varying evolution of the linear dependence able to depict the co-movement between the risky asset and the market portfolio. The standard deviations $\sigma_{\tilde{R}_i,t+1}$ and $\sigma_{\tilde{P}_i,t+1}$ describe the levels of uncertainty regarding the changes of the risky asset and the market portfolio.

3. Econometric Methodology

The framework proposes that the natural logarithmic variation of the daily *benchmark* interest rate, $R_{z,t+1}$, follows a reduced form of an Exponential GARCH(1,1) specification (Bollerslev 1987, Nelson 1991). As such, it is possible to write the following expressions:

$$R_{z,t+1}=\frac{z_{t+1}-z_t}{z_t}\simeq log\left(\frac{z_{t+1}}{z_t}\right)=c+\epsilon_{z,t+1} \tag{15}$$

$$E\left[\epsilon^2_{z,t+1}\,|\,F_t\right]=\sigma^2_{z,t+1} \tag{16}$$

$$log\left(\sigma^2_{z,t+1}\right)=d+f\cdot log\left(\sigma^2_{z,t}\right)+g\cdot\left(|\,innov_t\,|\;-E\,|\,innov_t\,|\right)+h\cdot innov_t \tag{17}$$

where, c is the constant of the conditional mean equation and ϵ_z is the residual component. If the estimated coefficient g is positive, a deviation of the absolute value of the innovations from their expected values implies the variance of the process to be greater than otherwise. The term h, if smaller than zero, accounts for asymmetry effects, i.e. negative surprises ($innov_{i,t}\leq 0$) raise the future variance of the *benchmark* interest rate more than otherwise. Further, d and f respectively represent the constant and the persistence of the conditional variance process[5].

Therefore, the expected *excess* rates of return for a risky asset $\tilde{R}_i$ as well as for the market portfolio $\tilde{P}_i$ are modeled as AR(1)/GJR-GARCH(1,1) processes, in the following way:

$$\tilde{R}_{i,t+1} = \alpha_i + \theta_i \cdot \tilde{R}_{i,t} + \xi_{i,t+1} \tag{18}$$

$$\tilde{P}_{i,t+1} = \lambda_i + \phi_i \cdot \tilde{P}_{i,t} + \nu_{i,t+1}. \tag{19}$$

The excess rates of return for a risky asset $\left(\tilde{R}_{i,t+1}\right)$, as well as for the related market portfolio $\left(\tilde{P}_{i,t+1}\right)$, are respectively defined as the difference between the rate of return for a risky asset and the market portfolio with respect to the expected level of the *benchmark* interest rate[6]; α_i and λ_i are respectively the constants of the conditional mean equations; θ_i and ϕ_i are the coefficients of the auto-regressive components; ξ_i and v_i are the residuals of the conditional mean equations. The conditional variances for the excess rates of return for a risky asset $\left(\tilde{R}_i\right)$ and the excess rates of return for the market portfolio $\left(\tilde{P}_i\right)$ are respectively described in the following way:

$$E\left[\xi^2_{i,t+1} \,|\, F_t\right] = \sigma^2_{\tilde{R}_i,t+1} = \psi_i + \Phi_i \cdot \xi^2_{i,t} + \tau_i \cdot \sigma^2_{\tilde{R}_i,t} + \varrho_i \cdot S^-_t \cdot \xi^2_{i,t} \tag{20}$$

$$E\left[v^2_{i,t+1} \,|\, F_t\right] = \sigma^2_{\tilde{P}_i,t+1} = k_i + b_i \cdot v^2_{i,t} + \varpi_i \cdot \sigma^2_{\tilde{P}_i,t} + \gamma_i \cdot Z^-_t \cdot v^2_{i,t} \tag{21}$$

where, ψ_i and k_i are respectively the estimated coefficients that depict the long term component of the variance processes able to describe the dynamics for the risky asset and the market portfolio; Φ_i and b_i are the estimated coefficients that depict the squared residuals; τ_i and ϖ_i are the estimated coefficients that depict the persistence of the variance processes; ϱ_i and γ_i are the estimated coefficients that depict the asymmetry effects; S^-_t and Z^-_t are respectively the indicator functions that have a value equal to 1, if their residuals are smaller than 0 and a value that is equal to 0 otherwise. The conditional covariance between $\tilde{R}_{i,t+1}$ and $\tilde{P}_{i,t+1}$, provided the information set F at time t, can be described as follows:

$$E\left[\xi_{i,t+1} \cdot v_{i,t+1} \,|\, F_t\right] = \sigma_{\tilde{R}_i\tilde{P}_i,t+1} = \rho_{\tilde{R}_i\tilde{P}_i,t+1} \cdot \sigma_{\tilde{R}_i,t+1} \cdot \sigma_{\tilde{P}_i,t+1} \tag{22}$$

where, the conditional correlation $\left(\rho_{\tilde{R}_i\tilde{P}_i,t+1}\right)$ between $\tilde{R}_{i,t+1}$ and $\tilde{P}_{i,t+1}$ relies on the DCC(1,1) model

[5] The standardized residuals for the conditional variance processes are modeled as Student's T distributions, in order to compensate for the fat tails, concerned about the natural logarithmic variations.

[6] The expected level for the U.S. *benchmark* interest rate relies on the estimation of the quantity derived in equations (15) - (17). It is important to remark the difference between the natural logarithmic VARIATION of the benchmark interest rate (R_z) and the LEVEL of this quantity (z).

proposed by Engle (2002a, 2009), that is in line with the methodology proposed by Tse and Tsui (2002). Therefore, the time-varying correlation for a bivariate case, at time $t+1$, can be written in the following way:

$$\rho_{\tilde{R}_i\tilde{P}_i,t+1} = \frac{\sigma_{\tilde{R}_i\tilde{P}_i,t+1}}{\sigma_{\tilde{R}_i,t+1}\cdot\sigma_{\tilde{P}_i,t+1}} \tag{23}$$

where, $\sigma_{\tilde{R}_i\tilde{P}_i,t+1}$ represents the conditional covariance, at time $t+1$; $\sigma_{\tilde{R}_i,t+1}$ and $\sigma_{\tilde{P}_i,t+1}$ respectively represent the conditional standard deviations related to each risky asset and the market portfolio, at time $t+1$.

The flow of information available to market participants also impacts the dynamics of the fourth conditional co-moments that determines an adjustment to the dynamics of the conditional market beta, concerned about the evolution for the conditional co-spikes.

Therefore, it is possible to denominate the real solution of the polynomial equation $\beta_{1,t+1}$, as the Dynamic Conditional Quartic Beta $(DCQB)$. For each risky asset i, at time $t+1$, this quantity relies on three estimated components: $A_{i,t+1}$, $B_{i,t+1}$ and $C_{i,t+1}$. These components are defined in the following way:

$$A_{i,t+1} = \left(\frac{\sigma^2_{\tilde{P}_i,t+1}\cdot\sigma^2_{\tilde{R}_i,t+1}\cdot\left(1+2\cdot\rho^2_{\tilde{R}_i\tilde{P}_i,t+1}\right)}{3\cdot\sigma^4_{\tilde{P}_i,t+1}} - \frac{\left(3\cdot\rho_{\tilde{R}_i\tilde{P}_i,t+1}\cdot\sigma^3_{\tilde{P}_i,t+1}\cdot\sigma_{\tilde{R}_i,t+1}\right)^2}{\left(3\cdot\sigma^4_{\tilde{P}_i,t+1}\right)^2}\right) \tag{24}$$

$$\begin{aligned} B_{i,t+1} = &\left(\frac{\sigma^2_{\tilde{P}_i,t+1}\cdot\sigma^2_{\tilde{R}_i,t+1}\cdot\left(1+2\cdot\rho^2_{\tilde{R}_i\tilde{P}_i,t+1}\right)}{3\cdot\sigma^4_{\tilde{P}_i,t+1}} - \frac{\left(3\cdot\rho_{\tilde{R}_i\tilde{P}_i,t+1}\cdot\sigma^3_{\tilde{P}_i,t+1}\cdot\sigma_{\tilde{R}_i,t+1}\right)^2}{\left(3\cdot\sigma^4_{\tilde{P}_i,t+1}\right)^2}\right)^3 + \\ +&\left(\frac{3\cdot\rho_{\tilde{R}_i\tilde{P}_i,t+1}\cdot\sigma_{\tilde{P}_i,t+1}\cdot\sigma^3_{\tilde{R}_i,t+1}}{2\left(3\cdot\sigma^4_{\tilde{P}_i,t+1}\right)} + \frac{\left(3\cdot\rho_{\tilde{R}_i\tilde{P}_i,t+1}\cdot\sigma^3_{\tilde{P}_i,t+1}\cdot\sigma_{\tilde{R}_i,t+1}\right)^3}{\left(3\cdot\sigma^4_{\tilde{P}_i,t+1}\right)^3} - \frac{3\left(\sigma^2_{\tilde{P}_i,t+1}\cdot\sigma^2_{\tilde{R}_i,t+1}\cdot\left(1+2\cdot\rho^2_{\tilde{R}_i\tilde{P}_i,t+1}\right)\right)\left(3\cdot\rho_{\tilde{R}_i\tilde{P}_i,t+1}\cdot\sigma^3_{\tilde{P}_i,t+1}\cdot\sigma_{\tilde{R}_i,t+1}\right)}{2\left(3\cdot\sigma^4_{\tilde{P}_i,t+1}\right)^2}\right)^2 \end{aligned} \tag{25}$$

$$\begin{aligned} C_{i,t+1} = &\frac{3\cdot\rho_{\tilde{R}_i\tilde{P}_i,t+1}\cdot\sigma_{\tilde{P}_i,t+1}\cdot\sigma^3_{\tilde{R}_i,t+1}}{2\left(3\cdot\sigma^4_{\tilde{P}_i,t+1}\right)} + \frac{\left(3\cdot\rho_{\tilde{R}_i\tilde{P}_i,t+1}\cdot\sigma^3_{\tilde{P}_i,t+1}\cdot\sigma_{\tilde{R}_i,t+1}\right)^3}{\left(3\cdot\sigma^4_{\tilde{P}_i,t+1}\right)^3} + \\ &- \frac{3\left(\sigma^2_{\tilde{P}_i,t+1}\cdot\sigma^2_{\tilde{R}_i,t+1}\cdot\left(1+2\cdot\rho^2_{\tilde{R}_i\tilde{P}_i,t+1}\right)\right)\left(3\cdot\rho_{\tilde{R}_i\tilde{P}_i,t+1}\cdot\sigma^3_{\tilde{P}_i,t+1}\cdot\sigma_{\tilde{R}_i,t+1}\right)}{2\left(3\cdot\sigma^4_{\tilde{P}_i,t+1}\right)^2}\ . \end{aligned} \tag{26}$$

The DCQB, for each risky asset i, at time $t+1$, can be written in the following way:

$$DCQB_{i,t+1} = \frac{3 \cdot \rho_{\tilde{R}_i \tilde{P}_i,t+1} \cdot \sigma^3_{\tilde{P}_i,t+1} \cdot \sigma_{\tilde{R}_i,t+1}}{3 \cdot \sigma^4_{\tilde{P}_i,t+1}} - \frac{A_{i,t+1}}{\left(B^{\frac{1}{2}}_{i,t+1} + C_{i,t+1}\right)^{\frac{1}{3}}} + \left(B^{\frac{1}{2}}_{i,t+1} + C_{i,t+1}\right)^{\frac{1}{3}} \tag{27}$$

or, simply:

$$DCQB_{i,t+1} = \frac{\rho_{\tilde{R}_i \tilde{P}_i,t+1} \cdot \sigma_{\tilde{R}_i,t+1}}{\sigma_{\tilde{P}_i,t+1}} - \frac{A_{i,t+1}}{\left(B^{\frac{1}{2}}_{i,t+1} + C_{i,t+1}\right)^{\frac{1}{3}}} + \left(B^{\frac{1}{2}}_{i,t+1} + C_{i,t+1}\right)^{\frac{1}{3}}. \tag{28}$$

The right hand side (RHS) of the equality can be re-compacted as follows:

$$DCQB_{i,t+1} = DCB_{i,t+1} - \frac{A_{i,t+1}}{\left(B^{\frac{1}{2}}_{i,t+1} + C_{i,t+1}\right)^{\frac{1}{3}}} + \left(B^{\frac{1}{2}}_{i,t+1} + C_{i,t+1}\right)^{\frac{1}{3}} \tag{29}$$

The quantity DCB for each risky asset i, at time $t+1$, is The Black Dynamic Conditional Beta (Corvasce 2015) that incorporates the expected value of the *benchmark* interest rate and able to evaluate the dynamics of the sensitivity for a risky asset with respect to the market portfolio[7].

3.1 Some measures of divergence between The *Dynamic Conditional Quartic Beta* and The *Black Dynamic Conditional Beta*

This section proposes three different metrics for comparing the divergence between The Dynamic Conditional Quartic Beta (DCQB) and The Black Dynamic Conditional Beta (DCB) for each risky asset i, where, the estimated quantities take into account the dynamics of the *benchmark* interest rate. As such, these metrics are respectively the Root Mean Square Divergence (RMSD), the Mean Absolute Percentage Divergence (MAPD) and the Symmetric Mean Absolute Percentage Divergence (SMAPD). The metrics of divergence, for each risky asset i, can be defined in the following way:

$$RMSD_i = \sqrt{\frac{1}{n} \cdot \sum_{t=1}^{n} \left(DCQB_{i,t} - DCB_{i,t}\right)^2} \tag{30}$$

[7]Following Black (1972) as well as Ross (1973), this framework is based on the assumption that the conditional covariance between the *benchmark* interest rate and the market portfolio is equal to 0. This assumption holds from an empirical standpoint. Indeed, a plot of the conditional covariance, estimated with a DCC(1,1) specification, shows a quite stable evolution around 0 and a slightly increase during the recent financial crisis period. In this framework, The *Black Dynamic Conditional Beta* (DCB) as well as the *Dynamic Conditional Quartic Beta* (DCQB) incorporate the dynamics of the *benchmark* interest rate modeled as a reduced form of an EGARCH(1,1) specification as well as the evolution of the fourth conditional (co)-moments between the risky asset and the market portfolio. In this framework, the investors are concerned about extreme shocks that might create dramatic variations to the evolution of the risky asset and the market portfolio.

$$MAPD_i = \frac{1}{n} \cdot \sum_{t=1}^{n} abs\left(\frac{DCQB_{i,t} - DCB_{i,t}}{DCB_{i,t}}\right) \tag{31}$$

$$SMAPD_i = \frac{1}{n} \cdot \sum_{t=1}^{n} \frac{abs\left(DCQB_{i,t} - DCB_{i,t}\right)}{\left(DCQB_{i,t} + DCB_{i,t}\right)/2} \tag{32}$$

The RMSD is the square root of the average squared differences between the estimated DCQB and The Black DCB, computed over a period of time[8]; the MAPD is the average for the absolute values (abs) of the variations between the estimated DCQB and The Black DCB; whereas, the SMAPD is the average of the ratios between the absolute values of the differences between the estimated DCQB and The Black DCB[9], for the average values between the estimated DCQB and The Black DCB. These metrics of divergence evaluate the departure from a benchmark (The Black DCB) and this alternative estimator (DCQB) that also incorporates the dynamics of the fourth conditional co-moments that might increase the estimated value of the market beta, during the periods of dramatic variations of the conditional volatility as well as correlation between the risky asset and the market portfolio.

3.2 A multivariate bootstrapping procedure for the *Dynamic Conditional Quartic Beta* and *The Black Dynamic Conditional Beta*

This section proposes a simple exercise for simulating the dynamics of the conditional quartic beta (DCQB) and constructs the forecast of this quantity at different time horizons ahead. Therefore, the DCQB for a risky asset i, starting from day t, on day $t-1$ can be simulated with S return paths of length h. As such, it is possible to write the following expression:

$$\left(\tilde{R}^s_{i,t+\tau-1} \quad \tilde{P}^s_{i,t+\tau-1}\right)_{\tau=1}^{h} \quad s = 1, ..., S \tag{33}$$

The empirical distributions, for the *excess* rates of return $\left(\tilde{R}\right)$ for the asset i as well as for the market portfolio $\left(\tilde{P}\right)$, concerned about the asset i at time t, are constructed with a procedure able to draw with replacement from the sample of past standardized residuals, $\left(\tilde{\epsilon}_{i,t+\tau-1} \quad \tilde{v}_{i,t+\tau-1}\right)_{\tau=1}^{h}$. The random drawing relies on a bivariate uniform distribution that allows to choose which past standardized residuals to pick and describes the simulated empirical distribution densities[10] for the *excess* rates of return for the asset i and for

[8] The quantity n depicts the number of observations for each specific portfolio.

[9] The SMAPD has both a lower and an upper bound and provides a result between 0% and 200%.

[10] For simplicity, the framework considers a procedure in which it is possible to random draw with replacement from a bivariate uniform distribution computed on the excess rates of return for the industry portfolio i and the related market portfolio. The

the market portfolio. Further, the procedure selects the number of times S, drawing with replacement from the set of past standardized residuals and fixes the number of days h[11].

Therefore, the procedure relies on the current levels of volatility and correlation as starting conditions and simulates the dynamics of the conditional quartic beta $(DCQB)$, at h days ahead and for each risky asset i. This quantity relies on three simulated components, $A^h_{i,t-1}$, $B^h_{i,t-1}$ and $C^h_{i,t-1}$, defined in the following way:

$$A^h_{i,t-1} = \left(\frac{\sigma^{2,h}_{\tilde{P}_i,t-1} \cdot \sigma^{2,h}_{\tilde{R}_i,t-1} \cdot \left(1 + 2 \cdot \rho^{2,h}_{\tilde{R}_i \tilde{P}_i,t-1}\right)}{3 \cdot \sigma^{4,h}_{\tilde{P}_i,t-1}} - \frac{\left(3 \cdot \rho^h_{\tilde{R}_i \tilde{P}_i,t-1} \cdot \sigma^{3,h}_{\tilde{P}_i,t-1} \cdot \sigma^h_{\tilde{R}_i,t-1}\right)^2}{\left(3 \cdot \sigma^{4,h}_{\tilde{P}_i,t-1}\right)^2} \right) \tag{34}$$

$$\begin{aligned} B^h_{i,t-1} = & \left(\frac{\sigma^{2,h}_{\tilde{P}_i,t-1} \cdot \sigma^{2,h}_{\tilde{R}_i,t-1} \cdot \left(1 + 2 \cdot \rho^{2,h}_{\tilde{R}_i \tilde{P}_i,t-1}\right)}{3 \cdot \sigma^{4,h}_{\tilde{P}_i,t-1}} - \frac{\left(3 \cdot \rho^h_{\tilde{R}_i \tilde{P}_i,t-1} \cdot \sigma^{3,h}_{\tilde{P}_i,t-1} \cdot \sigma^h_{\tilde{R}_i,t-1}\right)^2}{\left(3 \cdot \sigma^{4,h}_{\tilde{P}_i,t-1}\right)^2} \right)^3 + \\ & + \left(\frac{3 \cdot \rho^h_{\tilde{R}_i \tilde{P}_i,t-1} \cdot \sigma^h_{\tilde{P}_i,t-1} \cdot \sigma^{3,h}_{\tilde{R}_i,t-1}}{2\left(3 \cdot \sigma^{4,h}_{\tilde{P}_i,t-1}\right)} + \frac{\left(3 \cdot \rho^h_{\tilde{R}_i \tilde{P}_i,t-1} \cdot \sigma^{3,h}_{\tilde{P}_i,t-1} \cdot \sigma^h_{\tilde{R}_i,t-1}\right)^3}{\left(3 \cdot \sigma^{4,h}_{\tilde{P}_i,t-1}\right)^3} - \frac{3\left(\sigma^{2,h}_{\tilde{P}_i,t-1} \cdot \sigma^{2,h}_{\tilde{R}_i,t-1} \cdot \left(1 + 2 \cdot \rho^{2,h}_{\tilde{R}_i \tilde{P}_i,t-1}\right)\right)\left(3 \cdot \rho^h_{\tilde{R}_i \tilde{P}_i,t-1} \cdot \sigma^{3,h}_{\tilde{P}_i,t-1} \cdot \sigma^h_{\tilde{R}_i,t-1}\right)}{2\left(3 \cdot \sigma^{4,h}_{\tilde{P}_i,t-1}\right)^2} \right)^2 \end{aligned} \tag{35}$$

$$\begin{aligned} C^h_{i,t-1} = & \frac{3 \cdot \rho^h_{\tilde{R}_i \tilde{P}_i,t-1} \cdot \sigma^h_{\tilde{P}_i,t-1} \cdot \sigma^{3,h}_{\tilde{R}_i,t-1}}{2\left(3 \cdot \sigma^{4,h}_{\tilde{P}_i,t-1}\right)} + \frac{\left(3 \cdot \rho^h_{\tilde{R}_i \tilde{P}_i,t-1} \cdot \sigma^{3,h}_{\tilde{P}_i,t-1} \cdot \sigma^h_{\tilde{R}_i,t-1}\right)^3}{\left(3 \cdot \sigma^{4,h}_{\tilde{P}_i,t-1}\right)^3} + \\ & - \frac{3\left(\sigma^{2,h}_{\tilde{P}_i,t-1} \cdot \sigma^{2,h}_{\tilde{R}_i,t-1} \cdot \left(1 + 2 \cdot \rho^{2,h}_{\tilde{R}_i \tilde{P}_i,t-1}\right)\right)\left(3 \cdot \rho^h_{\tilde{R}_i \tilde{P}_i,t-1} \cdot \sigma^{3,h}_{\tilde{P}_i,t-1} \cdot \sigma^h_{\tilde{R}_i,t-1}\right)}{2\left(3 \cdot \sigma^{4,h}_{\tilde{P}_i,t-1}\right)^2}. \end{aligned} \tag{36}$$

Therefore, the simulated DCQB at h days ahead and for each risky asset i, at time $t-1$, can be written in the following way:

$$DCQB^h_{i,t-1} = \frac{3 \cdot \rho^h_{\tilde{R}_i \tilde{P}_i,t-1} \cdot \sigma^{3,h}_{\tilde{P}_i,t-1} \cdot \sigma^h_{\tilde{R}_i,t-1}}{3 \cdot \sigma^{4,h}_{\tilde{P}_i,t-1}} - \frac{A^h_{i,t-1}}{\left(B^{\frac{1}{2},h}_{i,t-1} + C^h_{i,t-1}\right)^{\frac{1}{3}}} + \left(B^{\frac{1}{2},h}_{i,t-1} + C^h_{i,t-1}\right)^{\frac{1}{3}} \tag{37}$$

or, simply:

simulations can also incorporate each component of the excess rates of return and so jointly describe the potential simulated evolution of the *benchmark* interest rate, as a further source of risk.

This section describes a bootstrapping procedure where the observations are approximately independent and identically distributed (i.i.d). This property is crucial for bootstrapping and allows the sampling procedure to safely avoid the pitfalls of sampling from a population in which successive observations are serially dependent. Therefore, in order to produce i.i.d. observations, the framework relies on a first order auto-regressive model (AR(1)) that compensates for the auto-correlation of each financial time series. An alternative procedure for describing the simulated empirical distributions relies on a multivariate procedure based on a bootstrapping technique for non i.i.d. models (Wu 1986 and Liu 1988) or multivariate block bootstrapping procedures based on non i.i.d observations.

[11] The studies developed by Christoffersen et al. (1998), Figlewski (2004) as well as Andersen et al. (2005) recommend weekly and monthly returns for the long run simulations.

$$DCQB_{i,t-1}^{h} = \frac{\rho_{\tilde{R}_i \tilde{P}_i,t-1}^{h} \cdot \sigma_{\tilde{R}_i,t-1}^{h}}{\sigma_{\tilde{P}_i,t-1}^{h}} - \frac{A_{i,t-1}^{h}}{\left(B_{i,t-1}^{\frac{1}{2},h} + C_{i,t-1}^{h}\right)^{\frac{1}{3}}} + \left(B_{i,t-1}^{\frac{1}{2},h} + C_{i,t-1}^{h}\right)^{\frac{1}{3}}. \tag{38}$$

The previous equality can be written as a function of the simulated $DCQB$ computed at h days ahead and for each risky asset i, in the following way:

$$DCQB_{i,t-1}^{h} = DCB_{i,t-1}^{h} - \frac{A_{i,t-1}^{h}}{\left(B_{i,t-1}^{\frac{1}{2},h} + C_{i,t-1}^{h}\right)^{\frac{1}{3}}} + \left(B_{i,t-1}^{\frac{1}{2},h} + C_{i,t-1}^{h}\right)^{\frac{1}{3}}. \tag{39}$$

The quantities $\sigma_{\tilde{P}_i,t-1}^{h}$ and $\sigma_{\tilde{R}_i,t-1}^{h}$ respectively depict the simulated conditional standard deviations computed at h days ahead, for the market portfolio as well as for each risky asset i, provided the information set F, at time $t-2$; whereas, $\rho_{\tilde{R}_i \tilde{P}_i,t-1}^{h}$ and $DCB_{i,t-1}^{h}$ respectively represent the simulated conditional correlation between each risky asset i and the market portfolio as well as the simulated Black Dynamic Conditional Beta (DCB), at h days ahead, provided the information set F, at time $t-2$.

4. Data

The analysis relies on the U.S. data downloaded from Kenneth French's website, based on daily returns for the 49 Fama-French value weighted U.S. industry portfolios, with the aim to study the relationship between each industry portfolio and the market portfolio, that is the value weighted return for all CRSP firms incorporated in the U.S. and listed on the NYSE, AMEX and NASDAQ stock exchanges[12]. The *benchmark* interest rate is the 3 months U.S. Treasury constant maturity rate, available on the Board of Governors of the Federal Reserve System website[13]. The potential missing values concerned about the variation of the daily *benchmark* interest rate and for the 49 Fama-French value weighted U.S. industry portfolios are interpolated with a cubic spline technique[14].

[12]Kenneth French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html) reports a detailed definition of the U.S. market portfolio. Bali and Engle (2010) relies on several definitions of the market portfolio and finds how, for their analysis, the New York Stock Exchange (NYSE), the Standard and Poor's 500 (S&P 500), the Standard and Poor's 100 (S&P 100) and the Dow Jones Industrial Average (DJIA) indexes provide similar results.

[13]The reason for considering the 3 months U.S. Treasury constant maturity rate is to model the dynamics of the *benchmark* interest rate. The data reported by Ibbotson and Associates only considers 3 decimal digits. This limitation imposes some constraints for describing the evolution of the *benchmark* interest rate.

[14]The textbook "A Practical Guide to Splines" (De Boor 1978) provides some techniques for interpolating data.

[Please Insert Appendix B around here]

For the purpose of the analysis, Appendix B reports the descriptive statistics regarding the dynamics of the industry portfolios, considering the period from January 1982 to December 2014. The average return for industry portfolios is equal to 0.054%; whereas, the median is equal to 0.050%. The average of the standard deviations across these portfolios is equal to 1.45% and respectively achieves a level that is equal to 2.49% and 2.47% for GOLD and COAL industry portfolios. The cross-sectional standard deviation ranges from 2.066% for the first quintile to 1.054% for the last quintile; whereas, the average cross-sectional kurtosis is equal to 14.38, ranging from 23.95 for the first quintile with the highest level of kurtosis to 8.82, for the last quintile with the lowest level of kurtosis.

[Please Insert Appendix C around here]

Appendix C shows the dynamics of the daily conditional volatility for the 3 months U.S. *benchmark* interest rate that ranges from 0 to 0.2 (second quarter of 2007) and sharply increases around the financial crisis period (July 2007 to March 2009) as well as around the first and second phases of the recent sovereign debt crisis. The increase of the uncertainty reflects the deterioration of the market conditions and increasing concerns about the illiquidity for the 3 months U.S. *benchmark* interest rate, where, investors preferred to rely on more liquid assets as well as longer term U.S. treasury bonds.

5. Empirical Results

The section is devoted to the discussion of the empirical findings concerned about the methodology proposed in Section 2 and Section 3 . The analysis requires the estimation of the coefficients for the daily dynamics of the U.S. *benchmark* interest rate. The coefficient f is equal to 0.9923 and depicts the persistence of the variance component; whereas, the term h that accounts for asymmetry effects is equal to -0.0953. This means that negative news also impact the innovations ($innov_t$) of the U.S. *benchmark* interest rate and so raise the future variance of the *benchmark* interest rate more than good news. Further, the term g is positive and statistically significant with a value that is equal to 0.3114, implying a sharp increase of the variance process, when, the absolute values of the innovations that drive the evolution of the *benchmark* interest rate are greater than their expected values.

[Please Insert Table 1 around here]

The dynamics of the U.S. benchmark interest rate is an important component able to determine the excess rates of return concerned about the industry and the market portfolios. Table 1 respectively reports the estimated coefficients for AR(1)/GJR-GARCH(1,1) specifications and the coefficients for the Dynamic Conditional Correlation (DCC(1,1)) able to describe the dynamics of the conditional volatility for the 49 Fama-French industry portfolios and the time-varying evolution of the conditional correlation between the excess rates of return for each industry portfolio and the excess rates of return for the market portfolio[15].

The median value across the coefficients τ is equal to 0.925. In particular, the values for the coefficients concerned about AGRIC, FOOD, GOLD and OIL industries are respectively equal to 0.947, 0.903, 0.944 and 0.939; therefore, they depict the persistence of the conditional variance processes. A high level of the conditional variance at time t respectively determines an increase of the conditional variance at time $t+1$;

The coefficient (Φ) that depicts the influence of the squared residuals, has a median value that is equal to 0.037. A high level of this quantity at time t determines an increase of the conditional variance, at time $t+1$. In particular, the values for the coefficients concerned about the estimation of the conditional volatility for AGRIC, FOOD, GOLD and OIL industry portfolios are respectively equal to 0.011, 0.038, 0.043 and 0.034; the coefficient ϱ, that accounts for the asymmetry effect, has a median value that is equal to 0.059[16], showing a level that is respectively equal to 0.064, 0.069, 0.012 and 0.043 for AGRIC, FOOD, GOLD and OIL industry portfolios. Therefore, negative surprises, regarding the dynamics of the conditional variances for industry portfolios at time t, increase the variance processes at time $t+1$, more than positive news.

[Please Insert Figure 1 around here]

The median values of the coefficients for DCC(1,1) specifications, i.e. α_DCC and β_DCC, are respectively equal to 0.023 and 0.948, showing a high level of persistence for the conditional covariance across the

[15] In line with the studies proposed by Blume (1970) as well as Fama and French (2004), the empirical results rely on portfolios rather than individual securities for providing unbiased analysis and improving the precision of the estimated market beta.

[16] The "leverage effect" is a topic of discussion among researchers. Black 1976, Christie 1982, Bollerslev et al. 2006, Ait-Sahalia et al. 2013 focus on this puzzle that usually refers to the relationship between volatility and stock prices. The "leverage effect" suggests that a negative return should make a firm more levered and therefore leads to a higher level of volatility.

49 Fama-French U.S. industry portfolios. Figure 1 respectively shows the evolution of the cross-sectional median for the conditional volatility and correlation as well as the time-varying cross-sectional median for The Black DCB and the DCQB. The descriptive statistics concerned about the main components of this alternative estimator of the market beta are reported in Table 2. In particular, the percentage variations of the Dynamic Conditional Volatility (DCV) and Correlation (DCC) for industry portfolios are computed for three main sub-periods: (i) January 1982 to June 2007; (ii) July 2007 to March 2009; (iii) March 2009 to December 2014.

[Please Insert Table 2 around here]

The financial crisis period (July 2007 to March 2009) respectively shows a median increase of the conditional volatility for 224.11% and 23.92%, reflecting an increase of the uncertainty for industry portfolios and dependence with the market portfolio. From March 2009 to December 2014, the conditional volatility for the 49 Fama-French U.S. industry portfolios decreases for about 63%. In particular, AGRIC and FOOD industry portfolios respectively experienced a reduction of the conditional volatility for 66% and 59%; whereas, GOLD and OIL industry portfolios reported a decrease of the conditional volatility for 21% and 35%. The change of the conditional volatility reflects the *flight-to-safety* phenomenon, also characterized by a tendency of the investors to allocate capital for stocks related to industries with a lower level of volatility, despite to stocks characterized by a higher level volatility.

The behavior of investors, moving their capital away from riskier investments to safer investment stocks that mainly belong to AGRIC, FOOD, GOLD and OIL industries, is also linked to the tendency of the stock markets to provide lower returns for safer investment stocks. The average conditional correlation, between the rates of return for the 49 Fama-French U.S. industry portfolios and the market portfolio, is equal to -17.49%, implying a negative relationship between the industry and the market portfolios. Conversely, the conditional dependence between the returns for GOLD industry and the returns for the market portfolio is equal to 83.54%, provided the tendency of investors to allocate capital on stocks involved in GOLD industry, during the sovereign debt crisis, characterized by a reduction of the conditional volatility for about 63.18%.

[Please Insert Figure 2 and Figure 3 around here]

The conditional correlation and volatility are the main components able to describe the dynamics of the cross-sectional median for the fourth conditional centered (co)-moments (Figure 2). These quantities represent the conditional co-spikes, computed across the 49 Fama-French U.S. industry portfolios. In particular, Figure 2.1 reports the cross-sectional evolution of the median for the fourth conditional centered **moments**; whereas, Figure 2.2 shows the evolution of the fourth conditional centered **mixed moments**, between the industry and the market portfolios. The relevant statistics concerned about the numerosity of the conditional centered co-spikes are summarized in Figure 3. The histograms report the number of the conditional centered co-spikes and the magnitude for the cross-sectional evolution of the 49 Fama-French U.S. industry portfolios, from January 1982 to December 2014. The figure shows n. 8000 conditional co-spikes, with a magnitude between 1.00E-06 and 2.00E-06 as well as n. 1000-1500 conditional co-spikes, with a magnitude greater than 1.80E-05.

During the financial crisis (July 2007 to March 2009), the median values for the conditional co-spikes are respectively equal to 1.81E-07 (for the conditional co-spikes with **more weight to industry portfolios**), 1.28E-07 (for the conditional co-spikes with **equal weights to the industry and the market portfolios**), 9.90E-08 (for the conditional co-spikes with **more weight to the market portfolio**) and 5.62E-07 (for the conditional spikes regarding the market portfolio). The median values, computed across industry portfolios, decrease during the first and the second phases of the sovereign debt crisis (March 2009 to December 2014), characterized by an average decrease of the conditional correlation for about 17.5% and a drastic reduction of the conditional volatility (Table 3 and Table 4).

[Please Insert Table 3 and Table 4 around here]

The median values for the conditional co-spikes are respectively equal to 2.93E-08 (for the conditional co-spikes with **more weight to industry portfolios**), 1.91E-08 (for the conditional co-spikes with **equal weights to the industry and the market portfolios**), 1.34E-08 (for the conditional co-spikes with **more weight to the market portfolio**) and 1.04E-07 (for the conditional spikes regarding the market portfolio), during the period from March 2009 to December 2014. The difference between the median magnitude for the conditional co-spikes, computed during the sovereign debt crisis and before the financial crisis, is positive.

The excess of volatility for industry portfolios and the correlation with the market portfolio are the main components able to explain this departure. The difference between the average percentage variations for the conditional volatility computer after and before the financial crisis is equal to -38.26%; whereas, the difference for the conditional correlation is equal to -25.72%, with a positive level of dependence before the financial crisis (January 1982 to June 2007).

[Please Insert Figure 4 around here]

In particular, GOLD and OIL industry portfolios respectively experienced an increase of the correlation with respect to the market portfolio for about 200% and 28.26%, justifying a general tendency of investors to allocate capital for companies involved in GOLD and OIL industries, particularly during some days characterized by a higher level of volatility for the market portfolio, such as October 19th, 1987 and September 15th, 2008.

The average percentage variations for the fourth conditional centered co-moments, computed across the 49 Fama-French U.S. industry portfolios and 5 days before October 19th, 1987, are **greater than 175%;** whereas, the average percentage variations for the fourth conditional centered co-moments, computed 5 days before September 15th, 2008, are greater than 46%. Figure 4 reports the DCQB and its components for AGRIC industry portfolio. From November 2011, the DCQB decreases from a level of 1.27 to 0.33 (around August 20th, 2013) and sharply increases to 0.89 (at the end of December 2014). The Black DCB increases from the first quarter of 2000 to the first quarter of 2011, reaching a level above 1, around October 2010. Further, it decreases to the level of 0.55, at the end of December 2014, due to a dramatic decline of the conditional volatility for AGRIC industry portfolio that decreases from 6.169% (October 16th, 2008) to 0.967% (December 2014). Further, there is an increase of the conditional correlation between the returns associated to AGRIC industry portfolio and the returns for the market portfolio, from 0.187 to 0.457 (end of December 2014), reflecting the time-varying level of the dependence between the agriculture industry portfolio and the market portfolio.

[Please Insert Figure 5 around here]

Figure 5 shows the DCQB for FOOD industry portfolio. It declines from a level of 1.185 (January 1982) to 0.06 (April 14th, 2000), reaching a level of 3.185 (around October 16th, 1989). The sensitivity of FOOD industry portfolio with respect to the market portfolio reaches a level of 2.035 and further declines to 1.070. The Black DCB reaches a level of 1.882 (around April 5th, 1993) and declines to 0.034 (around April 14th, 2000); whereas, the downward pattern of the conditional volatility for FOOD industry portfolio creates a reduction of The Black DCB that reaches a level of 0.543, from October 16th, 2008.

[Please Insert Figure 6 around here]

The evolution of the DCQB for GOLD industry portfolio is reported in Figure 6. It declines from 0.14 to -0.39, around August 15th, 2008, increasing to the level of 0.313, at the end of December 2014. The Black DCB represents the main component of the DCQB. It reaches a level of 2.346 (around March 21st, 1985) and drastically declines to -0.836 (June 30th, 1987), fluctuating between -0.8 and 1.2, from the last quarter of June 1987. The time-varying behavior of The Black DCB can be explained by a reduction of the correlation between GOLD industry portfolio and the U.S. market portfolio, from 0.038 to -0.245 (around the middle of November 2001), reaching a level of 0.18 at the end of December 2014. The conditional volatility for GOLD industry portfolio sharply declines from 0.08 (March 25th, 2008) to 0.032 (December 31st, 2014), justifying a different risk aversion of investors moving capital to firms involved in previous metals such as gold and silver ores (*flight-to-safety*), particularly on some days characterized by an increasing level of the conditional volatility for the market portfolio.

[Please Insert Figure 7 around here]

The DCQB for OIL industry portfolio reaches a level of 2.707 (around October 16th, 1989) and further declines to 1.065 (December 31st, 2014), after reaching the lowest level of -0.130 around March 8th, 2000 (see Figure 7). The time-varying nature of the DCQB is mainly characterized by The Black DCB that decreases from 0.59 (January 1982) to -0.129 (March 8th, 2000) and further increases to 1.378 (January 2007).

The evolution of The Black DCB is mainly related to the conditional volatility for OIL industry portfolio that dramatically declines from 0.074 (October 23rd, 2008) to 0.02 (December 31st, 2014); whereas, the conditional correlation sharply increases from -0.075 to 0.524, at the end of December 2014.

5.1 The statistical relationship between the fourth conditional co-moments and the "stylized facts"

This subsection is devoted to the description of the statistical relationship between the fourth conditional centered co-moments and the estimated coefficients of the processes able to describe the dynamics of industry portfolio. The main goal is to test the *"stylized facts"* for financial time series, such as the asymmetry regarding the distribution of errors able to depict the skewness that also consists of losses having a distribution with a heavier tail than gains (French et al. 1987) and the so-called *"leverage effect"* (Black 1976, Christie 1982, French et al. 1987, Campbell and Hentschel 1992, Ait-Sahalia et al. 2013), where, the fall of asset prices is associated to the increase of the riskiness, that is usually measured in terms of conditional volatility for a company that compensates the higher level of leverage, with the relative value of the debt that tends to increase with respect to the equity value. Therefore, a dramatic variation of the fourth conditional centered co-moments is also linked to the *"asymmetry effect"* (Nelson 1991, Engle and Ng 1993) and related to drastic declines in stock prices that are accompanied by larger increases in volatility than declines in volatility that are related to rising stick prices.

[Please Insert Appendix D around here]

The estimation of AR(1)/GJR-GARCH(1,1) specifications (Glosten et al. 1993) allows to accommodate both these stylized facts and so define the dynamics regarding the industry portfolios. Appendix D reports the correlation matrix among the estimated coefficients for the asymmetry GARCH specifications, with the aim to study the statistical relationship with the percentage variations of the fourth conditional centered co-moments

For the purpose of the analysis, these quantities are computed for the period between January 1st, 1982 and October 19th, 1987 (**Panel 5.1**) as well as for the period between January 1st, 1982 and September 15th, 2008 (**Panel 5.2**). The statistical exercise relies on the Ordinary Least Squares (OLS) technique that is based on a Newey-West estimator (Newey and West 1987) able to provide a more efficient estimation of the covariance matrix and consider the auto-correlation as well as the heteroskedasticity for the error terms.

[Please Insert Table 5 around here]

Table 5 shows a positive and statistical relationship between the coefficient (ϱ) and the percentage variations for the fourth conditional centered co-moments. An increase of the coefficient that depicts the *"asymmetry effect"* provocates an increase of the percentage variations for the fourth conditional centered co-moments, since the fall of asset prices is associated to the increase of the riskiness that compensates the higher level of leverage, during some days characterized by an increase of the conditional volatility. The component that depicts the estimated degree of freedom ($t - DoF$) is not statistically significant for explaining the percentage variations of the fourth conditional centered co-moments, computed between January 1st, 1982 and October 19th, 1987; whereas, the statistical effect is not significant, if the estimation of the unknown parameters relies on the MM estimation technique and considers the period between January 1st, 1982 and September 15th, 2008.

The MM estimation technique (Yohai 1987) allows to estimate the regression parameters using the S estimation procedure which minimizes the scale of the residuals from the M estimation technique. This alternative procedure allows to obtain estimates with a high breakdown value, more efficient and robust to outliers, providing accurate results when the sample of observations is small. Therefore, Table 5 reports the unknown parameters based on the MM estimation procedure that relies on the pseudo random number generator (RNG) of the seeds based on the paper proposed by Knuth (1998) and a Tukey's bisquare weighted function with a breakdown value that is equal to 4.684. The last four columns of Table 5 report a general increase of the goodness-of-fit computed across the estimated percentage variations of the fourth conditional centered co-moments; whereas, the dummy variable (DUMMY), able to distinguish between industry portfolios with stocks involved in commodities (AGRIC, FOOD, OIL and GOLD) and non commodities, is not statistically significant for explaining the percentage increase of the fourth conditional centered co-moments, between January 1st, 1982 and October 19th, 1987.

5.2 Some measures of divergence: the DCQB vs. The Black DCB

This subsection proposes the discussion of some metrics able to depict the divergence between the DCQB and The Black DCB as well as study the forecasting power and the asymptoticity of the simulated values, computed at several horizons before some critical event dates, characterized by a dramatic reduction of the market portfolio and a sharp increase of the conditional volatility.

[Please Insert Table 6 around here]

Table 6 respectively reports the time-varying behavior of the DCQB and The Black DCB, with the aim to evaluate the characteristics of the proposed estimator. The DCQB respectively increases on average for about 10.93% and 2.01%, 5 days before October 19th, 1987 and September 15th, 2008. The upward movement of the DCQB is mainly motivated by an increase of the fourth conditional centered co-moments, regarding an average increase of the conditional volatility, measured across the 49 Fama-French U.S. industry portfolios and a reduction of The Black DCB for -1.85% and -6.46%. The evolution of the DCQB emphasizes the important role of the fourth conditional centered co-moments able to evaluate the amount of capital that investors would like to allocate on the market portfolio rather than the benchmark interest rate.

From July 2007 to March 2009, the average increase of the DCQB is equal to 41.51%, mainly due to an average increase of The Black DCB for about 39.36%. In particular, the DCQB for GOLD industry portfolio decreases for about -62.15%, mainly due to a sharp decrease of The Black DCB for about -65.94%; whereas, the increase of the DCQB for FOOD industry portfolio is mainly characterized by a variation for about 26.09%, regarding the ratios between the fourth conditional centered co-moments. From March 2009 to December 2014, the average percentage variation of the DCQB is equal to -26.36%, with a contribution from the variation of The Black DCB that is equal to -4.03%. In particular, the percentage variation of The Black DCB for GOLD industry portfolio is equal to 337.05%; whereas, it is equal to 64.11% for FOOD industry portfolio.

[Please Insert Table 7 around here]

The information gain created by the fourth conditional centered co-moments is measured with some metrics of divergence, between the DCQB and The Black DCB. The Root Mean Square Divergence (RMSD), the Mean Absolute Percentage Divergence (MAPD) as well as the Symmetric Mean Absolute Percentage Divergence (SMAPD) evaluate the departure between the DCQB and The Black DCB. The average value of the RMSD, across the 49 Fama-French U.S. industry portfolios, is equal to 65.04%. These values are respectively equal to 81.52% and 49.88%, for the MAPD and the SMAPD. A low value for these metrics of divergence implies that the information gain received by the fourth conditional centered co-moments is negligible and the DCQB that determines the proportion of capital that an investor allocates for replicating the dynamics of the industry portfolio provides the same results of The Black DCB.

Therefore, the fourth conditional centered co-moments do not change the investor decisions concerned about the allocation of resources also based on extreme fluctuations, experienced by the industry portfolio as well as the market portfolio, that might provocate dramatic changes for the risk adjusted returns. A high value concerned about the proposed metrics of divergence implies that the DCQB reports a value that is extremely different than The Black DCB. Therefore, the allocation of resources that also takes into account the extreme fluctuations of the industry portfolios and the market portfolio imply a higher amount of capital allocated to the market portfolio, in order to replicate the dynamics for industry portfolios.

5.3 The forecasting power and the asymptoticity of the DCQB

The subsection reports the pseudo out-of-sample results concerned about the DCQB with the aim to depict the forecasting power of the proposed methodology before some event dates, characterized by a dramatic decline of the U.S. market portfolio. The procedure relies on a multivariate bootstrapping technique (Corvasce 2015, Giannopoulos and Tunaru 2005) and further depicts the divergence, between the *estimated* and the *simulated* values of the *Dynamic Conditional Quartic Beta* (DCQB). For the purpose of the analysis, Table 8 reports the Symmetric Mean Absolute Percentage Divergence (SMAPD) and the Root Mean Square Divergence (RMSD) and evaluates the asymptoticity (Andrews and Buchinsky 1997) concerned about the simulated estimator, increasing the number of iterations from n. 50000 to n. 100000.

[Please Insert Table 8 around here]

The comparison between **Panel 8.1** and **Panel 8.2** allows to derive some statistical conclusions regarding the forecasting power of the DCQB, across industry portfolios. In particular, the average values for the SMAPD and the RMSD, that compare the simulated and the estimated values for this estimator, at several days before some event dates, allow to conclude that such metrics of accuracy respectively computed with

n. 100000 trials and **16 days** before October 15th, 2008, October 26th, 1987 and September 29th, 2008 provide higher values than the same metrics of accuracy computed with n. 50000 trials. These results imply that the simulation results tend to diverge around these event dates, characterized by a lower increase of the conditional volatility for the market portfolio.

Conversely, the same metrics of accuracy respectively computed with n. 100000 and **16 days** before October 19th, 1987 and December 1st, 2008, characterized by a higher increase of the conditional volatility and a higher decline of the market portfolio, provide almost the same levels of accuracy than the metrics of accuracy computed with n. 50000 trials. These statistical results imply that on average the simulated values provide the same level of adequacy and do not depend on the number of iterations. Further, the length of the memory regarding the wide sense stationary processes does not have a statistical influence, when the exercise for simulating the DCQB is performed at **16 days** before some event dates, characterized by a dramatic increase of the conditional volatility and a decline of the market portfolio; conversely, the length of the memory might be a crucial component, when the simulation exercise is performed at **16 days** before some event dates and characterized by a lower decline of the conditional volatility for the market portfolio.

The multivariate bootstrapping procedure for the DCQB, computed with n. 100000 and n. 50000 trials and performed with **2 days** before some event dates, characterized by a dramatic increase of the conditional volatility and a reduction of the market portfolio provides on average the same levels of accuracy[17].

6. Conclusions

The aim for providing a framework based on a market beta for the assets that varies over time and incorporates the dynamics of the fourth conditional centered co-moments, is mainly driven by *extreme fluctuations* concerned about the evolution of a risky asset as well as the market portfolio. These concerns might create a different allocation of resources able to replicate the dynamics for a risky asset. The *Dynamic Conditional Quartic Beta* (DCQB) is able to incorporate the information concerned about the relative risk for a firm's cash flows, linked to changes of the business cycle, motivated by the flow of the available information re-

[17] The simulated results might provide a different conclusion, when the mean equations that describe the dynamics for the 49 Fama-French U.S. industry portfolios also include some factors able to incorporate some concerns for the market participants that are justified by a theoretical framework.

garding each individual asset and the entire market portfolio. This flow of information, available to market participants, also impacts the dynamics of the higher moments, particularly the fourth conditional centered (co)-moments that represent the conditional (co)-spikes that are possible to observe along the time, showing the superiority of this alternative indicator of the market beta with the aim to replicate the dynamics for the risky assets and portfolios.

The pseudo out-of-sample results corroborate the statistical findings concerned about this estimator of the market beta and its forecasting power performed at several horizons before some event dates, characterized by a dramatic decline of the market portfolio and a sharp increase of the conditional volatility.

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Table 1.
Estimation Results for the Fama-French Industry portfolios

The table reports the estimated coefficients for **AR(1)-GJR-GARCH(1,1)/DCC(1,1)** specifications, able to model the dynamics of the conditional beta between the excess returns for the Fama-French U.S. industry portfolios and the excess returns for the market portfolio. **(1)** α is the constant of the mean equation. It is multiplied by **100**; **(2)** θ is the coefficient of the auto-regressive component for the mean equation; **(3)** ψ is the constant of the conditional variance process. It is multiplied by **10000**; **(4)** τ is the coefficient that depicts the persistence of the variance component; **(5)** Φ is the coefficient for the squared residuals; **(6)** ϱ is the coefficient that accounts for the asymmetry effect. **(7)** α_DCC and β_DCC are the estimated coefficients of the Dynamic Conditional Correlation (DCC(1,1)) that depicts the conditional correlation between the excess returns for each industry portfolio and the excess returns for the value weighted CRSP U.S. stock market portfolio. The estimated results are developed for the **49** value weighted (**VW**) Fama-French U.S. industry portfolios. All coefficients are statistically significant at 5% level.

PORTFOLIOS	α	θ	ψ	Φ	τ	ϱ	α_DCC	β_DCC
AGRIC	**0.013**	**0.008**	**0.020**	**0.011**	**0.947**	**0.064**	**0.015**	**0.960**
FOOD	**0.032**	**0.029**	**0.021**	**0.038**	**0.903**	**0.069**	**0.020**	**0.964**
SODA	0.035	-0.019	0.023	0.044	0.937	0.022	0.011	0.975
BEER	0.026	0.008	0.009	0.035	0.941	0.041	0.025	0.956
SMOKE	0.047	0.013	0.044	0.055	0.899	0.057	0.013	0.974
TOYS	0.017	0.054	0.040	0.037	0.912	0.066	0.022	0.942
FUN	0.038	0.069	0.030	0.041	0.912	0.071	0.017	0.946
BOOKS	0.009	0.069	0.010	0.037	0.931	0.050	0.013	0.944
HSHLD	0.020	0.008	0.016	0.032	0.925	0.059	0.020	0.960
CLTHS	0.032	0.093	0.013	0.037	0.930	0.050	0.024	0.946
HLTH	0.031	0.124	0.034	0.044	0.904	0.061	0.025	0.945
MEDEQ	0.031	0.081	0.031	0.030	0.907	0.078	0.023	0.949
DRUGS	0.032	0.048	0.024	0.032	0.913	0.074	0.026	0.953
CHEMS	0.025	0.067	0.011	0.029	0.934	0.061	0.028	0.945
RUBBR	0.031	0.064	0.024	0.041	0.897	0.086	0.025	0.937
TXTLS	0.024	0.096	0.009	0.032	0.935	0.059	0.016	0.964
BLDMT	0.026	0.083	0.017	0.039	0.911	0.079	0.025	0.952
CNSTR	0.008	0.097	0.017	0.051	0.921	0.048	0.027	0.939
STEEL	-0.002	0.114	0.013	0.041	0.930	0.052	0.023	0.948
FABPR	0.001	0.043	0.012	0.035	0.940	0.045	0.026	0.943
MACH	0.020	0.111	0.013	0.035	0.920	0.076	0.030	0.928
ELCEQ	0.030	0.042	0.025	0.021	0.929	0.076	0.025	0.942
AUTOS	0.018	0.049	0.028	0.028	0.932	0.056	0.027	0.940
AERO	0.038	0.060	0.020	0.028	0.928	0.064	0.024	0.939
SHIPS	0.030	0.020	0.031	0.042	0.926	0.040	0.010	0.977
GUNS	0.038	-0.012	0.025	0.037	0.929	0.041	0.013	0.972
GOLD	**-0.077**	**-0.013**	**0.050**	**0.043**	**0.944**	**0.012**	**0.012**	**0.979**
MINES	-0.009	0.083	0.009	0.042	0.939	0.036	0.024	0.945
COAL	-0.014	0.010	0.013	0.042	0.947	0.022	0.021	0.953
OIL	**0.019**	**0.023**	**0.011**	**0.034**	**0.939**	**0.043**	**0.028**	**0.939**
UTIL	0.031	0.084	0.008	0.069	0.898	0.049	0.032	0.929
TELCM	0.028	0.018	0.016	0.033	0.919	0.071	0.026	0.935
PERSV	0.025	0.074	0.028	0.037	0.916	0.055	0.025	0.940
BUSSV	0.026	0.118	0.017	0.040	0.892	0.105	0.025	0.930
HARDW	0.035	0.038	0.026	0.034	0.928	0.057	0.021	0.949
SOFTW	0.043	0.085	0.027	0.050	0.912	0.057	0.021	0.942
CHIPS	0.032	0.086	0.016	0.033	0.933	0.056	0.026	0.940
LABEQ	0.023	0.086	0.012	0.041	0.925	0.060	0.026	0.933
PAPER	0.024	0.070	0.014	0.035	0.925	0.055	0.025	0.941
BOXES	0.030	0.034	0.030	0.039	0.914	0.059	0.018	0.954
TRANS	0.018	0.079	0.023	0.022	0.918	0.089	0.025	0.948
WHLSL	0.028	0.097	0.014	0.044	0.903	0.081	0.025	0.938
RTAIL	0.032	0.079	0.016	0.026	0.926	0.074	0.022	0.952
MEALS	0.032	0.056	0.023	0.035	0.914	0.065	0.021	0.955
BANKS	0.022	0.099	0.014	0.038	0.913	0.088	0.023	0.955
INSUR	0.026	0.102	0.015	0.045	0.903	0.082	0.018	0.958
RLEST	-0.001	0.038	0.012	0.052	0.915	0.054	0.018	0.961
FIN	0.042	0.105	0.007	0.047	0.926	0.053	0.018	0.959
OTHER	0.018	0.074	0.017	0.047	0.918	0.052	0.017	0.968
MEDIAN	**0.026**	**0.069**	**0.017**	**0.037**	**0.925**	**0.059**	**0.023**	**0.948**

Table 2.

Dynamic Conditional Volatility and Dynamic Conditional Correlation

The table reports the percentage variations for the dynamic conditional volatility (**DCV**) as well as the dynamic conditional correlation (**DCC**) for the **49** Fama-French U.S. industry portfolios. The estimated values consider the following sub-periods: (i) January 1982 - June 2007; (ii) July 2007 - March 2009; (iii) March 2009 - December 2014.

PORTFOLIOS	**Dynamic Conditional Volatility (DCV)**			**Dynamic Conditional Correlation (DCC)**		
	Jan. 1982 June 2007	**July 2007 March 2009**	**March 2009 Dec. 2014**	**Jan. 1982 June 2007**	**July 2007 March 2009**	**March 2009 Dec. 2014**
AGRIC	**-14.83%**	**138.50%**	**-65.58%**	**-2.46%**	**46.06%**	**-9.66%**
FOOD	**-17.96%**	**154.18%**	**-58.80%**	**6.75%**	**18.43%**	**-24.48%**
SODA	-30.61%	169.97%	-60.43%	9.72%	44.12%	-57.43%
BEER	-35.63%	181.92%	-55.49%	24.14%	11.67%	-43.25%
SMOKE	-29.76%	54.68%	-34.53%	9.46%	17.33%	-23.77%
TOYS	-33.89%	191.36%	-40.89%	0.35%	23.55%	-32.86%
FUN	-43.56%	291.88%	-62.13%	-7.53%	33.68%	-16.51%
BOOKS	-27.06%	283.02%	-71.27%	0.49%	-0.27%	-0.60%
HSHLD	-31.05%	190.55%	-65.22%	-5.16%	34.71%	-29.47%
CLTHS	-16.29%	199.14%	-69.09%	-1.92%	34.49%	-32.48%
HLTH	-41.80%	301.38%	-69.60%	13.04%	15.70%	-25.95%
MEDEQ	-22.58%	181.37%	-62.07%	13.05%	1.77%	-8.12%
DRUGS	-13.10%	114.83%	-37.92%	10.73%	10.36%	-32.21%
CHEMS	-27.59%	280.42%	-65.66%	3.68%	24.18%	-12.95%
RUBBR	-28.57%	357.47%	-77.25%	1.30%	27.37%	-10.56%
TXTLS	-23.90%	353.80%	-77.61%	-6.38%	38.01%	-18.98%
BLDMT	-22.81%	321.17%	-72.40%	1.45%	30.37%	-12.25%
CNSTR	-10.45%	203.01%	-69.96%	12.19%	21.21%	-13.41%
STEEL	-11.57%	286.18%	-73.03%	-0.93%	38.28%	-14.28%
FABPR	-25.56%	271.65%	-49.52%	7.32%	51.04%	-19.89%
MACH	-24.67%	278.28%	-70.31%	-0.72%	23.92%	-12.35%
ELCEQ	-33.05%	284.32%	-65.03%	1.40%	24.83%	-16.04%
AUTOS	-33.90%	288.56%	-68.83%	-2.72%	36.83%	-10.32%
AERO	-32.79%	236.83%	-66.96%	3.32%	26.23%	-24.04%
SHIPS	-32.19%	230.10%	-48.46%	-5.63%	54.25%	-14.85%
GUNS	-17.71%	154.41%	-64.52%	-19.16%	59.74%	-24.37%
GOLD	**-35.05%**	**158.00%**	**-20.94%**	**200.15%**	**-59.67%**	**83.54%**
MINES	-21.09%	237.82%	-54.74%	-2.95%	59.35%	-22.79%
COAL	-11.57%	198.79%	-60.18%	28.71%	38.20%	-22.81%
OIL	**-2.37%**	**133.16%**	**-34.65%**	**28.26%**	**19.95%**	**-19.15%**
UTIL	43.20%	84.44%	-58.04%	17.90%	16.11%	-17.22%
TELCM	-27.79%	220.14%	-70.04%	4.01%	20.70%	-27.77%
PERSV	-32.34%	177.78%	-53.98%	6.00%	12.66%	10.61%
BUSSV	-28.67%	237.32%	-69.07%	4.45%	10.09%	-13.77%
HARDW	-39.18%	192.86%	-60.15%	5.82%	28.49%	-18.77%
SOFTW	-42.23%	185.31%	-56.41%	6.34%	17.61%	-22.90%
CHIPS	-37.84%	179.99%	-63.70%	1.10%	26.34%	-19.29%
LABEQ	-41.89%	258.40%	-66.54%	6.98%	16.94%	-14.41%
PAPER	-26.89%	255.62%	-71.04%	5.47%	21.62%	-22.20%
BOXES	-24.25%	177.63%	-64.92%	2.12%	27.99%	-17.62%
TRANS	-14.38%	224.11%	-71.76%	0.09%	31.52%	-33.16%
WHLSL	-22.31%	290.24%	-74.05%	4.15%	11.43%	-9.32%
RTAIL	-29.98%	150.24%	-61.61%	0.72%	17.02%	-28.55%
MEALS	-17.31%	115.33%	-56.88%	4.38%	19.63%	-24.10%
BANKS	-34.68%	494.16%	-83.69%	8.61%	5.76%	-9.15%
INSUR	-35.34%	484.35%	-81.09%	6.36%	11.23%	-15.63%
RLEST	8.88%	225.58%	-82.34%	7.05%	36.20%	-17.20%
FIN	-23.75%	301.18%	-78.00%	1.42%	9.96%	-13.84%
OTHER	-41.29%	436.08%	-79.29%	-9.40%	41.04%	-20.41%

Table 3.
Fourth Conditional (Co)-moments

The table shows some descriptive statistics for the dynamics of the fourth conditional (co)-moments centered around 0. In particular, $\mu_{13,t+1}$ represents the fourth conditional co-moment centered around 0, with more weight to the **49** Fama-French U.S. industry portfolios (**Panel 3.1**); $\mu_{22,t+1}$ is the fourth conditional co-moment centered around 0, with equal weights to the industry portfolios and the market portfolio (**Panel 3.2**); $\mu_{31,t+1}$ is the fourth conditional co-moment centered around 0, with more weight to the market portfolio (**Panel 3.3**); $\mu_{04,t+1}$ is the fourth conditional moment centered around 0, for U.S. industry portfolios (**Panel 3.4**). The values are estimated for the following sub-periods: (i) **January 1982 - June 2007**; (ii) **July 2007 - March 2009**; (iii) **March 2009 - December 2014**.

Panel 3.1: Fourth conditional co-moments with more weight to the 49 U.S. Industry Portfolios (IPs)

PORTFOLIOS	Jan 1982 to June 2007		July 2007 to March 2009		March 2009 to Dec. 2014	
	Mean	**Median**	**Mean**	**Median**	**Mean**	**Median**
AGRIC	**2.92E-08**	**1.12E-08**	**1.75E-06**	**2.89E-07**	**8.07E-08**	**2.54E-08**
FOOD	**5.50E-08**	**6.76E-09**	**3.25E-07**	**3.61E-08**	**2.29E-08**	**6.35E-09**
SODA	1.09E-07	2.35E-08	6.86E-07	7.04E-08	5.18E-08	1.27E-08
BEER	6.81E-08	1.29E-08	3.71E-07	2.58E-08	1.98E-08	5.59E-09
SMOKE	8.28E-08	1.96E-08	5.12E-07	4.53E-08	2.69E-08	1.14E-08
TOYS	1.35E-07	2.45E-08	9.26E-07	1.78E-07	1.06E-07	2.62E-08
FUN	2.24E-07	2.00E-08	3.33E-06	2.48E-07	2.38E-07	5.31E-08
BOOKS	6.71E-08	1.51E-08	2.76E-06	2.25E-07	2.52E-07	4.45E-08
HSHLD	4.90E-08	9.38E-09	4.69E-07	3.50E-08	2.85E-08	7.54E-09
CLTHS	9.10E-08	1.37E-08	1.64E-06	2.62E-07	1.39E-07	2.42E-08
HLTH	7.72E-08	1.55E-08	6.72E-07	3.81E-08	8.69E-08	1.81E-08
MEDEQ	6.83E-08	1.41E-08	5.59E-07	3.92E-08	6.09E-08	1.43E-08
DRUGS	9.85E-08	1.39E-08	4.61E-07	3.98E-08	3.46E-08	1.11E-08
CHEMS	9.52E-08	1.24E-08	2.22E-06	1.50E-07	1.55E-07	2.77E-08
RUBBR	8.08E-08	9.42E-09	1.06E-06	1.53E-07	1.24E-07	2.01E-08
TXTLS	9.80E-08	1.07E-08	2.13E-06	1.98E-07	3.79E-07	4.59E-08
BLDMT	9.42E-08	1.07E-08	1.89E-06	1.83E-07	2.37E-07	3.97E-08
CNSTR	1.38E-07	2.13E-08	4.27E-06	5.01E-07	3.11E-07	5.74E-08
STEEL	1.66E-07	1.86E-08	6.42E-06	3.86E-07	4.31E-07	6.43E-08
FABPR	5.73E-08	1.53E-08	2.16E-06	1.55E-07	2.75E-07	6.33E-08
MACH	1.19E-07	1.31E-08	2.89E-06	1.72E-07	2.28E-07	3.66E-08
ELCEQ	1.34E-07	2.23E-08	2.02E-06	1.42E-07	1.66E-07	3.17E-08
AUTOS	1.12E-07	2.41E-08	2.56E-06	3.33E-07	2.32E-07	4.14E-08
AERO	1.08E-07	1.56E-08	1.17E-06	1.14E-07	1.08E-07	2.07E-08
SHIPS	5.66E-08	2.24E-08	8.41E-07	1.06E-07	2.23E-07	6.42E-08
GUNS	7.08E-08	1.60E-08	7.42E-07	5.47E-08	4.97E-08	1.45E-08
GOLD	**3.61E-08**	**8.82E-09**	**1.08E-06**	**6.39E-08**	**7.40E-08**	**4.27E-08**
MINES	9.11E-08	1.57E-08	6.01E-06	5.08E-07	3.19E-07	6.82E-08
COAL	1.64E-07	2.60E-08	1.06E-05	4.39E-07	5.69E-07	1.30E-07
OIL	**7.63E-08**	**1.29E-08**	**2.83E-06**	**1.23E-07**	**1.13E-07**	**2.41E-08**
UTIL	3.25E-08	3.22E-09	1.14E-06	4.88E-08	3.47E-08	5.93E-09
TELCM	9.30E-08	1.11E-08	1.44E-06	1.06E-07	6.35E-08	1.20E-08
PERSV	7.70E-08	1.42E-08	7.95E-07	1.73E-07	1.02E-07	2.94E-08
BUSSV	1.09E-07	8.99E-09	1.15E-06	9.16E-08	9.63E-08	1.69E-08
HARDW	2.35E-07	3.13E-08	1.16E-06	1.55E-07	8.72E-08	2.55E-08
SOFTW	2.58E-07	3.79E-08	1.11E-06	1.01E-07	7.72E-08	1.96E-08
CHIPS	2.36E-07	2.81E-08	1.39E-06	1.38E-07	1.06E-07	2.43E-08
LABEQ	1.75E-07	2.07E-08	1.27E-06	7.74E-08	1.42E-07	2.37E-08
PAPER	8.26E-08	9.85E-09	8.86E-07	1.01E-07	8.86E-08	1.66E-08
BOXES	1.01E-07	1.65E-08	1.12E-06	1.44E-07	8.95E-08	2.07E-08
TRANS	6.66E-08	1.36E-08	1.12E-06	1.56E-07	1.13E-07	1.96E-08
WHLSL	4.45E-08	8.27E-09	9.15E-07	8.42E-08	7.91E-08	1.30E-08
RTAIL	1.19E-07	1.47E-08	8.09E-07	1.47E-07	4.71E-08	1.16E-08
MEALS	7.14E-08	1.33E-08	6.40E-07	1.11E-07	4.97E-08	1.16E-08
BANKS	6.62E-08	1.24E-08	3.93E-06	6.55E-07	5.01E-07	3.06E-08
INSUR	3.72E-08	8.19E-09	2.16E-06	1.89E-07	1.73E-07	1.82E-08
RLEST	4.90E-08	7.44E-09	3.19E-06	3.94E-07	4.00E-07	3.12E-08
FIN	1.22E-07	1.09E-08	3.24E-06	6.24E-07	3.30E-07	3.87E-08
OTHER	1.21E-07	1.35E-08	1.13E-06	7.05E-08	1.15E-07	1.41E-08

Panel 3.2: Fourth conditional co-moments with equal weights to the 49 IPs and the market portfolio

PORTFOLIOS	Jan. 1982 to June 2007		July 2007 to March 2009		March 2009 to Dec. 2014	
	Mean	Median	Mean	Median	Mean	Median
AGRIC	**3.34E-08**	**9.55E-09**	**1.34E-06**	**1.74E-07**	**6.69E-08**	**1.86E-08**
FOOD	**6.00E-08**	**6.68E-09**	**4.62E-07**	**4.62E-08**	**3.02E-08**	**7.21E-09**
SODA	9.10E-08	1.50E-08	7.19E-07	7.03E-08	5.01E-08	1.19E-08
BEER	6.79E-08	1.03E-08	4.83E-07	3.60E-08	2.66E-08	6.59E-09
SMOKE	7.37E-08	1.40E-08	5.84E-07	4.87E-08	3.23E-08	1.04E-08
TOYS	1.03E-07	1.52E-08	8.98E-07	1.31E-07	8.21E-08	1.83E-08
FUN	1.47E-07	1.30E-08	2.13E-06	1.72E-07	1.39E-07	2.85E-08
BOOKS	6.98E-08	1.33E-08	2.23E-06	1.86E-07	1.74E-07	3.15E-08
HSHLD	5.40E-08	8.07E-09	5.97E-07	4.51E-08	3.45E-08	7.97E-09
CLTHS	8.08E-08	1.01E-08	1.36E-06	1.76E-07	1.01E-07	1.72E-08
HLTH	6.99E-08	1.12E-08	7.46E-07	4.60E-08	7.31E-08	1.41E-08
MEDEQ	6.71E-08	1.06E-08	6.71E-07	4.67E-08	5.88E-08	1.25E-08
DRUGS	8.75E-08	1.06E-08	5.98E-07	4.95E-08	3.94E-08	1.04E-08
CHEMS	8.66E-08	9.80E-09	1.68E-06	1.19E-07	1.11E-07	1.90E-08
RUBBR	7.56E-08	8.17E-09	1.01E-06	1.21E-07	9.34E-08	1.58E-08
TXTLS	8.30E-08	8.49E-09	1.51E-06	1.43E-07	1.83E-07	2.64E-08
BLDMT	8.66E-08	8.90E-09	1.56E-06	1.39E-07	1.48E-07	2.54E-08
CNSTR	1.03E-07	1.37E-08	2.55E-06	2.51E-07	1.73E-07	3.02E-08
STEEL	1.18E-07	1.27E-08	3.32E-06	2.19E-07	2.10E-07	3.28E-08
FABPR	5.44E-08	1.08E-08	1.59E-06	1.15E-07	1.56E-07	3.20E-08
MACH	9.95E-08	1.02E-08	2.02E-06	1.28E-07	1.45E-07	2.19E-08
ELCEQ	1.07E-07	1.42E-08	1.59E-06	1.15E-07	1.18E-07	2.11E-08
AUTOS	9.14E-08	1.49E-08	1.85E-06	2.05E-07	1.42E-07	2.49E-08
AERO	8.59E-08	1.11E-08	1.09E-06	9.69E-08	8.64E-08	1.53E-08
SHIPS	5.24E-08	1.47E-08	8.30E-07	8.80E-08	1.33E-07	3.28E-08
GUNS	6.90E-08	1.16E-08	7.62E-07	5.80E-08	4.85E-08	1.25E-08
GOLD	**1.09E-07**	**2.78E-08**	**1.53E-06**	**1.35E-07**	**8.37E-08**	**4.03E-08**
MINES	7.78E-08	1.17E-08	3.10E-06	2.53E-07	1.64E-07	3.39E-08
COAL	9.87E-08	1.80E-08	4.44E-06	2.98E-07	2.41E-07	4.98E-08
OIL	**7.16E-08**	**9.86E-09**	**1.88E-06**	**1.07E-07**	**8.67E-08**	**1.69E-08**
UTIL	3.86E-08	3.89E-09	1.04E-06	5.28E-08	3.96E-08	6.55E-09
TELCM	8.39E-08	9.06E-09	1.29E-06	9.36E-08	6.16E-08	1.11E-08
PERSV	7.23E-08	1.06E-08	8.34E-07	1.25E-07	8.11E-08	1.96E-08
BUSSV	9.72E-08	8.07E-09	1.15E-06	9.04E-08	8.46E-08	1.45E-08
HARDW	1.37E-07	1.76E-08	1.07E-06	1.20E-07	7.26E-08	1.81E-08
SOFTW	1.60E-07	1.98E-08	1.05E-06	9.05E-08	6.89E-08	1.52E-08
CHIPS	1.38E-07	1.66E-08	1.23E-06	1.11E-07	8.56E-08	1.75E-08
LABEQ	1.22E-07	1.34E-08	1.17E-06	7.68E-08	1.04E-07	1.71E-08
PAPER	7.72E-08	8.28E-09	9.30E-07	9.25E-08	7.72E-08	1.36E-08
BOXES	8.62E-08	1.17E-08	1.05E-06	1.15E-07	7.51E-08	1.57E-08
TRANS	6.35E-08	1.04E-08	1.07E-06	1.22E-07	8.97E-08	1.51E-08
WHLSL	4.99E-08	7.39E-09	9.64E-07	8.44E-08	7.27E-08	1.19E-08
RTAIL	1.02E-07	1.09E-08	8.82E-07	1.18E-07	5.04E-08	1.08E-08
MEALS	7.03E-08	1.01E-08	7.38E-07	9.60E-08	5.07E-08	1.10E-08
BANKS	6.16E-08	9.86E-09	2.23E-06	3.27E-07	2.22E-07	2.08E-08
INSUR	4.24E-08	7.51E-09	1.58E-06	1.39E-07	1.17E-07	1.49E-08
RLEST	4.70E-08	6.94E-09	1.94E-06	2.24E-07	1.89E-07	1.99E-08
FIN	8.33E-08	8.88E-09	2.00E-06	3.19E-07	1.77E-07	2.39E-08
OTHER	9.60E-08	1.04E-08	1.03E-06	6.99E-08	8.83E-08	1.23E-08

Panel 3.3: Fourth conditional co-moments with more weight to the market portfolio

PORTFOLIOS	Jan. 1982 to June 2007		July 2007 to March 2009		March 2009 to Dec. 2014	
	Mean	**Median**	**Mean**	**Median**	**Mean**	**Median**
AGRIC	**3.58E-08**	**5.76E-09**	**9.80E-07**	**7.64E-08**	**5.46E-08**	**1.03E-08**
FOOD	**7.16E-08**	**6.75E-09**	**7.45E-07**	**6.42E-08**	**4.52E-08**	**8.30E-09**
SODA	7.46E-08	7.52E-09	8.14E-07	6.34E-08	4.93E-08	7.76E-09
BEER	6.84E-08	6.70E-09	7.07E-07	4.83E-08	3.77E-08	6.50E-09
SMOKE	6.18E-08	7.26E-09	7.16E-07	5.18E-08	3.80E-08	7.91E-09
TOYS	8.58E-08	9.23E-09	1.01E-06	1.05E-07	7.25E-08	1.32E-08
FUN	1.08E-07	8.94E-09	1.55E-06	1.26E-07	9.22E-08	1.63E-08
BOOKS	8.74E-08	1.19E-08	1.87E-06	1.58E-07	1.28E-07	2.33E-08
HSHLD	5.69E-08	7.34E-09	8.64E-07	6.35E-08	4.80E-08	8.63E-09
CLTHS	8.00E-08	8.06E-09	1.30E-06	1.28E-07	8.29E-08	1.34E-08
HLTH	7.14E-08	7.93E-09	9.43E-07	6.11E-08	6.89E-08	1.14E-08
MEDEQ	7.57E-08	8.31E-09	9.12E-07	6.38E-08	6.42E-08	1.15E-08
DRUGS	8.59E-08	8.22E-09	8.75E-07	6.58E-08	5.10E-08	1.04E-08
CHEMS	8.77E-08	8.19E-09	1.42E-06	1.02E-07	9.04E-08	1.46E-08
RUBBR	7.83E-08	7.16E-09	1.12E-06	1.09E-07	8.13E-08	1.33E-08
TXTLS	7.76E-08	6.89E-09	1.28E-06	1.14E-07	1.06E-07	1.70E-08
BLDMT	8.86E-08	7.83E-09	1.45E-06	1.21E-07	1.06E-07	1.77E-08
CNSTR	9.06E-08	8.90E-09	1.73E-06	1.50E-07	1.11E-07	1.75E-08
STEEL	8.97E-08	8.61E-09	1.92E-06	1.24E-07	1.19E-07	1.84E-08
FABPR	5.36E-08	6.65E-09	1.32E-06	9.03E-08	1.01E-07	1.71E-08
MACH	9.35E-08	8.36E-09	1.59E-06	1.06E-07	1.05E-07	1.62E-08
ELCEQ	9.62E-08	1.00E-08	1.42E-06	1.05E-07	9.45E-08	1.58E-08
AUTOS	8.52E-08	9.52E-09	1.53E-06	1.44E-07	1.01E-07	1.66E-08
AERO	8.21E-08	8.27E-09	1.15E-06	9.17E-08	7.83E-08	1.28E-08
SHIPS	4.99E-08	7.58E-09	9.23E-07	7.99E-08	8.96E-08	1.71E-08
GUNS	5.55E-08	6.74E-09	8.69E-07	5.61E-08	4.96E-08	9.48E-09
GOLD	**6.82E-09**	**1.20E-09**	**3.20E-07**	**2.40E-08**	**2.75E-08**	**6.81E-09**
MINES	7.07E-08	6.40E-09	1.71E-06	1.10E-07	9.66E-08	1.64E-08
COAL	5.87E-08	6.42E-09	1.79E-06	9.22E-08	1.05E-07	1.61E-08
OIL	**6.70E-08**	**6.20E-09**	**1.36E-06**	**7.39E-08**	**7.48E-08**	**1.23E-08**
UTIL	5.10E-08	4.70E-09	1.08E-06	6.42E-08	5.07E-08	7.96E-09
TELCM	8.75E-08	7.99E-09	1.29E-06	9.46E-08	6.78E-08	1.15E-08
PERSV	7.76E-08	8.18E-09	1.00E-06	1.04E-07	7.29E-08	1.39E-08
BUSSV	9.83E-08	8.17E-09	1.26E-06	9.97E-08	8.29E-08	1.40E-08
HARDW	9.56E-08	1.02E-08	1.11E-06	1.00E-07	6.81E-08	1.26E-08
SOFTW	1.14E-07	1.12E-08	1.12E-06	9.00E-08	6.98E-08	1.28E-08
CHIPS	1.01E-07	1.05E-08	1.23E-06	1.03E-07	7.88E-08	1.42E-08
LABEQ	1.01E-07	9.35E-09	1.21E-06	8.42E-08	8.74E-08	1.42E-08
PAPER	7.91E-08	7.60E-09	1.10E-06	9.65E-08	7.63E-08	1.28E-08
BOXES	8.02E-08	8.28E-09	1.10E-06	9.77E-08	7.05E-08	1.27E-08
TRANS	7.21E-08	8.40E-09	1.16E-06	1.10E-07	8.12E-08	1.32E-08
WHLSL	1.32E-07	1.09E-08	1.62E-06	1.32E-07	9.99E-08	1.73E-08
RTAIL	9.76E-08	9.11E-09	1.08E-06	1.10E-07	6.09E-08	1.12E-08
MEALS	7.74E-08	8.11E-09	9.67E-07	9.61E-08	5.89E-08	1.08E-08
BANKS	7.14E-08	8.55E-09	1.52E-06	1.80E-07	1.23E-07	1.54E-08
INSUR	4.92E-08	7.49E-09	1.29E-06	1.22E-07	9.27E-08	1.37E-08
RLEST	5.04E-08	5.17E-09	1.31E-06	1.42E-07	1.08E-07	1.44E-08
FIN	7.10E-08	8.19E-09	1.29E-06	1.78E-07	1.12E-07	1.66E-08
OTHER	8.58E-08	7.68E-09	1.10E-06	7.87E-08	7.85E-08	1.16E-08

Panel 3.4: Fourth conditional moments with more weight to the 49 IPs

PORTFOLIOS	Jan. 1982 to June 2007		July 2007 to March 2009		March 2009 to Dec. 2014	
	Mean	**Median**	**Mean**	**Median**	**Mean**	**Median**
AGRIC	**9.34E-08**	**4.44E-08**	**5.64E-06**	**1.91E-06**	**2.40E-07**	**1.07E-07**
FOOD	**7.85E-08**	**1.26E-08**	**3.06E-07**	**4.12E-08**	**2.68E-08**	**1.01E-08**
SODA	3.80E-07	1.07E-07	1.16E-06	1.63E-07	1.20E-07	4.11E-08
BEER	1.56E-07	4.24E-08	4.44E-07	4.06E-08	2.97E-08	1.29E-08
SMOKE	4.15E-07	8.15E-08	8.38E-07	9.71E-08	5.81E-08	3.44E-08
TOYS	3.47E-07	7.62E-08	1.44E-06	3.87E-07	2.17E-07	6.66E-08
FUN	5.94E-07	5.37E-08	7.59E-06	5.14E-07	6.63E-07	1.69E-07
BOOKS	7.92E-08	1.76E-08	3.54E-06	2.83E-07	3.96E-07	6.42E-08
HSHLD	1.05E-07	1.85E-08	4.74E-07	4.25E-08	3.69E-08	1.31E-08
CLTHS	1.72E-07	3.17E-08	2.63E-06	5.48E-07	2.80E-07	5.85E-08
HLTH	1.91E-07	4.38E-08	8.21E-07	5.71E-08	1.60E-07	4.32E-08
MEDEQ	1.15E-07	3.05E-08	6.05E-07	4.99E-08	9.09E-08	2.56E-08
DRUGS	1.81E-07	3.18E-08	4.49E-07	4.71E-08	4.74E-08	1.86E-08
CHEMS	1.59E-07	2.45E-08	3.92E-06	3.22E-07	2.91E-07	5.77E-08
RUBBR	1.41E-07	1.90E-08	1.46E-06	2.84E-07	2.28E-07	3.87E-08
TXTLS	2.15E-07	2.65E-08	4.51E-06	4.50E-07	1.23E-06	1.27E-07
BLDMT	1.52E-07	2.09E-08	2.75E-06	3.17E-07	4.88E-07	8.96E-08
CNSTR	3.36E-07	5.90E-08	9.91E-06	1.52E-06	7.94E-07	1.77E-07
STEEL	4.85E-07	4.84E-08	1.79E-05	1.19E-06	1.30E-06	2.07E-07
FABPR	1.61E-07	5.39E-08	4.29E-06	4.42E-07	7.62E-07	2.38E-07
MACH	2.22E-07	2.59E-08	5.31E-06	3.38E-07	4.71E-07	8.26E-08
ELCEQ	2.70E-07	5.22E-08	3.25E-06	2.71E-07	3.08E-07	6.86E-08
AUTOS	2.36E-07	6.92E-08	4.61E-06	7.14E-07	5.25E-07	1.04E-07
AERO	2.51E-07	3.80E-08	1.69E-06	2.05E-07	1.90E-07	4.42E-08
SHIPS	2.14E-07	9.05E-08	1.35E-06	2.28E-07	6.26E-07	2.12E-07
GUNS	2.58E-07	6.05E-08	1.20E-06	1.24E-07	1.14E-07	4.22E-08
GOLD	**1.65E-06**	**6.14E-07**	**1.31E-05**	**1.28E-06**	**1.11E-06**	**6.80E-07**
MINES	2.42E-07	5.86E-08	1.99E-05	2.75E-06	1.10E-06	3.27E-07
COAL	1.29E-06	1.74E-07	5.60E-05	4.83E-06	3.07E-06	1.06E-06
OIL	**1.86E-07**	**4.31E-08**	**6.87E-06**	**4.03E-07**	**2.39E-07**	**6.04E-08**
UTIL	5.98E-08	5.01E-09	1.75E-06	9.18E-08	4.74E-08	1.00E-08
TELCM	1.55E-07	2.06E-08	1.99E-06	1.60E-07	8.76E-08	1.83E-08
PERSV	1.42E-07	3.27E-08	1.07E-06	3.51E-07	2.09E-07	7.17E-08
BUSSV	1.64E-07	1.33E-08	1.36E-06	1.12E-07	1.35E-07	2.51E-08
HARDW	8.90E-07	1.03E-07	1.79E-06	3.28E-07	1.82E-07	7.65E-08
SOFTW	7.16E-07	1.29E-07	1.58E-06	1.82E-07	1.24E-07	3.94E-08
CHIPS	8.10E-07	7.87E-08	2.06E-06	2.60E-07	1.84E-07	5.58E-08
LABEQ	4.30E-07	5.45E-08	1.78E-06	1.13E-07	2.64E-07	4.73E-08
PAPER	1.45E-07	1.86E-08	1.06E-06	1.51E-07	1.34E-07	2.79E-08
BOXES	2.20E-07	4.30E-08	1.69E-06	3.08E-07	1.63E-07	4.67E-08
TRANS	1.18E-07	2.89E-08	1.50E-06	2.68E-07	1.93E-07	3.87E-08
WHLSL	6.49E-08	1.29E-08	1.05E-06	1.10E-07	1.10E-07	2.06E-08
RTAIL	2.05E-07	2.89E-08	9.18E-07	2.38E-07	6.03E-08	1.84E-08
MEALS	1.17E-07	2.85E-08	7.24E-07	1.69E-07	7.15E-08	2.14E-08
BANKS	1.23E-07	2.37E-08	1.06E-05	1.65E-06	1.69E-06	6.81E-08
INSUR	6.44E-08	1.30E-08	4.44E-06	3.21E-07	3.52E-07	3.15E-08
RLEST	1.28E-07	2.10E-08	8.85E-06	1.07E-06	1.33E-06	8.31E-08
FIN	3.22E-07	1.98E-08	9.50E-06	1.71E-06	8.72E-07	8.93E-08
OTHER	3.05E-07	3.57E-08	1.69E-06	1.13E-07	2.13E-07	2.75E-08

Table 4.

The dynamics of the Fourth Conditional Co-moments 5 DAYS before **Oct. 19th, 1987** and **Sept. 15th, 2008**

The table shows the percentage variation of the fourth conditional co-moments centered around 0, **5 DAYS** before two event dates: **October 19th, 1987** (The Black Monday) and **September 15th, 2008** (The Chapter 11 for Lehman Brothers). In particular, the table reports the following estimated conditional co-moments: $\mu_{04,t+1}$ is the fourth conditional moment centered around 0, for U.S. industry portfolios; $\mu_{13,t+1}$ is the fourth conditional co-moment centered around 0, with more weight to U.S. industry portfolios; $\mu_{31,t+1}$ is the fourth conditional co-moment centered around 0, with more weight to the market portfolio; $\mu_{22,t+1}$ is the fourth conditional co-moment centered around 0, with equal weights to the industry and the market portfolios. The values are estimated for the period that spans from **January 1982** to **December 2014**.

	$\mu_{04,t+1}$		$\mu_{13,t+1}$		$\mu_{31,t+1}$		$\mu_{22,t+1}$	
	Oct. 19th, 1987	**Sept. 15th, 2008**	**Oct. 19th, 1987**	**Sept. 15th, 2008**	**Oct. 19th, 1987**	**Sept. 15th, 2008**	**Oct. 19th, 1987**	**Sept. 15th, 2008**
AGRIC	**-15.47%**	**35.17%**	**25.51%**	**63.70%**	**156.60%**	**93.14%**	**75.45%**	**62.64%**
FOOD	**300.66%**	**-2.42%**	**294.67%**	**19.21%**	**270.63%**	**65.54%**	**283.03%**	**39.67%**
SODA	242.98%	-20.24%	264.62%	-0.17%	270.09%	53.33%	264.34%	23.33%
BEER	14.91%	-3.61%	56.70%	17.41%	174.79%	64.04%	107.31%	37.26%
SMOKE	215.28%	-30.02%	240.44%	-7.00%	260.40%	52.49%	246.96%	16.99%
TOYS	41.09%	-15.71%	85.04%	3.32%	192.83%	54.37%	132.26%	26.26%
FUN	272.60%	-6.99%	302.81%	8.94%	292.26%	54.95%	292.21%	30.12%
BOOKS	119.15%	25.96%	147.57%	39.55%	214.36%	70.56%	179.19%	54.37%
HSHLD	285.74%	-8.83%	281.37%	11.38%	265.00%	60.00%	273.28%	33.54%
CLTHS	27.72%	-8.37%	69.50%	11.67%	181.94%	60.01%	118.43%	33.76%
HLTH	199.14%	11.08%	234.28%	34.43%	263.30%	74.96%	245.66%	52.73%
MEDEQ	141.57%	-26.26%	175.86%	-5.18%	233.63%	51.46%	203.28%	19.72%
DRUGS	296.99%	12.33%	308.46%	29.66%	285.36%	67.81%	297.60%	47.50%
CHEMS	190.46%	106.29%	215.03%	94.25%	247.46%	85.51%	231.70%	92.63%
RUBBR	621.46%	33.65%	568.56%	46.01%	367.88%	73.24%	455.01%	59.06%
TXTLS	86.73%	-3.27%	128.62%	14.97%	214.48%	60.34%	166.20%	35.69%
BLDMT	359.27%	53.50%	339.67%	62.08%	285.64%	79.45%	312.48%	70.60%
CNSTR	226.08%	275.53%	264.27%	226.77%	279.19%	131.30%	267.60%	172.21%
STEEL	78.23%	113.22%	129.21%	89.01%	222.74%	77.55%	169.04%	94.17%
FABPR	56.79%	120.73%	126.35%	112.29%	239.80%	96.00%	149.46%	103.92%
MACH	109.52%	132.97%	148.20%	120.78%	222.33%	98.42%	183.59%	109.32%
ELCEQ	49.81%	116.71%	88.25%	100.80%	189.11%	87.11%	133.34%	94.28%
AUTOS	235.27%	23.59%	265.89%	37.89%	275.63%	70.13%	269.99%	53.14%
AERO	87.20%	-14.94%	130.83%	2.52%	217.13%	52.48%	169.62%	25.05%
SHIPS	144.65%	-16.25%	186.96%	0.90%	244.87%	51.23%	207.61%	23.94%
GUNS	56.18%	-0.74%	104.83%	19.32%	208.10%	64.28%	144.69%	39.13%
GOLD	**7.50%**	**58.71%**	**130.48%**	**14.04%**	**317.86%**	**24.17%**	**95.06%**	**69.08%**
MINES	265.98%	75.10%	320.02%	82.85%	312.71%	89.54%	299.41%	82.54%
COAL	88.38%	98.38%	141.78%	39.24%	231.13%	35.60%	173.40%	92.41%
OIL	**80.40%**	**128.08%**	**132.60%**	**221.33%**	**225.53%**	**191.86%**	**171.72%**	**117.37%**
UTIL	399.25%	142.32%	362.98%	224.80%	289.50%	186.20%	324.34%	148.89%
TELCM	93.18%	-6.75%	131.09%	11.85%	212.53%	58.88%	169.15%	33.37%
PERSV	167.52%	141.30%	209.80%	138.47%	256.05%	110.58%	230.49%	124.02%
BUSSV	402.40%	77.12%	369.15%	81.98%	293.44%	87.57%	330.51%	85.01%
HARDW	54.15%	-9.83%	91.03%	8.98%	189.23%	57.42%	134.90%	30.85%
SOFTW	-16.70%	-22.91%	23.58%	-10.07%	154.53%	40.49%	76.35%	13.21%
CHIPS	235.53%	12.10%	250.21%	24.70%	259.39%	61.56%	254.93%	42.78%
LABEQ	58.99%	9.54%	101.66%	25.09%	200.63%	63.95%	145.87%	43.22%
PAPER	108.94%	-3.16%	150.03%	15.77%	225.15%	61.37%	185.63%	36.83%
BOXES	162.52%	73.10%	195.28%	67.32%	242.58%	74.45%	217.38%	72.01%
TRANS	387.07%	117.30%	399.08%	99.94%	325.08%	86.05%	355.07%	93.12%
WHLSL	348.36%	31.54%	342.77%	44.47%	253.34%	88.16%	318.59%	58.04%
RTAIL	207.65%	25.34%	227.79%	42.14%	251.29%	74.14%	240.17%	57.64%
MEALS	274.99%	-19.73%	283.96%	-0.44%	272.72%	52.43%	278.37%	23.21%
BANKS	171.57%	33.48%	206.69%	49.89%	249.82%	77.96%	227.82%	63.32%
INSUR	260.45%	138.64%	281.57%	126.98%	277.78%	101.55%	278.55%	114.01%
RLEST	130.29%	42.64%	215.83%	56.91%	291.22%	80.21%	216.75%	67.68%
FIN	190.52%	124.89%	217.32%	116.98%	249.96%	98.47%	233.39%	107.53%
OTHER	59.05%	62.33%	99.74%	73.67%	197.71%	86.97%	143.12%	80.35%

Table 5.

The relationship between the fourth conditional (co)-moments and the asymmetry effect

The table shows the statistical relationships between the percentage variations of the fourth conditional (co)-moments computed from January 1st, 1982 to October 19th, 1987 (**Panel 5.1**) as well as from January 1st, 1982 to September 15th, 2008 (**Panel 5.2**), with the estimated coefficients related to the AR(1)/GJR-GARCH(1,1) processes and a fixed effect dummy (**DUMMY**) able to differentiate between AGRIC, FOOD, GOLD and OIL U.S. industry portfolios with other U.S. industry portfolios. For simplicity the percentage variation is multiplied by **100**. **(1)** α is the constant of the mean equation. It is multiplied by **100000**; **(2)** θ is the coefficient of the auto-regressive component for the mean equation. It is multiplied by **10000**; **(3)** ψ is the constant of the conditional variance process. It is multiplied by **10000000**; **(4)** τ is the coefficient that depicts the persistence of the variance component. It is multiplied by **10000**; **(5)** Φ is the coefficient for the squared residuals. It is multiplied by **10000**; **(6)** ϱ is the coefficient that accounts for asymmetry effects. It is multiplied by **10000; (7)** t-DoF is the estimated quantity that depicts the degree of freedom for the distribution of the returns. It is multiplied by **100**. **(8) DUMMY** is a variable that takes a value equals to 1 if it includes AGRIC, FOOD, GOLD and OIL industry portfolios and a value that is equal to 0, otherwise. The coefficients are estimated via OLS procedure with a Newey-West covariance matrix as well as via MM-procedure (Yohai 1987), where the *(pseudo) random number generator* (RNG) of the seeds is based on the paper proposed by Knuth (1998) and a Tukey's bisquare weighted function with a breakdown value that is equal to 4.684. The **Adjusted R^2** represents the goodness of fit for the OLS-estimation procedure; whereas, the **Adjusted R(w)^2** represents the goodness of fit for the MM-estimation procedure. The brackets report the standard errors. ***, ** and * respectively indicate significance at 1%, 5% and 10% levels.

Panel 5.1: January 1st, 1982 – October 19th, 1987

Coefficients (Parameters)	**OLS-Estimation**				**MM-Estimation**			
	μ_{13}	μ_{31}	μ_{22}	μ_{04}	μ_{13}	μ_{31}	μ_{22}	μ_{04}
α	0.110*** (0.031)	0.038*** (0.012)	0.073*** (0.018)	0.136*** (0.041)	0.063*** (0.021)	0.037*** (0.011)	0.054*** (0.017)	0.071*** (0.023)
θ	-0.001 (0.001)	0.000 (0.000)	0.000 (0.001)	-0.001 (0.002)	0.000 (0.001)	0.000 (0.001)	0.000 (0.001)	0.000 (0.001)
ψ	-0.050* (0.029)	-0.014 (0.010)	-0.032* (0.017)	-0.076* (0.042)	-0.028 (0.026)	-0.014 (0.013)	-0.026 (0.021)	-0.041 (0.029)
Φ	0.021*** (0.004)	0.008*** (0.001)	0.014*** (0.003)	0.025*** (0.005)	0.016*** (0.003)	0.008*** (0.002)	0.013*** (0.003)	0.019*** (0.004)
τ	0.000 (0.001)	0.001*** (0.000)	0.001 (0.000)	0.000 (0.001)	0.000 (0.000)	0.001*** (0.000)	0.001 (0.000)	0.000 (0.000)
ϱ	**0.027***** **(0.007)**	**0.009***** **(0.002)**	**0.016***** **(0.003)**	**0.035***** **(0.010)**	**0.015***** **(0.003)**	**0.008***** **(0.001)**	**0.012***** **(0.002)**	**0.018***** **(0.003)**
t-DoF	**-0.013** **(0.010)**	**-0.004** **(0.003)**	**-0.007** **(0.005)**	**-0.016** **(0.014)**	**-0.005** **(0.006)**	**-0.004** **(0.003)**	**-0.004** **(0.005)**	**-0.005** **(0.006)**
DUMMY	1.041 (2.887)	-1.434 (0.982)	-0.188 (1.716)	3.255 (3.948)	-2.053 (2.517)	-1.614 (1.305)	-1.077 (2.015)	-0.738 (2.854)
Adj-R(w)^2	**66.79%**	**64.16%**	**70.21%**	**63.81%**	**71.41%**	**71.28%**	**72.41%**	**72.62%**
N. obs	**49**	**49**	**49**	**49**	**49**	**49**	**49**	**49**

Panel 5.2: January 1st, 1982 – September 15th, 2008

Coefficients (Parameters)	OLS-Estimation				MM-Estimation			
	μ_{13}	μ_{31}	μ_{22}	μ_{04}	μ_{13}	μ_{31}	μ_{22}	μ_{04}
α	-0.113*	0.067	-0.151***	-0.371***	-0.267***	0.010	-0.248***	-0.415***
	(0.064)	(0.046)	(0.031)	(0.106)	(0.054)	(0.026)	(0.042)	(0.059)
θ	-0.001	-0.002	0.000	0.002	-0.001	0.001	0.000	0.001
	(0.003)	(0.002)	(0.002)	(0.004)	(0.003)	(0.001)	(0.002)	(0.003)
ψ	-0.437***	-0.184***	-0.262***	-0.664***	-0.135	-0.063	-0.154**	-0.161
	(0.152)	(0.070)	(0.084)	(0.244)	(0.098)	(0.046)	(0.076)	(0.107)
Φ	0.022	0.005	0.013	0.036*	0.012	-0.003	0.012	0.014
	(0.019)	(0.010)	(0.009)	(0.022)	(0.010)	(0.005)	(0.008)	(0.011)
τ	0.001	0.001	0.001**	0.002	0.001	0.001***	0.001*	0.002*
	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)	(0.000)	(0.001)	(0.001)
ϱ	**0.034*** **	**0.016*** **	**0.019*** **	**0.031** **	**0.017*** **	**0.009*** **	**0.015*** **	**0.014* **
	(0.013)	**(0.005)**	**(0.006)**	**(0.015)**	**(0.007)**	**(0.003)**	**(0.005)**	**(0.007)**
t-DoF	**-0.020*** **	**-0.008** **	**-0.013** **	**-0.029** **	**-0.009**	**-0.009** **	**-0.009**	**-0.013**
	(0.007)	**(0.004)**	**(0.006)**	**(0.015)**	**(0.009)**	**(0.004)**	**(0.007)**	**(0.010)**
DUMMY	-6.409***	-6.378***	-2.506	-3.142	-2.651	-3.618**	-0.565	6.983*
	(2.435)	(1.875)	(1.544)	(5.868)	(3.492)	(1.659)	(2.708)	(3.819)
Adj-R(w)^2	**37.37%**	**58.38%**	**36.78%**	**26.62%**	**41.67%**	**38.87%**	**51.38%**	**57.10%**
N. obs	**49**	**49**	**49**	**49**	**49**	**49**	**49**	**49**

Table 6.

Dynamic Conditional Quartic Beta and **The Black *Dynamic Conditional Beta***

The table reports the percentage variations for the *Dynamic Conditional Quartic Beta* (**DCQB**) and The Black *Dynamic Conditional Beta* (**DCB**) for the **49** Fama-French U.S. industry portfolios. The values are estimated for the following sub-periods: (i) **5 DAYS** before **October 19th, 1987;** (ii) **5 DAYS** before **September 15th, 2008;** (iii) January 1982 - June 2007; (iv) July 2007 - March 2009; (v) March 2009 - December 2014.

	DCQB					**The Black DCB**				
PORT.	**Oct. 19th, 1987**	**Sept. 15th, 2008**	**Jan. 1982 June 2007**	**July 2007 March 2009**	**March 2009 Dec. 2014**	**Oct. 19th, 1987**	**Sept. 15th, 2008**	**Jan. 1982 June 2007**	**July 2007 March 2009**	**March 2009 Dec. 2014**
AGRIC	**4.40%**	**12.13%**	**-2.82%**	**58.35%**	**-7.85%**	**-27.38%**	**2.65%**	**-1.60%**	**21.39%**	**-5.62%**
FOOD	**4.17%**	**5.83%**	**10.42%**	**30.98%**	**-34.78%**	**4.89%**	**-12.02%**	**3.34%**	**4.89%**	**0.61%**
SODA	8.87%	1.33%	11.82%	64.08%	-65.07%	4.74%	-18.51%	-9.21%	38.62%	-47.67%
BEER	5.69%	3.99%	33.58%	16.13%	-51.82%	-22.23%	-12.82%	-5.08%	5.06%	-19.67%
SMOKE	7.53%	4.61%	11.35%	14.00%	-25.31%	2.00%	-18.95%	-9.90%	-32.83%	41.63%
TOYS	7.86%	0.49%	0.48%	36.00%	-41.94%	-17.13%	-17.96%	-20.59%	24.81%	16.82%
FUN	15.66%	-3.08%	-10.40%	54.66%	-24.99%	11.01%	-17.65%	-37.91%	83.05%	-3.94%
BOOKS	4.88%	2.70%	11.03%	-6.20%	-10.45%	-11.03%	-9.35%	-14.45%	29.32%	-11.86%
HSHLD	2.36%	4.03%	-7.36%	56.46%	-41.29%	3.30%	-14.96%	-23.40%	27.14%	-23.52%
CLTHS	5.58%	3.95%	-2.84%	66.59%	-47.67%	-20.21%	-14.96%	-2.80%	35.31%	-36.11%
HLTH	12.17%	11.02%	18.86%	22.27%	-35.36%	2.82%	-7.01%	-24.62%	56.67%	-29.69%
MEDEQ	7.71%	3.15%	21.91%	-3.09%	-12.61%	-5.58%	-19.50%	3.09%	-3.35%	6.50%
DRUGS	13.33%	2.93%	17.19%	12.41%	-43.51%	9.06%	-10.81%	13.80%	-16.71%	27.36%
CHEMS	8.11%	-4.63%	6.02%	48.55%	-23.39%	-1.67%	-1.41%	-11.01%	57.14%	-7.00%
RUBBR	19.74%	0.65%	1.97%	47.78%	-16.98%	32.41%	-7.93%	-15.55%	74.24%	-34.76%
TXTLS	6.89%	1.08%	-8.49%	60.21%	-27.88%	-11.00%	-14.78%	-15.75%	125.91%	-44.14%
BLDMT	5.43%	0.87%	2.42%	74.57%	-26.18%	9.14%	-4.63%	-7.24%	84.19%	-22.20%
CNSTR	16.11%	4.74%	19.03%	40.57%	-22.39%	7.32%	22.93%	20.18%	32.94%	-22.06%
STEEL	13.90%	-9.50%	-1.32%	69.09%	-23.42%	-8.66%	-5.63%	4.23%	76.03%	-28.09%
FABPR	20.19%	0.12%	9.53%	85.10%	-29.38%	-3.83%	4.17%	-4.60%	87.29%	25.36%
MACH	9.51%	-0.04%	-1.18%	49.28%	-22.38%	-8.78%	5.45%	-8.32%	56.67%	-16.85%
ELCEQ	2.91%	-7.13%	2.31%	55.84%	-29.86%	-18.18%	-0.56%	-19.20%	53.62%	-8.09%
AUTOS	15.20%	0.83%	-3.97%	66.75%	-18.61%	6.31%	-9.58%	-23.89%	77.63%	-13.28%
AERO	9.40%	-2.26%	4.95%	48.16%	-34.75%	-10.25%	-18.96%	-18.67%	42.54%	-20.99%
SHIPS	9.45%	-2.36%	-6.80%	80.90%	-20.18%	-2.40%	-19.62%	-22.87%	64.73%	38.61%
GUNS	8.96%	3.49%	-21.72%	74.85%	-26.44%	-12.80%	-12.69%	-19.34%	40.02%	-18.27%
GOLD	**59.27%**	**-31.53%**	**209.50%**	**-62.15%**	**82.23%**	**18.26%**	**-34.01%**	**134.66%**	**-65.94%**	**337.05%**
MINES	22.59%	2.74%	-3.51%	83.58%	-29.61%	16.80%	0.74%	-8.14%	79.84%	8.42%
COAL	12.73%	-28.95%	33.68%	50.33%	-26.95%	-6.29%	-27.93%	35.61%	40.18%	-3.10%
OIL	**14.72%**	**49.81%**	**39.33%**	**30.85%**	**-26.06%**	**-7.87%**	**55.12%**	**45.91%**	**-5.33%**	**64.11%**
UTIL	1.97%	51.25%	27.10%	23.48%	-21.28%	10.23%	52.11%	96.54%	-29.36%	37.86%
TELCM	6.57%	1.53%	6.99%	44.26%	-44.20%	-11.55%	-15.56%	-11.27%	31.40%	-30.96%
PERSV	14.81%	10.47%	9.10%	18.93%	21.32%	0.76%	11.92%	-15.26%	-0.10%	40.79%
BUSSV	5.24%	3.28%	9.88%	25.73%	-29.04%	11.35%	-0.31%	-11.35%	21.92%	-13.58%
HARDW	1.23%	0.86%	8.37%	50.04%	-29.02%	-18.15%	-16.33%	-24.45%	26.49%	2.96%
SOFTW	5.27%	-10.99%	10.01%	33.02%	-34.95%	-27.97%	-25.33%	-27.66%	6.95%	3.91%
CHIPS	6.36%	-3.21%	1.71%	51.90%	-31.98%	1.71%	-14.14%	-25.30%	18.92%	-10.25%
LABEQ	7.36%	-0.47%	11.69%	31.12%	-24.57%	-14.92%	-12.87%	-25.92%	39.87%	-9.01%
PAPER	11.02%	3.13%	9.30%	44.19%	-36.93%	-7.98%	-14.23%	-8.35%	44.98%	-28.56%
BOXES	8.25%	-8.59%	3.05%	45.47%	-26.90%	-3.05%	-7.29%	-7.80%	19.05%	-9.20%
TRANS	18.98%	-8.59%	0.14%	70.82%	-48.88%	20.30%	-1.12%	1.45%	38.09%	-39.87%
WHLSL	11.78%	1.00%	7.76%	20.36%	-17.76%	11.24%	-8.17%	-3.55%	44.99%	-26.34%
RTAIL	7.35%	6.09%	1.26%	33.88%	-43.96%	-0.58%	-7.45%	-15.81%	-1.26%	-16.80%
MEALS	8.10%	0.53%	6.86%	36.17%	-36.81%	5.48%	-18.99%	3.20%	-10.77%	3.84%
BANKS	12.21%	6.21%	15.58%	11.37%	-16.27%	-1.00%	-5.42%	-15.51%	112.87%	-53.62%
INSUR	12.03%	2.15%	11.39%	21.11%	-27.18%	6.92%	7.12%	-18.16%	113.48%	-50.25%
RLEST	28.55%	4.46%	9.17%	57.25%	-24.81%	10.72%	-4.22%	37.89%	54.55%	-55.75%
FIN	8.39%	1.80%	2.42%	18.44%	-22.83%	-0.96%	5.48%	-8.27%	40.32%	-43.51%
OTHER	4.85%	6.70%	-12.67%	69.62%	-30.84%	-15.74%	-0.63%	-36.37%	161.18%	-48.54%

Table 7.
Measures of Divergence between
The *Dynamic Conditional Quartic Beta* and The Black *Dynamic Conditional Beta*

The table reports some metrics (**RMSD**, **MAPD** and **SMAPD**) for depicting the divergence between The *Dynamic Conditional Quartic Beta* and The Black *Dynamic Conditional Beta*. (1) **RMSD** is the Root Mean Square Divergence; (2) **MAPD** is the Mean Absolute Percentage Divergence; (3) **SMAPD** is the Symmetric Mean Absolute Percentage Divergence. These values are computed for the **49** Fama-French U.S. industry portfolios.

PORTFOLIOS	RMSD	MAPD	SMAPD
AGRIC	**0.232**	**0.431**	**0.325**
FOOD	**0.738**	**1.286**	**0.744**
SODA	0.255	0.371	0.299
BEER	0.420	0.735	0.478
SMOKE	0.285	0.439	0.333
TOYS	0.380	0.431	0.327
FUN	0.370	0.409	0.321
BOOKS	6.401	5.816	1.455
HSHLD	0.678	1.065	0.661
CLTHS	0.562	0.713	0.501
HLTH	0.443	0.606	0.428
MEDEQ	0.615	0.852	0.559
DRUGS	0.652	0.887	0.571
CHEMS	0.688	0.847	0.575
RUBBR	0.627	0.919	0.605
TXTLS	0.439	0.649	0.454
BLDMT	0.750	0.928	0.607
CNSTR	0.401	0.431	0.332
STEEL	0.411	0.453	0.343
FABPR	0.314	0.438	0.335
MACH	0.685	0.815	0.554
ELCEQ	0.580	0.591	0.434
AUTOS	0.439	0.439	0.338
AERO	0.504	0.618	0.449
SHIPS	0.269	0.344	0.280
GUNS	0.270	0.403	0.307
GOLD	**0.199**	**0.332**	**0.207**
MINES	0.262	0.399	0.309
COAL	0.308	0.282	0.308
OIL	**0.350**	**0.513**	**0.384**
UTIL	0.691	1.545	0.826
TELCM	0.766	1.013	0.648
PERSV	0.521	0.676	0.476
BUSSV	1.086	1.395	0.809
HARDW	0.360	0.353	0.286
SOFTW	0.436	0.430	0.325
CHIPS	0.509	0.488	0.368
LABEQ	0.558	0.637	0.449
PAPER	0.764	1.046	0.662
BOXES	0.467	0.591	0.432
TRANS	0.655	0.812	0.553
WHLSL	0.935	1.346	0.787
RTAIL	0.798	0.962	0.621
MEALS	0.634	0.870	0.573
BANKS	0.703	0.848	0.563
INSUR	0.806	1.156	0.712
RLEST	0.388	0.750	0.484
FIN	0.712	0.868	0.553
OTHER	0.554	0.718	0.491

Table 8.

Forecasting Power of The *Dynamic Conditional Quartic Beta*

The Table reports the *pseudo-out-of-sample* results of the methodology and goodness-of-fit statistics for the ***Dynamic Conditional Quartic Beta*** related to the Value Weighted (**VW**) **49** Fama-French U.S. industry portfolios. The table reports the median estimated values across U.S. industry portfolios and the performance measures (Symmetric Mean Absolute Percentage Divergence **(SMAPD)** and Root Mean Square Divergence **(RMSD)**) that relate the *estimated values* with the *simulated values* around some important dates in which the CRSP value weighted U.S. market portfolio sharply decreased. The performance measures are computed at 2, 4, 8 and 16 days, *before the event dates*. **Panel 8.1** shows the ***pseudo out-of-sample*** results, relying on **50000** Trials; whereas, **Panel 8.2** shows the ***pseudo out-of-sample*** results, relying on **100000** simulated trials. The **SMAPD** ranges between **0%** and **200%**. The event dates are selected during the period that spans from **January 1982** to **December 2014**.

Panel 8.1: *Pseudo out-of-sample* results for the **49 Value Weighted** U.S. industry portfolios (N. Trials = **50000**)

EVENT DATE	**MARKET RETURN**	**N. OBS**	**MED. EST. VALUES**	**SIMULATED VALUES**							
				2 DAYS		**4 DAYS**		**8 DAYS**		**16 DAYS**	
				SMAPD	**RMSD**	**SMAPD**	**RMSD**	**SMAPD**	**RMSD**	**SMAPD**	**RMSD**
10/19/1987	-17.41%	1467	1.3792	59.70%	63.19%	56.34%	58.02%	56.96%	56.49%	56.34%	58.33%
12/01/2008	-8.95%	6792	1.1652	63.62%	57.83%	65.34%	60.14%	62.60%	56.89%	62.91%	57.80%
10/15/2008	-8.78%	6760	1.1616	74.43%	65.18%	74.56%	64.92%	74.84%	64.94%	68.06%	61.09%
10/26/1987	-8.28%	1472	1.1592	69.01%	60.43%	69.91%	60.25%	67.64%	61.79%	51.98%	50.86%
9/29/2008	-8.25%	6756	1.1607	73.81%	65.00%	75.44%	65.47%	66.45%	61.94%	66.40%	61.01%

Panel 8.2: *Pseudo out-of-sample* results for the **49 Value Weighted** U.S. industry portfolios (N. Trials = **100000**)

EVENT DATE	**MARKET RETURN**	**N. OBS**	**MED. EST. VALUES**	**SIMULATED VALUES**							
				2 DAYS		**4 DAYS**		**8 DAYS**		**16 DAYS**	
				SMAPD	**RMSD**	**SMAPD**	**RMSD**	**SMAPD**	**RMSD**	**SMAPD**	**RMSD**
10/19/1987	-17.41%	1467	1.3792	**59.74%**	**63.27%**	56.34%	58.01%	57.00%	56.54%	**56.37%**	**58.36%**
12/01/2008	-8.95%	6792	1.1652	**63.64%**	**57.86%**	65.40%	60.21%	62.69%	56.97%	**62.89%**	**57.80%**
10/15/2008	-8.78%	6760	1.1616	**74.45%**	**65.18%**	74.57%	64.92%	74.83%	64.97%	**68.20%**	**61.22%**
10/26/1987	-8.28%	1472	1.1592	**69.01%**	**60.43%**	69.95%	60.28%	67.69%	61.89%	**52.09%**	**50.98%**
9/29/2008	-8.25%	6756	1.1607	**73.82%**	**64.99%**	75.41%	65.46%	66.49%	62.01%	**66.46%**	**61.03%**

Figure 1.

Indicators of Dynamic Conditional Volatility, Correlation, Quartic Beta and Beta

The figure shows the cross-sectional median concerned about the *Dynamic Conditional Volatility* and the *Dynamic Conditional Correlation* (**Figure 1.1**) as well as The Black *Dynamic Conditional Beta* and The *Dynamic Conditional Quartic Beta* (**Figure 1.2**), across the **49** Fama-French value weighted U.S. industry portfolios, from **January 1982** to **December 2014**.

Figure 1.1 Cross-sectional Median (Conditional Volatility and **Correlation)**

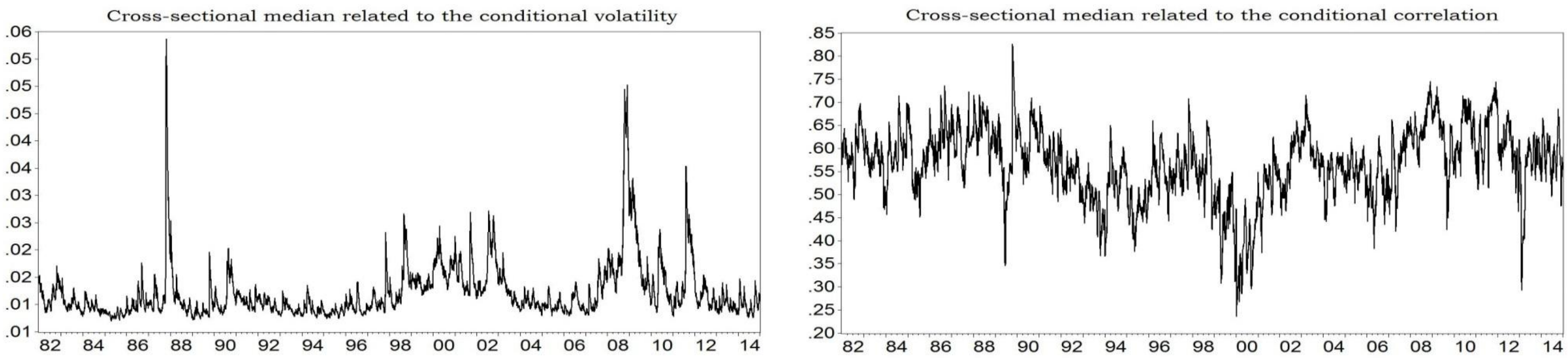


Figure 1.2 Cross-sectional Median (The Black *Dynamic Conditional Beta* and **The *Dynamic Conditional Quartic Beta*)**

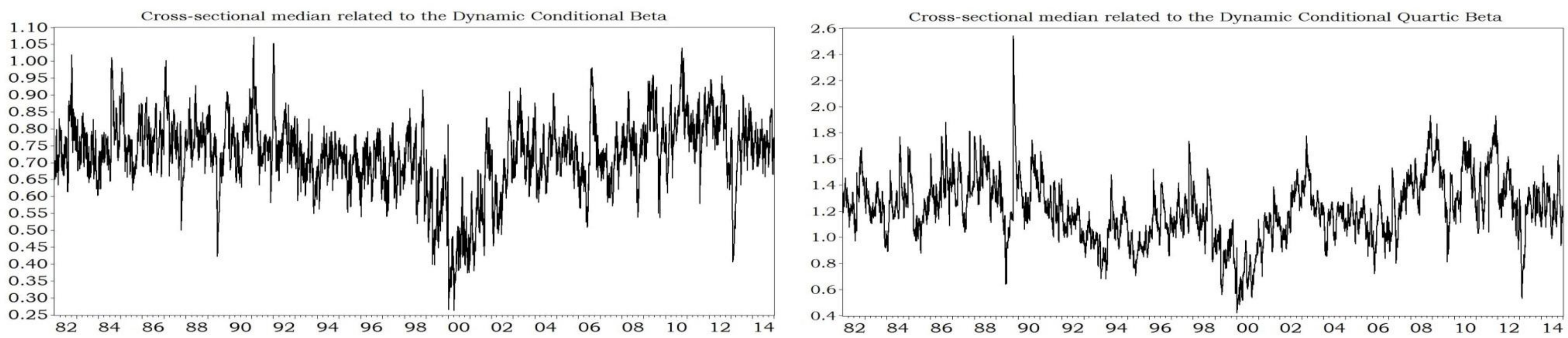
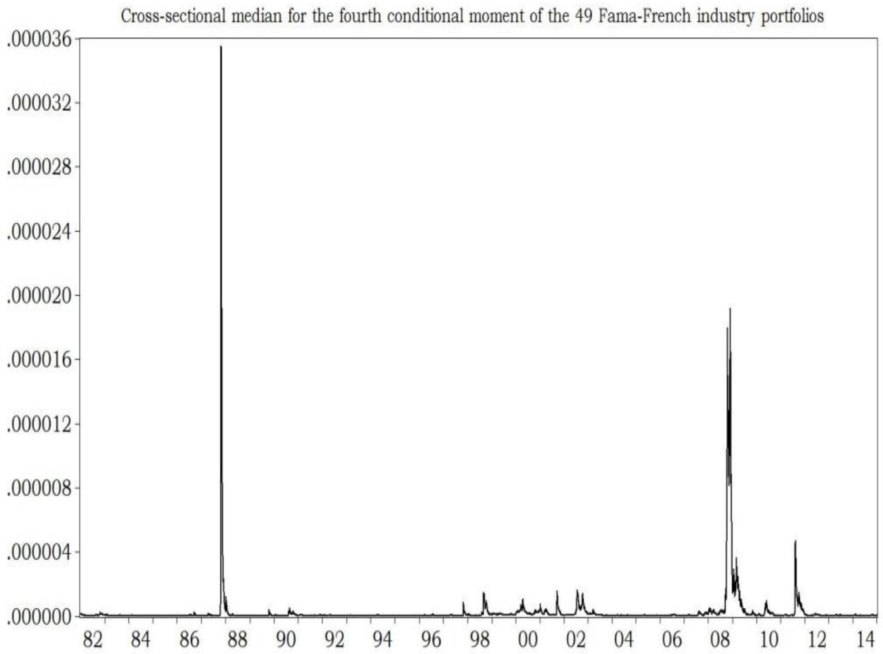


Figure 2.

The dynamics for the fourth conditional (co)-moments

The figure shows the dynamics of the cross-sectional median for the *fourth conditional (co)-moments* centered around 0; the dynamics of the cross-sectional median for the **49** Fama-French U.S. industry portfolios (**IPs**) and the dynamics for the *fourth conditional moment* centered around 0, regarding the U.S. market portfolio (**Figure 2.1**). **Figure 2.2** shows the evolution of the cross-sectional median for the *fourth conditional* (co)-moments, centered around 0.

Figure 2.1: Cross-sectional median for the fourth centered conditional moments



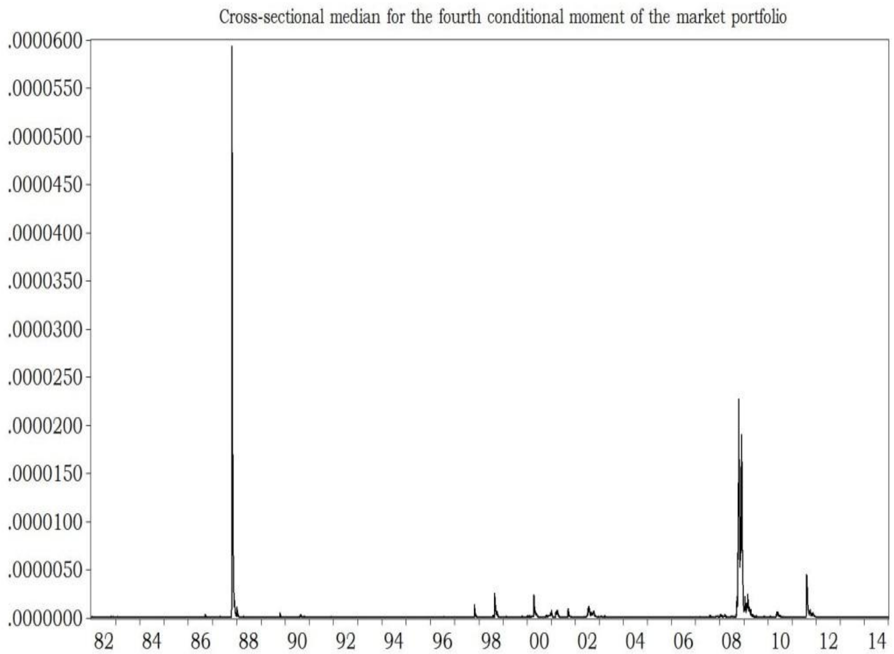


Figure 2.2: Cross-sectional median for the fourth conditional centered co-moments

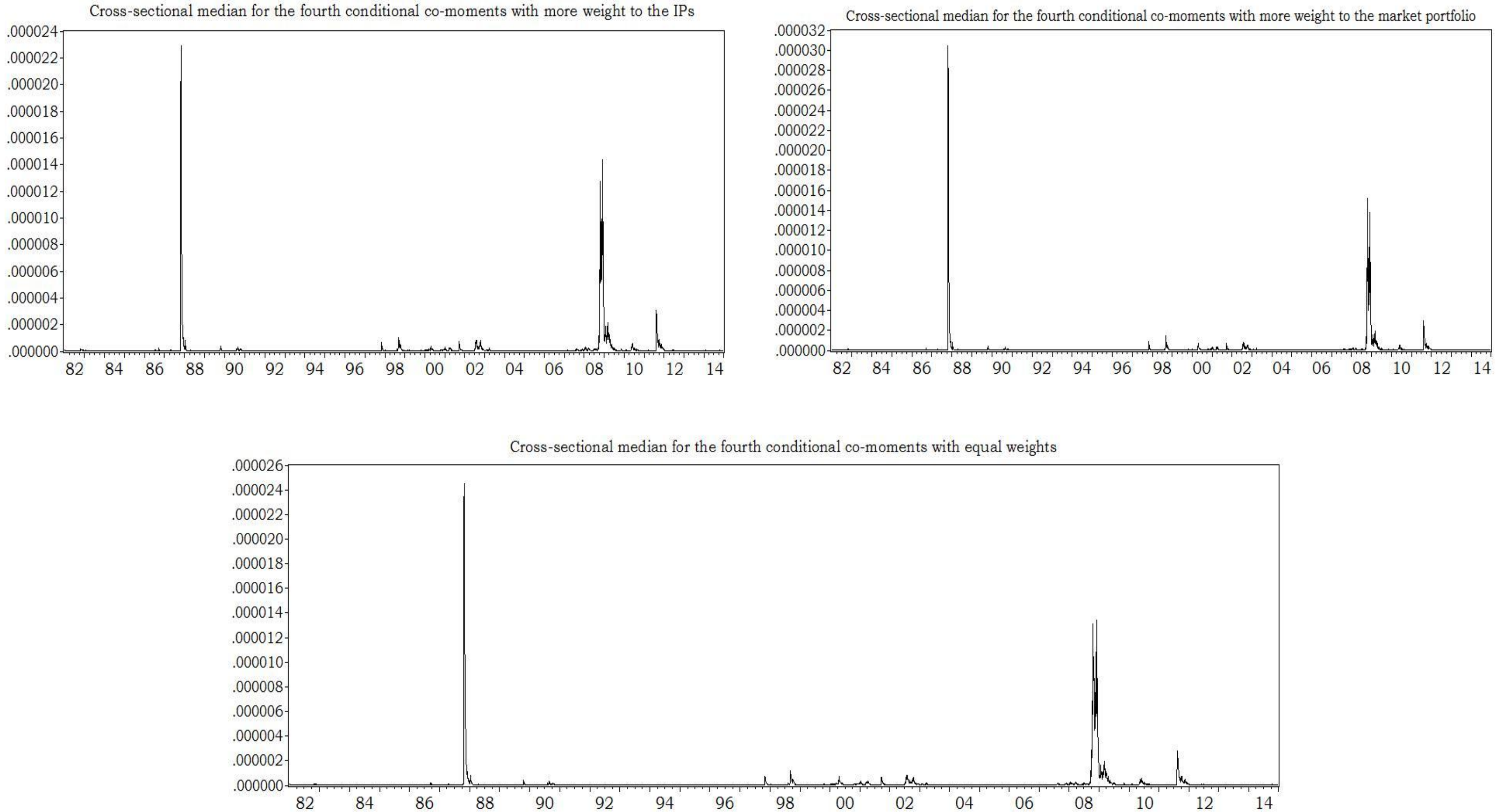


Figure 3.
Number of conditional (CO)-SPIKES

The figure shows the number of conditional (co)-spikes across the **49** U.S. industry portfolios (**IPs**) as well as the U.S. market portfolio. The *conditional* (**CO)-SPIKES** are the fourth conditional (co)-moments, centered around 0. The estimated values are multiplied by **1000000** and the period is from **January 1982** to **December 2014**.

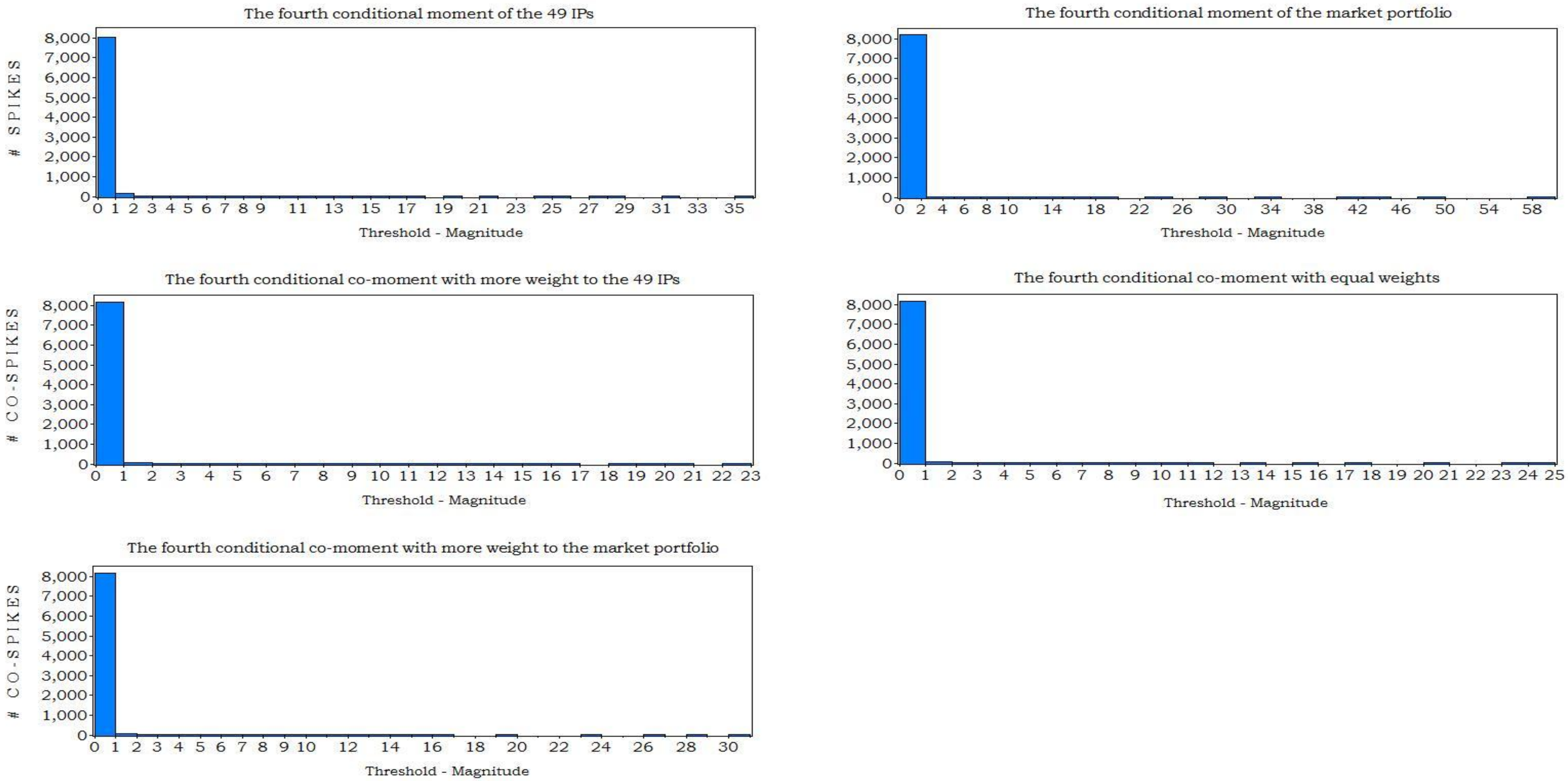


Figure 4.

AGRIC industry: DCQB and its components

The figure shows the evolution of The *Dynamic Conditional Quartic Beta*, The Black *Dynamic Conditional Beta* as well as the dynamics for the fourth conditional (co)-moments centered around 0, for **AGRIC** U.S. industry portfolio, from **January 1982** to **December 2014**. **MU_04** corresponds to the quantity $\mu_{04,t+1}$ and represents the fourth conditional moment centered around 0; **MU_13** corresponds to the quantity $\mu_{13,t+1}$ and represents the fourth conditional co-moment centered around 0, with more weight to the U.S. industry portfolio; **MU_31** corresponds to the quantity $\mu_{31,t+1}$ and represents the fourth conditional co-moment centered around 0, with more weight to the market portfolio; **MU_22** corresponds to the quantity $\mu_{22,t+1}$ and represents the fourth conditional co-moment centered around 0, with equal weights to the industry and the market portfolios.

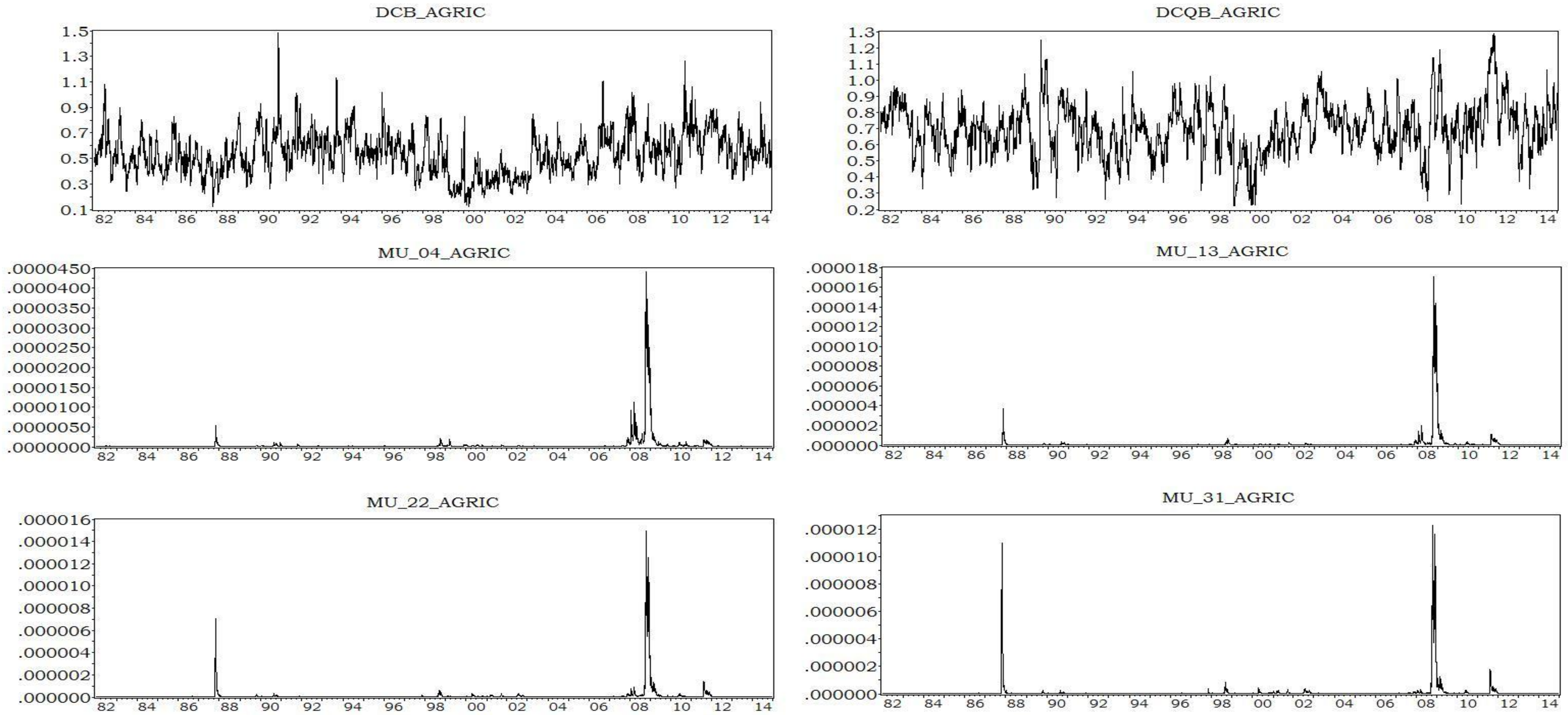


Figure 5.
FOOD industry: DCQB and its components

The figure shows the evolution of The *Dynamic Conditional Quartic Beta*, The Black *Dynamic Conditional Beta* as well as the dynamics for the fourth conditional (co)-moments centered around 0, for **FOOD** U.S. industry portfolio, from **January 1982** to **December 2014**.

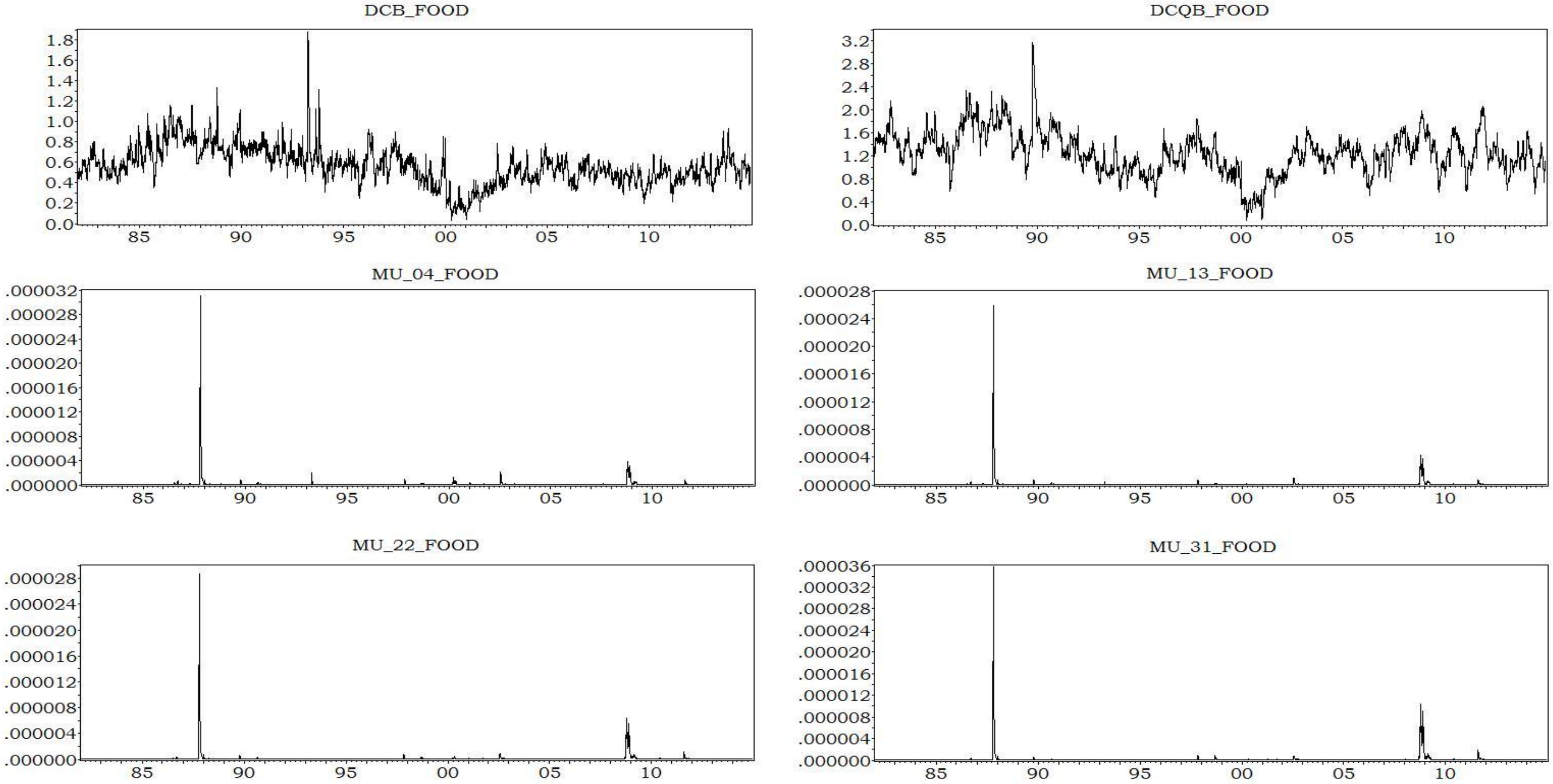


Figure 6.

GOLD industry: DCQB and its components

The figure shows the evolution of The *Dynamic Conditional Quartic Beta*, The Black *Dynamic Conditional Beta* as well as the dynamics for the fourth conditional (co)-moments centered around 0, for **GOLD** U.S. industry portfolio, from **January 1982** to **December 2014**.

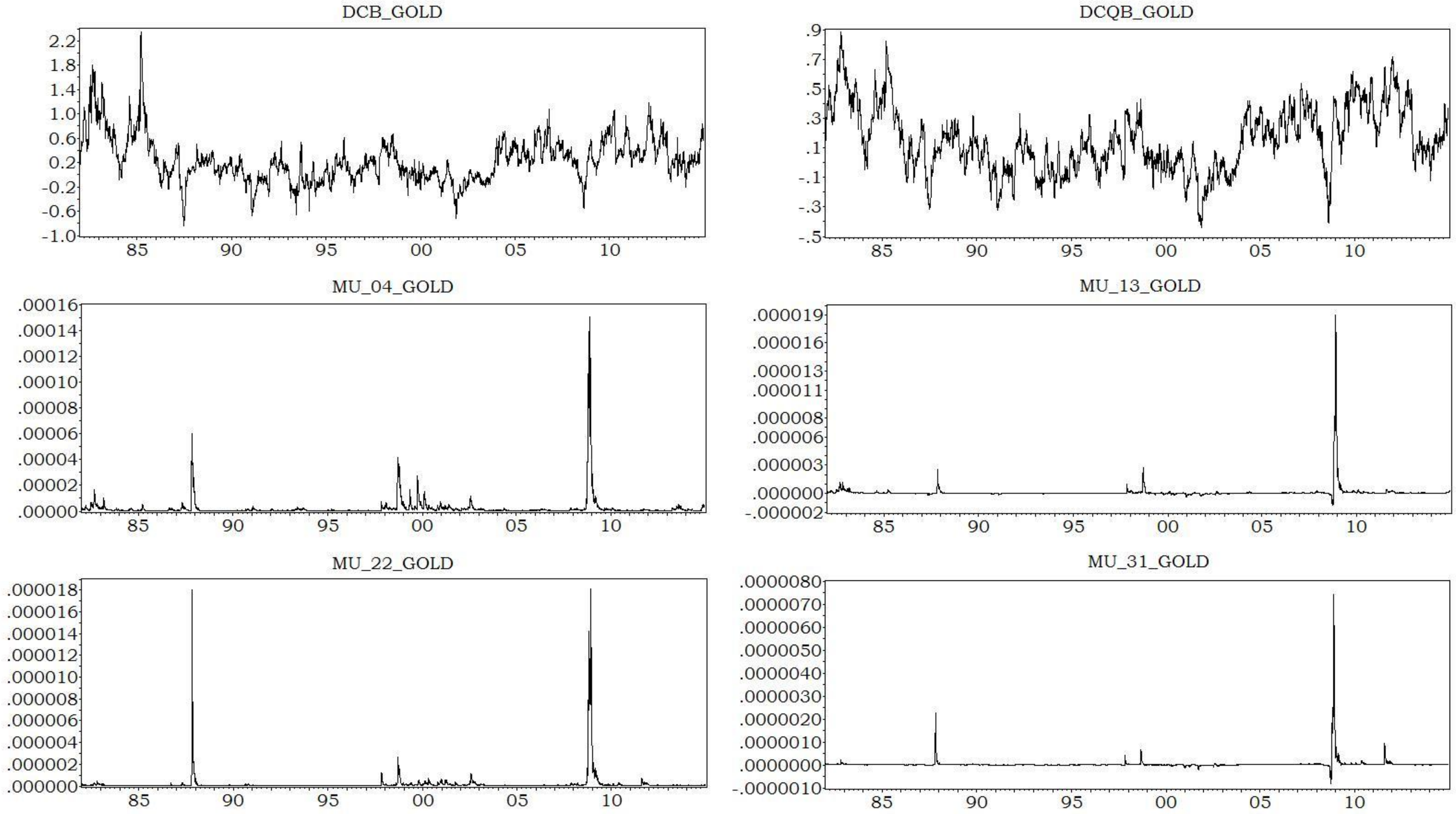
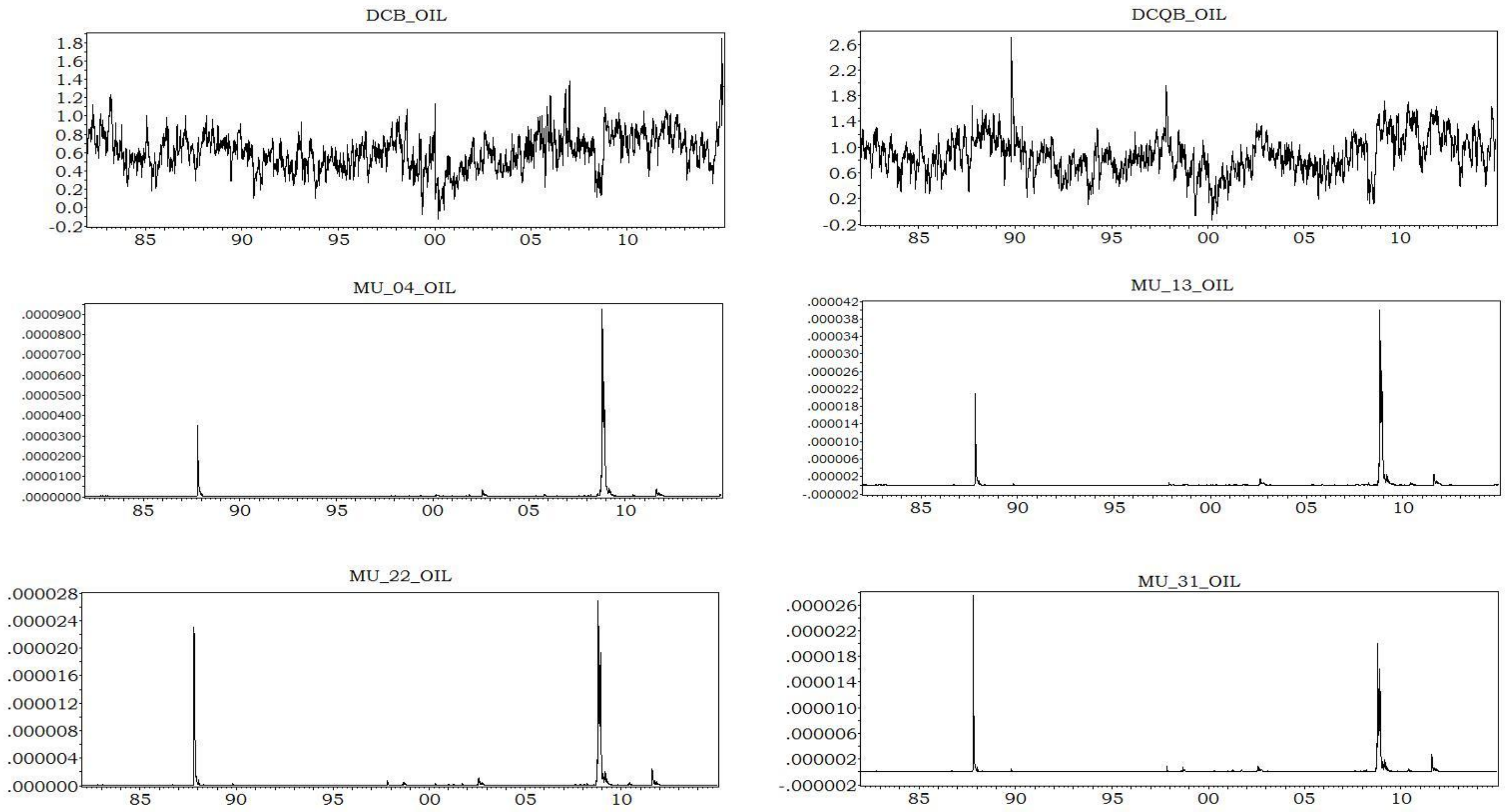


Figure 7.
OIL industry: DCQB and its components

The figure shows the evolution for The *Dynamic Conditional Quartic Beta*, The Black *Dynamic Conditional Beta* as well as the dynamics for the fourth conditional (co)-moments centered around 0, for **OIL** U.S. industry portfolio, from **January 1982** to **December 2014**.



Appendix A.

The imaginary solutions of the polynomial equation

Appendix A derives the imaginary solutions of the polynomial equation proposed in Section 2 and Section 3[1]. These solutions are constructed from the minimization of the fourth power related to the loss function, that is based on the error component $\eta_{i,t+1}$. The quantities A_{t+1}, B_{t+1} and C_{t+1} are estimated in the following way:

$$A_{t+1} = \left(\frac{\mu_{22,t+1}}{\mu_{40,t+1}} - \frac{\mu_{31,t+1}^2}{\mu_{40,t+1}^2} \right) \tag{1}$$

$$B_{t+1} = \left((A_{t+1})^3 + \left(\frac{\mu_{13,t+1}}{(2 \cdot \mu_{40,t+1})} + \frac{\mu_{31,t+1}^3}{\mu_{40,t+1}^3} - \frac{(3 \cdot \mu_{22,t+1} \cdot \mu_{31,t+1})}{\left(2 \cdot \mu_{40,t+1}^2\right)} \right)^2 \right)^{\frac{1}{2}} \tag{2}$$

$$C_{t+1} = \left(\frac{\mu_{13,t+1}}{(2 \cdot \mu_{40,t+1})} + \frac{\mu_{31,t+1}^3}{\mu_{40,t+1}^3} - \frac{(3 \cdot \mu_{22,t+1} \cdot \mu_{31,t+1})}{\left(2 \cdot \mu_{40,t+1}^2\right)} \right). \tag{3}$$

For simplicity, the quantities $\mu_{04,t+1}$ and $\mu_{40,t+1}$ respectively represent the fourth conditional moments, for the risky asset and the market portfolio; whereas, the quantities $\mu_{22,t+1}$, $\mu_{13,t+1}$ and $\mu_{31,t+1}$ represent the fourth mixed conditional moments, between the risky asset and the market portfolio, that are centered around the level of 0[2].

Section 2 and Section 3 derive the polynomial equation and compute the solutions that belong to the set of complex numbers. The first imaginary solution ($\beta_{2,t+1}$) of the polynomial equation is defined in the following way:

$$\beta_{2,t+1} = \frac{A_{t+1}}{\left(2 \cdot (B_{t+1} + C_{t+1})^{\frac{1}{3}}\right)} + \frac{\mu_{31,t+1}}{\mu_{40,t+1}} - \frac{(B_{t+1} + C_{t+1})^{\frac{1}{3}}}{2} - \left(3^{\frac{1}{2}} \cdot \left(\frac{A_{t+1}}{(B_{t+1} + C_{t+1})^{\frac{1}{3}}} + (B_{t+1} + C_{t+1})^{\frac{1}{3}} \right) \cdot \frac{j}{2} \right), \tag{4}$$

whereas, the second imaginary solution ($\beta_{3,t+1}$) of the polynomial equation is computed as follows:

$$\beta_{3,t+1} = \frac{A_{t+1}}{\left(2 \cdot (B_{t+1} + C_{t+1})^{\frac{1}{3}}\right)} + \frac{\mu_{31,t+1}}{\mu_{40,t+1}} - \frac{(B_{t+1} + C_{t+1})^{\frac{1}{3}}}{2} + \left(3^{\frac{1}{2}} \cdot \left(\frac{A_{t+1}}{(B_{t+1} + C_{t+1})^{\frac{1}{3}}} + (B_{t+1} + C_{t+1})^{\frac{1}{3}} \right) \cdot \frac{j}{2} \right). \tag{5}$$

Section 3 also provides the steps for estimating the imaginary solutions $\beta_{2,t+1}$ and $\beta_{3,t+1}$ that belong to the set of complex numbers.

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[1] The imaginary solutions contain the quantity j that represents the unit imaginary number. The term "imaginary" is used because there is no real number having a negative square. Imaginary numbers extend the set of real numbers to the set of complex numbers.

[2] The fourth conditional centered (co)-moments are based on the information set F, at time t.

Appendix B.

Summary and Descriptive Statistics

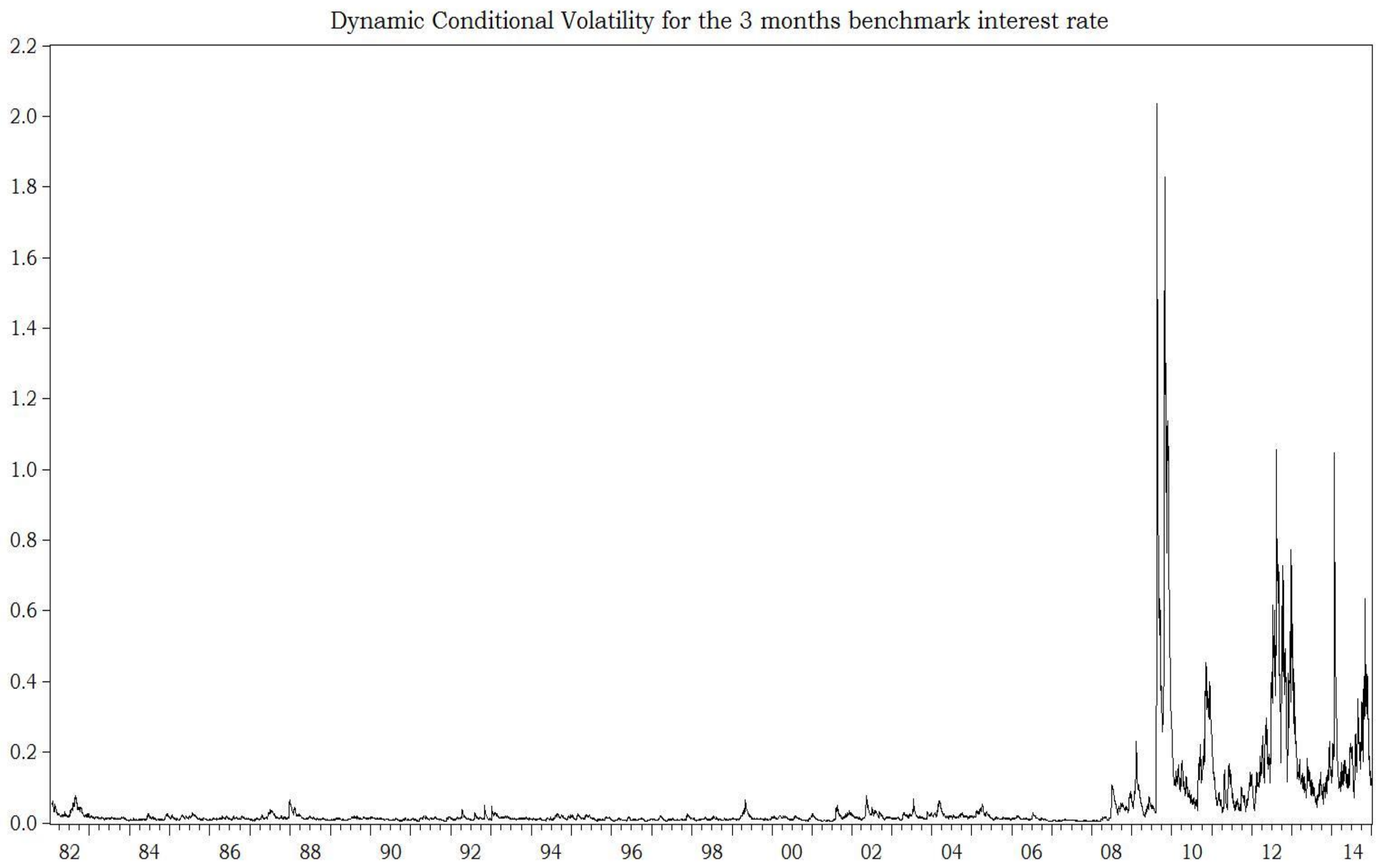
Appendix B reports the descriptive statistics (mean, median, max., min., standard deviation, kurtosis) for the **49** Fama-French value weighted (**VW**) U.S. industry portfolios, downloaded from Kenneth French's website, for the period from **January 1982** to **December 2014**.

PORTFOLIO	Mean	Median	Max	Min.	Std. Dev.	Kurt.
AGRIC	**0.05%**	**0.05%**	**20.32%**	**-15.27%**	**1.47%**	**20.14**
FOOD	**0.06%**	**0.08%**	**9.98%**	**-16.04%**	**0.98%**	**18.84**
SODA	0.06%	0.05%	11.68%	-19.22%	1.55%	13.54
BEER	0.07%	0.05%	10.12%	-14.72%	1.22%	10.35
SMOKE	0.08%	0.06%	14.99%	-13.99%	1.53%	12.84
TOYS	0.04%	0.06%	9.69%	-18.61%	1.51%	11.64
FUN	0.06%	0.07%	16.55%	-24.11%	1.72%	16.01
BOOKS	0.05%	0.04%	19.45%	-11.24%	1.30%	17.17
HSHLD	0.06%	0.05%	21.46%	-7.88%	1.11%	33.17
CLTHS	0.06%	0.08%	12.69%	-18.51%	1.37%	12.94
HLTH	0.05%	0.07%	8.29%	-15.44%	1.34%	11.29
MEDEQ	0.05%	0.08%	11.69%	-15.24%	1.20%	10.69
DRUGS	0.06%	0.07%	11.34%	-18.70%	1.20%	15.05
CHEMS	0.06%	0.05%	13.06%	-17.66%	1.34%	14.18
RUBBR	0.05%	0.08%	9.71%	-16.55%	1.21%	13.24
TXTLS	0.05%	0.07%	19.50%	-18.40%	1.51%	18.55
BLDMT	0.05%	0.06%	9.62%	-17.49%	1.33%	13.16
CNSTR	0.05%	0.05%	15.56%	-15.92%	1.71%	10.49
STEEL	0.04%	0.03%	20.06%	-23.94%	1.80%	16.54
FABPR	0.03%	0.04%	11.44%	-11.83%	1.54%	8.99
MACH	0.05%	0.07%	13.91%	-18.06%	1.44%	13.42
ELCEQ	0.06%	0.05%	14.08%	-19.70%	1.50%	12.14
AUTOS	0.05%	0.04%	11.70%	-19.71%	1.60%	11.27
AERO	0.06%	0.08%	13.57%	-18.37%	1.39%	14.65
SHIPS	0.06%	0.08%	10.62%	-13.20%	1.59%	7.10
GUNS	0.06%	0.06%	14.92%	-19.49%	1.45%	12.93
GOLD	**0.03%**	**-0.10%**	**25.50%**	**-23.54%**	**2.49%**	**9.59**
MINES	0.04%	0.02%	19.99%	-16.88%	1.76%	14.27
COAL	0.05%	0.00%	21.36%	-19.34%	2.47%	10.90
OIL	**0.05%**	**0.05%**	**19.27%**	**-19.50%**	**1.44%**	**18.34**
UTIL	0.05%	0.07%	14.43%	-12.86%	0.95%	23.98
TELCM	0.05%	0.07%	14.51%	-16.68%	1.24%	15.44
PERSV	0.04%	0.05%	9.82%	-14.45%	1.32%	10.10
BUSSV	0.05%	0.07%	8.29%	-16.34%	1.16%	13.94
HARDW	0.06%	0.08%	21.65%	-21.54%	1.80%	13.04
SOFTW	0.06%	0.09%	14.82%	-19.33%	1.70%	10.20
CHIPS	0.05%	0.08%	15.87%	-17.10%	1.77%	9.60
LABEQ	0.05%	0.07%	12.71%	-18.54%	1.52%	10.12
PAPER	0.05%	0.07%	8.63%	-20.53%	1.17%	19.72
BOXES	0.06%	0.06%	10.91%	-21.43%	1.38%	14.82
TRANS	0.05%	0.06%	9.33%	-14.03%	1.28%	9.60
WHLSL	0.05%	0.08%	13.20%	-8.49%	1.07%	11.68
RTAIL	0.06%	0.07%	11.75%	-18.00%	1.27%	13.52
MEALS	0.06%	0.07%	11.42%	-15.46%	1.19%	11.28
BANKS	0.07%	0.06%	16.96%	-16.98%	1.58%	19.76
INSUR	0.07%	0.08%	17.84%	-9.30%	1.26%	20.07
RLEST	0.04%	0.02%	21.90%	-12.07%	1.48%	22.41
FIN	0.09%	0.07%	17.94%	-9.99%	1.65%	16.50
OTHER	0.03%	0.04%	15.23%	-17.26%	1.42%	15.24

Appendix C.

The Dynamic Conditional Volatility for the U.S. *benchmark* interest rate

Appendix C reports the evolution of the conditional volatility for the 3 months U.S. *benchmark* interest rate, **from January 1982** to **December 2014**. The data are downloaded from the *Board of Governors of the Federal Reserve System*. For simplicity, the 3 months U.S. *benchmark* interest rate is the 3 months U.S. treasury bill rate.



Appendix D.
Correlation Matrix among Covariates

Appendix D reports the correlation matrix among variables, for regressions estimated via OLS as well as MM estimation techniques. The correlation matrix reports the p-values in brackets and considers the period from January 1st, 1982 to September 15th, 2008.

	α	θ	ψ	Φ	τ	ϱ	t-DoF	DUMMY
α	1.000							
θ	0.226 (0.119)	1.000						
ψ	-0.153 (0.294)	-0.330 (0.021)	1.000					
Φ	-0.115 (0.433)	0.167 (0.250)	0.109 (0.456)	1.000				
τ	-0.276 (0.055)	-0.416 (0.003)	-0.382 (0.007)	-0.491 (0.000)	1.000			
ϱ	0.382 (0.007)	0.550 (0.000)	0.073 (0.618)	-0.199 (0.170)	-0.681 (0.000)	1.000		
t-DoF	0.202 (0.164)	0.435 (0.002)	-0.351 (0.014)	-0.362 (0.011)	0.087 (0.554)	0.343 (0.016)	1.000	
DUMMY	-0.389 (0.006)	-0.358 (0.012)	0.177 (0.224)	-0.114 (0.435)	0.150 (0.304)	-0.168 (0.250)	-0.238 (0.100)	1.000