Forecasting Bitcoin risk measures: A robust approach

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June 2, 2018

Abstract

In the last few years, Bitcoin and other cryptocurrencies have attracted the interest of many investors, practitioners and researchers. However, little attention has been paid to the predictability of the risk measures of Bitcoin. In this paper, we compare the one-step-ahead volatility forecast of Bitcoin using several GARCH-type models and also evaluate the performance of several procedures when estimating the Value-at-Risk. We also take into account the presence of outliers and estimate the volatility and Value-at-Risk in a robust fashion. Our results show that robust procedures outperform the non-robust ones to forecast the volatility as well as to estimate the Value-at-Risk. These results suggest that the presence of outliers play an important role in modelling and forecasting Bitcoin risk measures.

Keywords: Cryptocurrency, GARCH, Model confidence set, Outliers, Realised volatility, Value-at-Risk.

JEL Classification: C51, C52, C53

2010 Mathematics Subject Classification: 62M10, 62F40, 62G35, 62P20, 91B84

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The author acknowledges financial support from São Paulo Research Foundation (FAPESP) grant 2016/18599-4 and is also grateful by the support of the Centre of Quantitative Studies in Economics and Finance (CEQEF) and Center of Applied Research on Econometrics, Finance and Statistics (CAREFS). The author thanks helpful comments of Luiz K. Hotta and João H. G. Mazzeu.

1 Introduction

Since its creation in 2008, Bitcoin has attracted the interest of many investors, practitioners and researchers. This interest has grown quickly in the last years, probably due to the decentralised nature of Bitcoin and by its large profits; see Nakamoto (2008).

Previous studies, such as Sapuric and Kokkinaki (2014), Baek and Elbeck (2015) and Brière et al. (2015), have observed that Bitcoin is highly volatile, thus forecasting the volatility as well as estimating the Value-at-Risk (VaR) as better as possible is crucial for better decisions of investors and practitioners.

Bitcoin daily volatility has been previously studied by Dyhrberg (2016), Balcilar et al. (2017), Chu et al. (2017), Katsiampa (2017), Liu et al. (2017), Pichl and Kaizoji (2017), Naimy and Hayek (2018) and Catania et al. (2018) among others. However, most of the studies have been focused on the in-sample framework and comparisons have been made based on information criteria. In this context, Katsiampa (2017) estimates the volatility of Bitcoin using several GARCH-type models assuming Gaussian errors and concludes that the best model to estimate the volatility is the AR(1)-CGARCH(1,1), Chu et al. (2017) analyse the seven most popular cryptocurrencies using GARCH-type models with different error distributions and conclude that the best model to estimate the Bitcoin volatility is the IGARCH(1,1). Liu et al. (2017) compare the GARCH model assuming the normal reciprocal inverse Gaussian (NRIG) distribution against the Gaussian and Student-t error distributions and conclude that the GARCH model with Student-t errors estimates better the volatilty. Charles and Darné (2018) replicate the study of Katsiampa (2017) and additionally take into account the presence of extreme observations. They find that, using the jump-filtered returns as in Laurent et al. (2016), the AR(1)-GARCH(1,1) model reports smaller information criteria than the models considered in Katsiampa (2017).

In an out-of-sample point of view, Naimy and Hayek (2018) compare the one-step-ahead volatility forecasts estimated by GARCH and EGARCH models with Gaussian, Student-t and generalised error distributions. The authors compare the predicted volatility with the realised volatility using the root mean square error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE) and they conclude that EGARCH models present the best performance. Catania et al. (2018) compare the Gaussian GARCH model with the generalised autoregressive score (GAS) models of Creal et al. (2013) and Harvey (2013). This comparison is based on the QLIKE measure and the predicted volatility is compared with the squared observed returns. They conclude that the GARCH model is outperformed by the GAS models. Peng et al. (2018) compare the volatility forecast obtained by GARCH, EGARCH and GJR models assuming symmetric and asymmetric Gaussian and Student-t errors against the Support Vector Regression GARCH model of Bezerra and Albuquerque (2017) and they find that the latter yields more accurate forecasts. Although out-of-sample comparisons are available in the literature, most of them are restrictive since they do not consider models which presented better performance in previous studies, and, additionally, they leave out several error distributions and models of the GARCH family. In this sense, a comprehensive out-of-sample comparison is needed.

On the other hand, most of the papers available in the literature do not consider the presence of outliers, which, as mentioned by, for instance, Carnero et al. (2012), Boudt et al. (2013) and Trucíos and Hotta (2016), affect drastically the volatility forecast and VaR estimation. As far as we know, only Catania and Grassi (2017), Charles and Darné (2018) and Catania et al. (2018) take into account the presence of outliers to estimate the Bitcoin volatility, being the last one the only work in an out-of-sample context. However, Catania et al. (2018) only consider the Gaussian GARCH model against the GAS models.

In a VaR context, Chu et al. (2017), Chan et al. (2017), Osterrieder and Lorenz (2017), Stavroyiannis (2018) and Gkillas and Katsiampa (2018) estimate the VaR of Bitcoin. However, none of these works consider the presence of outliers in a context of conditional heteroscedastic.

The contribution of this paper is threefold. First, we carry out an extensive out-of-sample comparison of the Bitcoin volatility forecast using several GARCH-type models with different error distributions, filling, then, a gap in the literature and also summarising the GARCH-type results found in the previous works. Second, we address the presence of outliers and show that the better volatility forecast is obtained using a robust procedure, suggesting that outliers play a crucial role when modelling and forecasting the volatility of Bitcoin. Finally, we evaluate the performance of the VaR estimation using of the best non-robust procedures to forecast the volatility, chosen by the Model Confidence Set (MCS), and the robust procedure proposed by Trucíos et al. (2017) showing that non-robust procedures underestimate the VaR while the robust procedure estimates the VaR with good results. These results are useful for academics, as an important reference for future research, as well as for practitioners, providing information about how to better measure the risk.

The rest of the paper is organized as follows: Section 2 describes the models, realised measures and loss functions used in the out-of-sample comparison. Section 3 describes the data and reports the main findings. Finally, Section 4 presents the conclusions and future researches.

2 Methodology

In this section, we briefly describe the GARCH-type models used to forecast the volatility of Bitcoin as well as the realised volatility measures used as volatility proxies. At the end of this section, we also describe the loss functions used to evaluate the out-of-sample performance and the robust bootstrap procedure of Trucíos et al. (2017) to estimate the VaR.

2.1 GARCH models

The GARCH model is commonly used for modelling and forecasting the second-order moments of return in economic and financial time series. Since its introduction by Bollerslev (1986), several extensions have been proposed in the literature. These extensions differ to each other in how the volatility equation is defined.

Let r_t be the observed returns at time t and ϵ_t the error term follows a white noise process. The GARCH(1,1) model is defined by

$$\begin{aligned} r_t &= \sigma_t \epsilon_t, \\ \sigma_t^2 &= \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2, \end{aligned}$$

with σ_t^2 being the conditional variance (or squared volatility) at time t and ω, α and β parameters satisfying some stationary conditions. Sufficient conditions for stationarity are given by $\omega > 0$, α, $\beta \geq 0$ and $\alpha + \beta < 1$.

As mentioned above, different extensions of the GARCH model involve different specifications of the volatility equation (σ_t^2). Table 1 describe the volatility equations of the GARCH-type models used in the comparison. [1] For good reviews of univarate GARCH-type models, see, for instance, Teräsvirta (2009) and Rodríguez and Ruiz (2012).

In all non-robust cases, several error distributions are assumed, namely, Normal, Skew Normal, Student-t, Skew Student-t, GED, Skew GED, Normal Inverse Gaussian, Generalized Hyperbolic and the Johnson's reparametrized SU innovation distribution; see Ghalanos (2018) for details of the parametrization distributions and GARCH-type models used in this paper.

Table 1: GARCH-type models

Model	Volatility equation	Proposed by
GARCH	$\sigma_{t+1}^2 = \omega + \alpha r_t^2 + \beta\sigma_t^2$	Bollerslev (1986)
IGARCH	$\sigma_{t+1}^2 = \omega + \alpha r_t^2 + (1-\alpha)\sigma_t^2$	Engle and Bollerslev (1986)
EGARCH	$log(\sigma_{t+1}^2) = \omega + \alpha z_t^2 + \gamma(\lvert z_t\rvert - E(\lvert z_t\rvert)) + \beta log(\sigma_t^2)$	Nelson (1991)
GJR	$\sigma_{t+1}^2 = \omega + \alpha r_t^2 + \gamma I(r_t < 0) r_t^2 + \beta\sigma_t^2$	Glosten et al. (1993)
APARCH	$\sigma_{t+1}^\delta = \omega + \alpha(\lvert r_t\rvert - \gamma r_t)^\delta + \beta\sigma_t^\delta$	Ding et al. (1993)
CGARCH	$\sigma_{t+1}^2 = q_{t+1} + \alpha(r_t^2 - q_t) + \beta(\sigma_t^2 - q_t)$ $q_{t+1} = \omega + \rho q_t + \phi(rt^2 - \sigma_t^2)$	Lee and Engle (1999)
TGARCH	$\sigma_{t+1} = \omega + \alpha\sigma_t(\lvert z_t\rvert - \eta_1 z_t) + \beta\sigma_t$	Zakoian (1994)
AVGARCH	$\sigma_{t+1} = \omega + \alpha\sigma_t(\lvert z_t - \eta_2\rvert - \eta_1(z_t - \eta_2)) + \beta\sigma_t$	Schwert (1990)
NGARCH	$\sigma_{t+1}^\delta = \omega + \alpha\sigma_t^\delta(\lvert z_t\rvert)^\delta + \beta\sigma_t^\delta$	Higgins and Bera (1992)
NAGARCH	$\sigma_{t+1}^2 = \omega + \alpha\sigma_t^2(\lvert z_t - \eta_2\rvert)^2 + \beta\sigma_t^2$	Engle and Ng (1993)
FGARCH	$\sigma_{t+1}^\delta = \omega + \alpha\sigma_t^\delta(\lvert z_t - \eta_2\rvert - \eta_1(z_t - \eta_2))^\delta + \beta\sigma_t^\delta$	Hentschel (1995)
Robust GARCH	$\sigma_{t+1}^2 = \omega + \gamma_c\alpha\rho\left(\frac{r_t^2}{\sigma_t^2}\right) + \beta\sigma_t^2,$ with $\rho(x) = \begin{cases} 1, & \text{if } x > c, \\ x, & \text{if } x \leq c, \end{cases}$ and γ_c being a constant to ensure consistency	Boudt et al. (2013)

2.2 Realised measures

Evaluate the predictive ability of different approaches to forecast the volatility is challenging since the volatility is a latent variable and consequently is not directly observable. A good proxy for the conditional variance is the realised variance (Andersen et al., 2003) which use intra-day data. In this paper we use the realised variance - RV and some alternative realised measures which are robust to microestructure noise. Specifically, we use the Bipower

[1]In Table 1, z_t corresponds to the devolatilised return $z_t = \frac{r_t}{\sigma_t}$.

variation - BV (Barndorff-Nielsen and Shephard, 2004), MinRV (Andersen et al., 2012) and MedRV (Andersen et al., 2012) as proxies of the true conditional variance. We prefer to use realised measures instead of square observed daily returns as in Catania et al. (2018) because realised measures have showed to be a better proxy (Alizadeh et al., 2002; McAleer and Medeiros, 2008; Patton, 2011) and are most widely used nowadays. Table 2 presents the realised measures used in this paper.

Table 2: Realised measures.

Realised measure	Formula
RV	$\sum_{i=1}^{N} r_i^2$
BV	$\frac{\pi}{2}\left(\frac{N}{N-1}\right)\sum_{i=1}^{N-1}\lvert r_i\rvert\lvert r_{i+1}\rvert$
MinRV	$\frac{\pi}{\pi-2}\left(\frac{N}{N-1}\right)\sum_{i=1}^{N-1} min\left(\lvert r_i\rvert, \lvert r_{i+1}\rvert\right)^2$
MedRV	$\frac{\pi}{6-4\sqrt{3}+\pi}\left(\frac{N}{N-2}\right)\sum_{i=1}^{N-1} med\left(\lvert r_{i-1}\rvert, \lvert r_i\rvert, \lvert r_{i+1}\rvert\right)^2$

For more details about realised measures as well as for asymptotic properties see McAleer and Medeiros (2008), Barndorff-Nielsen and Shephard (2004) and Andersen et al. (2012).

2.3 Robust loss function

The evaluation of the volatility forecast will be made using volatility proxies such as described in the previous section. However, these proxies are imperfect since they are estimates of the integrated variance. To avoid that imperfect volatility proxies lead to misleading results in the predictability comparison of the volatility forecast, Patton (2011) propose the use of robust loss functions, these functions are robust to the microestructure noise in the volatility proxy. The general class of the robust loss function is defined by Patton (2011) and given by

$$L(\hat{\sigma}^2, h, b) = \begin{cases} h - \hat{\sigma}^2 + \hat{\sigma}^2 log(\frac{\hat{\sigma}^2}{h}), & \textit{for b = -1,} \\ \frac{\hat{\sigma}^2}{h} - log(\frac{\hat{\sigma}^2}{h}) - 1, & \textit{for b = -2,} \\ \frac{(\hat{\sigma}^{2b+4} - h^{2b+4})}{(b+1)(b+2)} - \frac{h^{b+1}(\hat{\sigma}^2 - h)}{b+1}, & \text{otherwise,} \end{cases}$$

where $\hat{\sigma}^2$ is the squared forecasts volatility and h the squared volatility proxy. In this paper we consider the robust loss function of Patton (2011) with three different values of b: $b = -2$ (QLIKE), $b = -1$ (MSE) and $b = 0$ (hereafter denoted by RLF).

2.4 VaR estimation

Assuming that returns are zero mean, the one-step-ahead VaR in the non-robust models is estimated as usual by $\alpha\% VaR = Q_{\alpha\%}\hat{\sigma}_{T+1|T}$ where $Q_{\alpha\%}$ is the $\alpha\%$ quantile of the assumed error distribution (scaled to have unit variance) and $\hat{\sigma}_{T+1|T}$ is the one-step-ahead volatility forecast.

To estimate the VaR in a robust way, we use the robust bootstrap procedure recently proposed by Trucíos et al. (2017). The procedure is based on a residual-based bootstrap scheme combining with a robust estimator and robust filters for the volatility. The procedure can be summarized in the following steps:

- **Step 1:** Estimate the parameters ω, α and β in a robust way and obtain the standardized residuals $\hat{\epsilon}_t = \frac{r_t}{\hat{\sigma}_t}$. Denote by $\hat{F}_\epsilon$ the empirical distribution of these centred standardized residuals.

- **Step 2:** Using ϵ^*_t (bootstrap extractions from $\hat{F}_\epsilon$) generate bootstrap series through the following recursion.

$$\begin{aligned} r^*_t &= \sigma^*_t \epsilon^*_t, \\ \sigma^{*2}_{t+1} &= \hat{\omega} + \hat{\alpha}\sigma^{*2}_t c_\gamma r_c\left(\frac{r^{*2}_t}{\sigma^{*2}_t}\right) + \hat{\beta}\sigma^{*2}_t, \end{aligned}$$

  where $\sigma^{*2}_1 = \hat{\sigma}^2_1$ and the filter $r_c(\cdot)$ is defined in Table 3. Using the bootstrap series estimate the parameters $\hat{\omega}^*$, $\hat{\alpha}^*$ and $\hat{\beta}^*$ using the same estimator in Step 1.

- **Step 3:** Obtain h-steps-ahead forecast for returns as

$$\begin{aligned} \hat{r}^*_{T+h|T} &= \epsilon^*_{T+h}\hat{\sigma}^*_{T+h|T}, \\ \hat{\sigma}^{*2}_{T+h|T} &= \hat{\omega}^* + \hat{\alpha}^*\hat{\sigma}^{*2}_{T+h-1|T} c_\gamma r_c\left(\frac{r^{*2}_{T+h-1|T}}{\hat{\sigma}^{*2}_{T+h-1|T}}\right) + \hat{\beta}^*\hat{\sigma}^{*2}_{T+h-1|T}, \end{aligned} \tag{1}$$

  for $h = 1, ..., H$, and where $\hat{r}^*_{T|T} = r_T$, ϵ^*_{T+h} are bootstrap extractions from $\hat{F}_\epsilon$ and

$\hat{\sigma}^{*2}_{T|T}$ is obtained through the recursion $\hat{\sigma}^{*2}_{t|T} = \hat{\omega}^* + \hat{\alpha}^* \hat{\sigma}^{*2}_{t-1|T} c_\gamma r_c \left(\frac{r^2_{t-1}}{\hat{\sigma}^{*2}_{t-1|T}} \right) + \hat{\beta}^* \hat{\sigma}^{*2}_{t-1|T}$ for $t = 2, ..., T$, with $\hat{\sigma}^{*2}_{1|T} = \hat{\sigma}^2_1$, $r_c(x)$ equal to x if $x \leq c$ and ε^{*2}_t (squared bootstrap extractions from $\hat{F}_\epsilon$) otherwise and c being a cut-off value defined a priori.

- **Step 4:** Repeat steps 2 and 3 B times to obtain B bootstrap replicates $(\hat{r}^{*(1)}_{T+h|T}, ..., \hat{r}^{*(B)}_{T+h|T})$, the α % VaR is estimated as the α% empirical quantile of the bootstrap replicates.

3 Data and results

We use daily Bitcoin closing prices (in US dollar) traded on Bitstamp from September 13, 2011 to December 31, 2017 (2280 observations). Tick-by-tick data were obtained from *bitcoincharts*[2] and the daily closing prices were constructed from the tick-by-tick data as the last price traded at each day.

Returns are calculated as $r_t = log(P_t) - log(P_{t-1})$ with P_t being the closing price at day t. Table 3 reports descriptive statistics and Figure 1 shows the daily returns as well as the autocorrelation of returns and squared returns. We can observe that Bitcoin is highly volatile with an annualized standard deviation of 0.8350 ($0.0526 \times \sqrt{252}$), returns also present asymmetry and large Kurtosis. The large Kurtosis is probably explained by the presence of extreme returns, as observed in Figure 1. Because the returns series does not exhibit serial correlation no ARMA filter is applied to the data, the series is only centred to have zero mean.

Table 3: Descriptive statistics of Bitcoin daily returns

Mean	Std. Dev.	Min	Q_1	Med.	Q_3	Max	Skewness	Kurtosis
0.0034	0.0526	-0.6639	-0.0111	0.0024	0.0200	0.4455	-1.4011	28.4802

To evaluate the out-of-sample performance we use a rolling windows scheme with windows size equal to 1000 days. In each window the one-step-ahead conditional variance is estimated. The one-step-ahead conditional variance is compared with the four realised measures described in Section 2.2 using five minutes high-frequency data (results using 10 minutes high-frequency data were also evaluated and the conclusions are similar). The non-robust GARCH-type models were estimated using the R package *rugarch* of Ghalanos (2017)

[2]https://api.bitcoincharts.com/v1/csv/

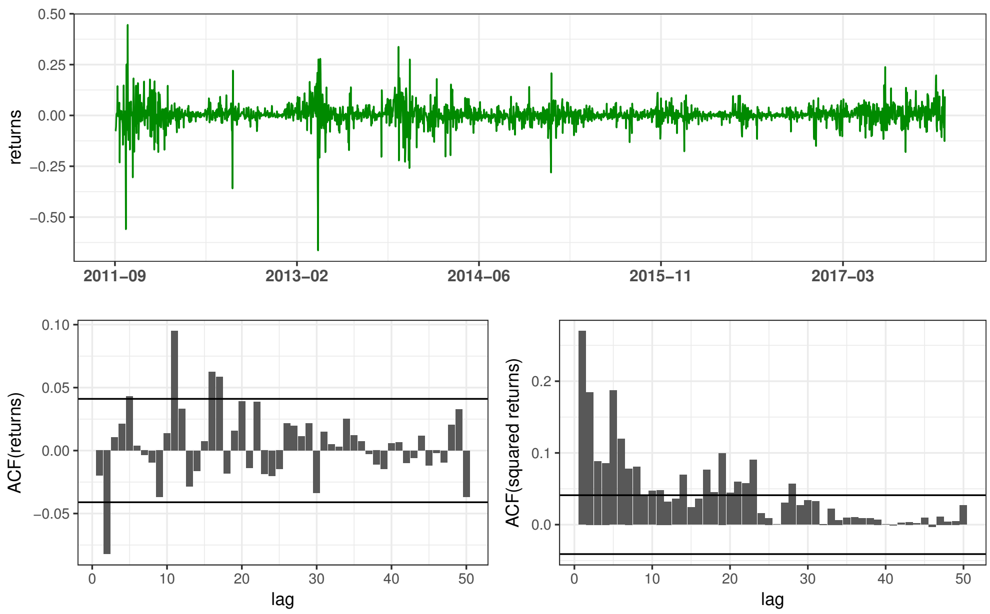

Figure 1: Daily returns, autocorrelation returns and autocorrelation squared returns

and the realised measures were computed using the R package *highfrequency* of Boudt et al. (2017).

Differently of Charles and Darné (2018) which takes into account the presence of outliers using jump-filtered returns as in (Laurent et al., 2016), we estimate the conditional variance in a robust way using the robust estimator of Boudt et al. (2013) with the modification introduced in Trucíos et al. (2017).

Table 4 reports the MSE, QLIKE and RLF between $\hat{\sigma}^2_{T+1}$ and the respective realised measure (RV, MinRV, MedRV, BP). The shadowed cells are the set of models with best out-of-sample performance obtained using the MCS approach (Hansen et al., 2011) at 75% significant level.[3] In bold, the best model (first position in the MCS rank) in each case.

The MSE selects a large set of models reflecting a low power to distinguish between different models, indeed using the MSE just 15 of 100 models were left out (16 when considering the MinRV as a volatility proxy). Other works have also found that the MSE has a low power to distinguish between different models, see for instance Patton and Sheppard (2009) and Liu et al. (2015). On the other hand, observe that the QLIKE and RLF loss functions are better to choose a small set of models with the best performance.

In general, note that the MCS methodology select the same models by loss function regardless the realised measure used. An exception is observed when the QLIKE measure

[3] We have used the R package MCS of Bernardi (2017)

is used in which case the CGARCH model belong to the group of models with the best performance only when the realised variance is used.

Considering all loss functions and realised measures only five models are in the set of better models in all cases, namely, TARCH and AVGARCH assuming generalized error distribution (standard and skew version) and the robust GARCH model. The CGARCH model which was the best model in Katsiampa (2017) appear in the set when using the MSE and RLF loss functions and also using the QLIKE loss function and the realised variance as volatility proxy. The IGARCH model which was the best model in the in-sample analyse in Chu et al. (2017) only appear in the set when considering the MSE loss functions.

In all cases, the GARCH model estimated in a robust way reports the best results regardless the loss function and realised measure used. This result is extremely important since most of analyses available in the literature about Bitcoin do not take into account the presence of outliers and, as we can see in Table 3, results can be substantially improved using a robust approach. A simple model such as the GARCH model estimated considering the presence of outliers outperforms more sophisticated models such as TARCH or AVGARCH. These results are in concordance with Charles and Darné (2018) which also find, in an in-sample context, that sophisticated models are outperformed by a GARCH model when the presence of outliers is considered in the analyse.

The 1% VaR is also computed and backtesting procedures for the VaR are carried out. We compute the VaR for the models chosen by the MCS as the models with the best performance in all criteria. Results are shown in Table 5 and report the proportion of fails (returns smaller than the 1% VaR), p-values of the unconditional coverage - UC (Kupiec, 1995), conditional coverage - CC (Christoffersen, 1998), dynamic quantile -DQ (Engle and Manganelli, 2004) tests as well as the average quantile loss function of González-Rivera et al. (2004).

Results reveal that the non-robust procedures underestimate the VaR and all backtesting procedures reject the null hypothesis that the proportion of fails is equal to 0.01. Using the robust procedure of Trucíos et al. (2017) all tests fail to reject the null hypothesis. Also, observe that the smallest average quantile loss function of González-Rivera et al. (2004) is obtained when the VaR is estimated in a robust way. Figure 2 reports the returns of the

Table 4: Average MSE, QLIKE and RHF between $\hat{\sigma}^2_{T+1}$ and the realised measures.

	Innov Distrib.	MSE ($\times 10^5$) RV	MinRV	MedRV	BP	QLIKE RV	MinRV	MedRV	BP	RLF ($\times 10^3$) RV	MinRV	MedRV	BP
GARCH	norm	0.6462	0.6924	0.6799	0.6932	4.6611	15.7753	12.8778	14.0071	0.2137	0.3374	0.3023	0.3356
	snorm	0.6420	0.6879	0.6755	0.6886	4.5618	15.5269	12.6780	13.7686	0.2110	0.3334	0.2987	0.3316
	std	0.6301	0.6750	0.6626	0.6756	4.3189	14.8890	12.1316	13.1683	0.2089	0.3293	0.2950	0.3271
	sstd	0.6301	0.6751	0.6627	0.6756	4.3255	14.9089	12.1485	13.1858	0.2091	0.3297	0.2952	0.3274
	ged	0.6223	0.6663	0.6541	0.6668	4.2173	14.6294	11.9493	12.9009	0.2039	0.3222	0.2885	0.3199
	sged	0.6233	0.6675	0.6553	0.6680	4.2448	14.7110	12.0190	12.9733	0.2046	0.3233	0.2895	0.3210
	nig	0.6298	0.6745	0.6622	0.6750	4.2573	14.7438	12.0325	13.0099	0.2067	0.3262	0.2921	0.3239
	ghyp	0.6301	0.6748	0.6624	0.6753	4.2558	14.7365	12.0229	13.0057	0.2068	0.3263	0.2922	0.3240
	jsu	0.6282	0.6729	0.6605	0.6734	4.2763	14.7824	12.0522	13.0590	0.2074	0.3272	0.2930	0.3250
IGARCH	norm	0.6848	0.7331	0.7202	0.7339	4.7743	16.1611	13.2284	14.3087	0.2237	0.3511	0.3150	0.3491
	snorm	0.6777	0.7255	0.7126	0.7262	4.6886	15.9360	13.0387	14.1000	0.2210	0.3472	0.3114	0.3451
	std	0.6347	0.6799	0.6675	0.6805	4.3386	14.9463	12.1776	13.2208	0.2101	0.3311	0.2965	0.3289
	sstd	0.6347	0.6799	0.6675	0.6805	4.3451	14.9660	12.1944	13.2380	0.2103	0.3314	0.2968	0.3292
	ged	0.6324	0.6769	0.6646	0.6774	4.2577	14.7373	12.0339	12.9998	0.2069	0.3262	0.2922	0.3238
	sged	0.6324	0.6770	0.6647	0.6775	4.2854	14.8159	12.0990	13.0714	0.2075	0.3272	0.2931	0.3249
	nig	0.6348	0.6797	0.6673	0.6802	4.2773	14.8017	12.0791	13.0627	0.2080	0.3280	0.2938	0.3257
	ghyp	0.6349	0.6798	0.6674	0.6803	4.2736	14.7908	12.0680	13.0539	0.2080	0.3280	0.2938	0.3257
	jsu	0.6329	0.6778	0.6654	0.6783	4.2947	14.8362	12.0955	13.1079	0.2087	0.3289	0.2946	0.3266
EGARCH	norm	0.3997	0.4384	0.4268	0.4397	4.2819	14.6638	11.9263	13.0381	0.1808	0.2942	0.2614	0.2934
	snorm	0.3816	0.4187	0.4073	0.4199	4.1724	14.2734	11.5432	12.7374	0.1756	0.2865	0.2542	0.2857
	std	1.3587	1.4412	1.4209	1.4434	8.3776	26.2082	21.2621	23.6404	0.4427	0.6451	0.5872	0.6438
	sstd	4.1221	4.2203	4.1960	4.2242	12.4856	33.6221	27.5656	31.3650	0.7206	0.9637	0.8926	0.9662
	ged	0.4687	0.5082	0.4964	0.5091	4.2841	14.7140	11.8625	13.1082	0.1953	0.3115	0.2778	0.3100
	sged	0.4772	0.5175	0.5055	0.5184	4.3620	14.9607	12.0615	13.3298	0.1989	0.3168	0.2826	0.3153
	nig	0.5874	0.6336	0.6205	0.6347	4.8527	16.4305	13.2608	14.6555	0.2302	0.3598	0.3223	0.3582
	ghyp	0.6182	0.6666	0.6531	0.6676	4.9544	16.7707	13.5804	14.9264	0.2372	0.3700	0.3317	0.3682
	jsu	0.7094	0.7620	0.7476	0.7632	5.4314	18.0941	14.5942	16.1862	0.2653	0.4078	0.3667	0.4061
GJR-GARCH	norm	0.6660	0.7127	0.7001	0.7135	4.7435	15.9572	12.9768	14.2207	0.2187	0.3438	0.3083	0.3420
	snorm	0.6389	0.6840	0.6717	0.6848	4.6482	15.6192	12.6296	13.9678	0.2145	0.3375	0.3024	0.3358
	std	0.6722	0.7172	0.7048	0.7177	4.5216	15.3809	12.4347	13.7138	0.2205	0.3438	0.3085	0.3417
	sstd	0.6751	0.7203	0.7078	0.7209	4.5321	15.4303	12.4847	13.7505	0.2211	0.3447	0.3094	0.3426
	ged	0.6565	0.7007	0.6884	0.7012	4.3948	15.0588	12.2121	13.3783	0.2138	0.3346	0.3001	0.3324
	sged	0.6636	0.7083	0.6960	0.7088	4.4448	15.2279	12.3615	13.5232	0.2161	0.3380	0.3033	0.3358
	nig	0.6720	0.7171	0.7047	0.7176	4.4515	15.2494	12.3697	13.5464	0.2179	0.3403	0.3054	0.3381
	ghyp	0.6676	0.7125	0.7001	0.7130	4.4402	15.2048	12.3250	13.5114	0.2173	0.3395	0.3046	0.3373
	jsu	0.6733	0.7183	0.7059	0.7188	4.4802	15.3081	12.3999	13.6200	0.2193	0.3422	0.3071	0.3400
APARCH	norm	0.5586	0.6027	0.5902	0.6038	4.7028	15.8339	12.8170	14.1272	0.2115	0.3350	0.2995	0.3337
	snorm	0.5675	0.6110	0.5986	0.6120	4.6737	15.6700	12.6122	14.0424	0.2126	0.3354	0.3000	0.3341
	std	1.7452	1.8356	1.8142	1.8373	8.3640	26.8273	21.8816	23.9507	0.4686	0.6800	0.6200	0.6771
	sstd	1.8165	1.9096	1.8877	1.9113	8.5378	27.4649	22.4993	24.4475	0.4809	0.6969	0.6357	0.6940
	ged	0.6816	0.7289	0.7158	0.7296	4.6978	16.0421	12.9647	14.2607	0.2317	0.3605	0.3235	0.3583
	sged	0.6922	0.7402	0.7270	0.7410	4.7671	16.2775	13.1611	14.4651	0.2352	0.3656	0.3282	0.3635
	nig	0.8510	0.9064	0.8918	0.9072	5.2732	17.8979	14.5316	15.8679	0.2698	0.4134	0.3723	0.4108
	ghyp	0.9272	0.9856	0.9705	0.9863	5.3893	18.3389	14.9618	16.2017	0.2803	0.4280	0.3859	0.4252
	jsu	1.0107	1.0730	1.0571	1.0739	5.8575	19.6262	15.9345	17.4342	0.3075	0.4645	0.4197	0.4619
CS-GARCH	norm	0.5051	0.5437	0.5324	0.5442	3.7672	13.2390	10.7400	11.7096	0.1779	0.2865	0.2553	0.2845
	snorm	0.5101	0.5494	0.5381	0.5499	3.7854	13.2734	10.7744	11.7376	0.1784	0.2872	0.2560	0.2852
	std	0.5592	0.5996	0.5881	0.5999	3.7305	13.2083	10.7250	11.6396	0.1846	0.2949	0.2633	0.2924
	sstd	0.5644	0.6056	0.5939	0.6058	3.8005	13.3810	10.8571	11.8077	0.1870	0.2986	0.2666	0.2960
	ged	0.5320	0.5713	0.5600	0.5715	3.7362	13.1697	10.7139	11.5997	0.1803	0.2888	0.2577	0.2863
	sged	0.5280	0.5672	0.5559	0.5674	3.7172	13.1183	10.6648	11.5505	0.1799	0.2883	0.2572	0.2858
	nig	0.5441	0.5842	0.5728	0.5844	3.7599	13.2428	10.7656	11.6695	0.1832	0.2929	0.2614	0.2903
	ghyp	0.5472	0.5876	0.5760	0.5878	3.7487	13.2312	10.7534	11.6463	0.1837	0.2937	0.2621	0.2911
	jsu	0.5493	0.5903	0.5786	0.5905	3.7040	13.1620	10.6864	11.5745	0.1837	0.2943	0.2626	0.2917
TARCH	norm	0.4556	0.4977	0.4855	0.4989	4.4336	15.1155	12.2453	13.4620	0.1931	0.3116	0.2773	0.3106
	snorm	0.4272	0.4673	0.4554	0.4685	4.3205	14.7282	11.8736	13.1577	0.1866	0.3021	0.2685	0.3012
	std	1.5828	1.6712	1.6500	1.6731	8.3242	26.6331	21.6692	23.8363	0.4582	0.6671	0.6075	0.6648
	sstd	1.6504	1.7416	1.7199	1.7436	8.5075	27.2939	22.2991	24.3616	0.4708	0.6845	0.6236	0.6821
	ged	0.3065	0.3125	0.3068	0.3128	3.0786	8.9822	6.6485	8.5162	0.1422	0.2063	0.1860	0.2055
	sged	6.2929	6.3005	6.2951	6.3018	4.2964	10.5292	8.2206	10.2862	0.2816	0.3505	0.3309	0.3530
	nig	0.7427	0.7960	0.7817	0.7969	5.1558	17.5626	14.2467	15.5755	0.2572	0.3974	0.3571	0.3952
	ghyp	0.8061	0.8622	0.8474	0.8631	5.2881	18.0411	14.7000	15.9507	0.2678	0.4122	0.3709	0.4098
	jsu	0.8974	0.9577	0.9420	0.9587	5.7644	19.3435	15.6862	17.1989	0.2962	0.4503	0.4061	0.4480
AVGARCH	norm	0.5495	0.5940	0.5813	0.5951	4.5948	15.2893	12.2950	13.7239	0.2074	0.3293	0.2941	0.3281
	snorm	0.5027	0.5443	0.5321	0.5454	4.4396	15.0180	12.1520	13.3952	0.1988	0.3169	0.2827	0.3158
	std	1.5283	1.6143	1.5937	1.6160	8.3379	26.7001	21.7781	23.8574	0.4505	0.6564	0.5978	0.6538
	sstd	1.5985	1.6881	1.6667	1.6900	8.6625	27.6488	22.6233	24.6775	0.4685	0.6805	0.6201	0.6781
	ged	0.2903	0.2957	0.2902	0.2960	3.0270	8.8674	6.5561	8.4003	0.1382	0.2010	0.1811	0.2002
	sged	0.3010	0.3067	0.3012	0.3071	3.1075	9.0838	6.7185	8.6091	0.1423	0.2063	0.1860	0.2055
	nig	0.7162	0.7690	0.7548	0.7699	5.1853	17.7134	14.4072	15.6786	0.2553	0.3951	0.3549	0.3929
	ghyp	0.8057	0.8616	0.8469	0.8624	5.3933	18.3932	15.0112	16.2479	0.2710	0.4163	0.3747	0.4138
	jsu	0.8512	0.9107	0.8953	0.9118	5.7763	19.1370	15.3250	17.1624	0.2912	0.4439	0.4001	0.4416
NGARCH	norm	0.5416	0.5858	0.5733	0.5869	4.6604	15.7700	12.7876	14.0398	0.2081	0.3310	0.2956	0.3296
	snorm	0.5319	0.5753	0.5629	0.5763	4.5226	15.3895	12.4799	13.6896	0.2030	0.3237	0.2890	0.3223
	std	1.6741	1.7651	1.7436	1.7668	7.9521	25.7754	21.2326	22.8155	0.4470	0.6538	0.5953	0.6508
	sstd	1.7375	1.8309	1.8089	1.8327	8.1307	26.3710	21.7885	23.2955	0.4593	0.6702	0.6105	0.6671
	ged	0.6240	0.6704	0.6575	0.6712	4.4331	15.3760	12.5222	13.5547	0.2159	0.3404	0.3046	0.3381
	sged	0.6309	0.6780	0.6650	0.6788	4.4978	15.5595	12.6575	13.7321	0.2191	0.3450	0.3089	0.3428
	nig	0.7848	0.8391	0.8248	0.8399	4.9346	16.9856	13.9009	14.9371	0.2505	0.3887	0.3493	0.3861
	ghyp	0.8490	0.9057	0.8910	0.9065	4.9718	17.2124	14.1918	15.0570	0.2560	0.3966	0.3567	0.3938
	jsu	0.9421	1.0036	0.9878	1.0045	5.4913	18.6339	15.2523	16.4191	0.2873	0.4389	0.3958	0.4362
NAGARCH	norm	0.6528	0.6993	0.6868	0.7001	4.6372	15.7110	12.8230	13.9435	0.2139	0.3375	0.3025	0.3357
	snorm	0.6234	0.6681	0.6559	0.6688	4.5686	15.4497	12.5384	13.7637	0.2100	0.3314	0.2968	0.3295
	std	0.6804	0.7263	0.7137	0.7269	4.7956	16.2179	13.0605	14.4966	0.2309	0.3581	0.3217	0.3559
	sstd	0.6805	0.7266	0.7139	0.7271	4.8023	16.2470	13.0900	14.5172	0.2311	0.3585	0.3220	0.3563
	ged	0.6477	0.6920	0.6797	0.6924	4.4874	15.3887	12.4775	13.6599	0.2158	0.3378	0.3030	0.3354
	sged	0.6462	0.6908	0.6785	0.6913	4.5119	15.4574	12.5342	13.7219	0.2162	0.3387	0.3037	0.3364
	nig	0.6691	0.7147	0.7021	0.7151	4.6438	15.8441	12.8098	14.0973	0.2243	0.3495	0.3136	0.3471
	ghyp	0.6664	0.7120	0.6994	0.7124	4.6296	15.7878	12.7553	14.0513	0.2238	0.3488	0.3130	0.3464
	jsu	0.6746	0.7203	0.7077	0.7207	4.7149	16.0246	12.9284	14.2904	0.2276	0.3538	0.3176	0.3515
Full GARCH	norm	0.6313	0.6769	0.6642	0.6779	4.6570	15.7984	12.8002	14.0545	0.2202	0.3456	0.3095	0.3440
	snorm	0.6314	0.6753	0.6628	0.6762	4.5357	15.3349	12.3748	13.7062	0.2181	0.3414	0.3058	0.3398
	std	1.5654	1.6517	1.6310	1.6535	8.4590	26.9654	21.9542	24.1603	0.4575	0.6647	0.6057	0.6622
	sstd	1.6544	1.7434	1.7221	1.7452	8.5986	27.4219	22.3873	24.4923	0.4713	0.6826	0.6223	0.6799
	ged	0.6032	0.6485	0.6357	0.6492	4.5789	15.7680	12.7706	13.9733	0.2213	0.3468	0.3105	0.3447
	sged	0.6112	0.6573	0.6443	0.6581	4.6636	16.0417	12.9991	14.2162	0.2249	0.3521	0.3153	0.3500
	nig	0.7458	0.7986	0.7844	0.7994	5.2114	17.7875	14.4494	15.7488	0.2592	0.3994	0.3591	0.3970
	ghyp	0.8244	0.8805	0.8657	0.8813	5.4332	18.5296	15.0815	16.3849	0.2744	0.4205	0.3786	0.4179
	jsu	0.8738	0.9329	0.9174	0.9338	5.7881	19.4739	15.8039	17.2912	0.2934	0.4461	0.4023	0.4436
Robust		**0.1027**	**0.1099**	**0.1041**	**0.1102**	**1.8795**	**7.2874**	**5.6621**	**6.4628**	**0.0644**	**0.1192**	**0.1020**	**0.1182**

out-of-sample period and the 1% VaR estimated in a robust way.

In general, our results show that the risk measures are better estimated when using robust procedures.

Table 5: Proportion of returns smaller than the 1% VaR, p-values of the UC, CC, DQ test and average quantile loss function (AQLF) ($\times 10^3$).

Models	Prop. Fails	UC	CC	DQ	AQLF
TGARCH GED	0.1039	0.0000	0.0000	0.0000	2.9755
TGARCH SGED	0.1039	0.0000	0.0000	0.0000	2.9805
AVGARCH GED	0.1023	0.0000	0.0000	0.0000	2.9776
AVGARCH SGED	0.1023	0.0000	0.0000	0.0000	2.9710
Robust	0.0063	0.1475	0.3331	0.6602	1.4283

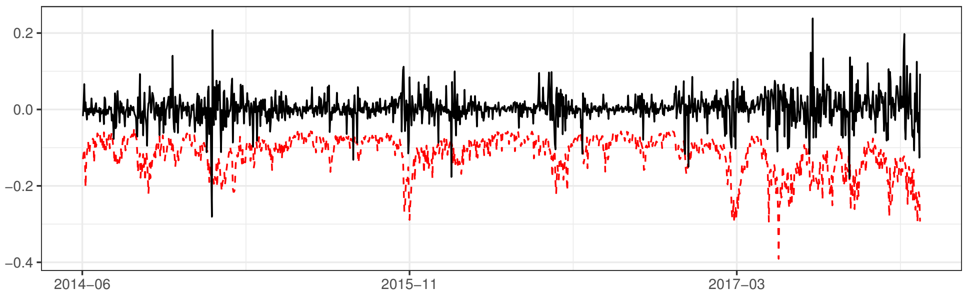

Figure 2: Return in the out-of-sample period (black solid line) and the estimated 1% VaR (red dashed line) obtained using the robust bootstrap procedure of Trucíos et al. (2017).

4 Conclusions and future works

In this paper, we have made a comprehensive out-of-sample comparison in a context of the daily volatility forecast of Bitcoin using GARCH-type models with different error distributions. Additionally, we included a robust GARCH procedure and compare it with the non-robust models. Our results reveal that better results are obtained when the presence of outliers is not neglected and estimation of the volatility is made in a robust way.

Among the non-robust procedures, the models with better out-of-sample performance to forecast the volatility are the TARCH and AVGARCH models both considering generalized error distribution (standard and skew version). However, these models are outperformed by the GARCH model when estimated in a robust fashion. In particular, we use the robust estimator of Boudt et al. (2013) with the modification used in Trucíos et al. (2017).

The VaR estimated in a non-robust way reports large values of the proportion of fails and all backtesting procedures reject the null hypothesis of that the proportion of fails is equal to 0.01 giving a misleading picture of what can be expected in the future. On the other hand, the VaR estimated using the robust bootstrap procedure of Trucíos et al. (2017) shown a good performance.

Additional comparisons considering different GARCH-type models estimated in a robust way as well as considering switch regime are interesting research topics. In the same spirit, to compare robust GARCH-type models with GAS models as the used in Catania et al. (2018) and some alternative procedures as the used in Peng et al. (2018) as well as analyse the performance of risk measures in a high-frequency context are interesting research topics.

Some papers such as Dyhrberg (2016), Balcilar et al. (2017) and Cermak (2017) have also consider explanatory variables to better estimate the volatility. Thus, one step further in this research is to analyse the performance of the out-of-sample volatility forecast considering explanatory variables and robust GARCH procedures.

This paper is in concordance with the results of Carnero et al. (2012), Trucíos and Hotta (2016) and Trucíos et al. (2017) which show the dramatic effect of additive outliers in the estimation and prediction of the volatility and VaR. For a good review about outliers in GARCH models, we refer to Hotta and Trucíos (2018).

In a multivariate framework Boudt et al. (2013), Grané et al. (2014) and Trucíos et al. (2018b) shown the dramatic effect of outliers in the estimation and prediction of the volatilities and co-volatilities. In this sense, it is important that future studies considering jointly Bitcoin with other cryptocurrencies take into account the presence of outliers. In this sense, the procedures proposed by, for instance, Croux et al. (2010), Boudt and Croux (2010), Boudt et al. (2013), Iqbal (2013), Trucíos et al. (2018a) and Trucíos et al. (2018a) could be useful.

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