Calculating Lifetime Expected Loss for IFRS 9: Which Formula is Correct?

Bernd Engelmann[1]

This version: April 30, 2018

ABSTRACT

IFRS 9, the new accounting rules for financial instruments require banks to build provisions for expected losses in their loan portfolios. A requirement which was not present in previous regulation is the necessity to provision the expected loss over a loan's lifetime in case a loan shows a deterioration in credit quality. The IFRS 9 rules are formulated in a qualitative way and no explicit formulas or precise parameter estimation methods are prescribed. In this article, lifetime expected loss is computed as the difference in present values of a loan's cash flows. It is assumed that cash flows are risk-free in the first step and expected present values including credit risk are subtracted in the second step to arrive at lifetime expected loss. This is done under different modeling assumptions and the outcome is compared to the weighted loss formula most commonly used in practice (e.g. Deloitte (2017) or PricewaterhouseCoopers (2015)). It turns out that the formula used in practice should at least be adjusted to be theoretically more sound or be replaced entirely by present value formulas to assure accuracy.

JEL Classification: G12
Keywords: IFRS 9, Provisioning, Lifetime Expected Loss

[1] Open University Ho Chi Minh City, e-mail: bernd.engelmann@ou.edu.vn and Open Source Investor Services B.V., e-mail: bernd.engelmann@os-is.com

The accounting rules IFRS 9, published in 2014 (IASB 2014), require banks to improve their quantitative modeling for calculating loan loss provisions. For loan exposures considered as normally performing, expected loss is provisioned on a one-year basis using the well-known formula $ECL = PD \cdot LGD \cdot EAD$ where ECL is the expected credit loss, PD the probability of a borrower default, LGD the loss given default and EAD the exposure at default. Note, that under IFRS 9 forward looking risk measures are required, i.e. the estimates for PD, LGD, and EAD have to be point-in-time estimates. This is in contrast to the Basel framework where for minimum capital calculations under the IRB approach through-the-cycle default probabilities are permitted and downturn LGD/EAD is required (Basel Committee on Banking Supervision 2006). For estimating point-in-time versus through-the-cycle PD see Aguais, Forest, Wong, and Diaz-Ledezma (2004) or Carlehed and Petrov (2012).

The more challenging part of IFRS 9 is the situation where the credit quality of a loan deteriorates. In this case it is required to provision expected loss over a loan's lifetime instead of one-year expected loss. A common way do compute lifetime expected loss in practice is using the formula for one-year ECL and computing lifetime ECL as the present value over all one-year ECLs in future periods:

$$ECL = E\left[\sum_{i=1}^{n} \frac{1}{(1+r)^i} \cdot Pr(\tau = i) \cdot LGD_i \cdot EAD_i\right], \tag{1}$$

where r is the discount rate, $i = 1, \ldots, n$ are a loan's periods, i.e. the years or quarters until it matures, and $Pr(\tau = i)$ is the probability that a borrower's default time τ is in period i implying that it survived the periods $j = 1, \ldots, i-1$, see e.g. Deloitte (2017), PricewaterhouseCoopers (2015), Skoglund (2017) and Xu (2016).

In the sequel, ECL will be computed in terms of a term-structure p_i which is the unconditional probability that a borrower defaults in one of the periods $j = 1, \ldots, i$. To link $Pr(\tau = i)$ with p_i, note that

$$\begin{aligned} Pr(\tau = i) &= Pr(\tau = i | \tau > i-1) \cdot Pr(\tau > i-1) = (1 - Pr(\tau > i | \tau > i-1)) \cdot Pr(\tau > i-1) \\ &= (1 - Pr(\tau > i)/Pr(\tau > i-1)) \cdot Pr(\tau > i-1) = Pr(\tau > i-1) - Pr(\tau > i) \\ &= Pr(\tau \leq i) - Pr(\tau \leq i-1) = p_i - p_{i-1}. \end{aligned}$$

Furthermore, the term-structure of survival probabilities q_i will play a major role which is defined as $q_i = 1 - p_i$. Denote the default probability in period i conditional on survival until period $i-1$ with $PD_i := Pr(\tau = i | \tau > i-1)$. The relation between q_i and PD_i is

$$q_i = Pr(\tau > i) = Pr(\tau > i | \tau > i-1) \cdot Pr(\tau > i-1) = \prod_{j=1}^{i} Pr(\tau > j | \tau > j-1) \cdot Pr(\tau > 0)$$

$$= \prod_{j=1}^{i} (1 - Pr(\tau = j | \tau > j-1)) \cdot 1 = \prod_{j=1}^{i} (1 - PD_i).$$

Unless no complex dependence structures are modeled between *PD*, *LGD* and *EAD* a scenario for term-structures of default probabilities, *LGD* and *EAD* in each period are estimated independently mostly using macroeconomic models and ECL is computed from these quantities as

$$ECL = \sum_{i=1}^{n} \frac{1}{(1+r)^i} \cdot (p_i - p_{i-1}) \cdot LGD_i \cdot EAD_i, \tag{2}$$

One reason for the popularity of this approach is its transparency. Expected loss is computed separately in each period and aggregated by a simple rule over lifetime. The estimation of risk parameters for this purpose is discussed in the aforementioned articles of Skoglund (2017) and Xu (2016).

The main purpose of this article is not to discuss the parameterization of (2) but to ask whether this formula is appropriate for computing lifetime *ECL* and to shed light on its theoretical foundation. The starting point for answering this question is paragraph B5.5.29 in IASB (2014). It says: ”For financial assets, a credit loss is the present value of the difference between: (a) the contractual cash flows that are due to an entity under the contract; and (b) the cash flows that the entity expects to receive.”. In the next section, two formulas will be derived that compute lifetime ECL for a simple bullet loan applying B5.5.29 literally using present values under different assumptions on the estimation of *LGD*. In the following section, a link between these two formulas and (2) will be derived. It will be shown that (2) which is used widely in practice has no theoretical justification but has to be adjusted for being consistent with a present values approach. In Section 3 it will be outlined how the formulas will change under the inclusion of prepayment probabilities. After that, the use of an effective instead of the contractual interest rate for discounting will be discussed. This is a requirement according to B5.5.44 in IASB (2014), where it says: ”Expected credit losses shall be discounted to the reporting date, not to the expected default or some other date, using the effective interest rate...”. It will be shown that the use of an interest rate different from the contractual rate for discounting is inconsistent with a weighted loss formula like (2). A numerical example will illustrate the impact of different assumptions and approaches.

1. Expected Lifetime Loss Based on Cash Flows

In this section a formula for *ECL* based on a loan's future cash flows is derived. To keep the exposition simple we use a bullet loan throughout this article. For the present value calculation all future cash flows of the loan have to be considered. Future cash flows are the interest rate payments, the payback of the loan's balance at maturity, and the liquidation proceeds of collateral in the case of a borrower default. We can split the present value into cash flows that will be paid if a borrower survives and cash flows a bank receives in the case of a default. For the survival part, the present value V_S is given as

$$V_S = \sum_{i=1}^{n} N \cdot z \cdot \delta_i \cdot q_i + N \cdot \delta_n \cdot q_n \tag{3}$$

where N is the outstanding loan balance, z is the interest rate, and δ_i is the discount factor corresponding to period i. The future interest periods until the end of a loan's lifetime are indexed $i = 1, \ldots, n$ as before. Since a borrower can pay interest and outstanding balance only if he survives all discounted cash flows are weighted with survival probabilities.

For the default part, we distinguish two cases: In Case I a recovery rate was estimated taking outstanding balance as the reference quantity, in the second case outstanding balance plus the interest was used as exposure at default in recovery estimation. In Case I the resulting recovery rate is denoted with R while in Case II we use $\overline{R}$. An analogous notation will be adopted for the corresponding LGD numbers. For Case I, we find

$$V_{D,I} = \sum_{i=1}^{n} N \cdot R_i \cdot \delta_i \cdot (q_{i-1} - q_i), \tag{4}$$

while in Case II we obtain

$$V_{D,II} = \sum_{i=1}^{n} N \cdot \overline{R}_i \cdot \delta_i \cdot (1+z) \cdot (q_{i-1} - q_i). \tag{5}$$

In both cases it was assumed that a bank can claim the recovery payment at the end of the interest period in case of a default. Recall that $q_{i-1} - q_i = p_i - p_{i-1} = Pr(\tau = i)$ is the probability that a borrower defaults in period i.

The expected present value of all future cash flows is the sum of V_S and V_D. According to B5.5.29 in IASB (2014) credit loss is defined as the difference of present values in contractual cash flows and expected cash flows. The present value of contractual cash flows V_C for a bullet loan is simply

$$V_C = \sum_{i=1}^{n} N \cdot z \cdot \delta_i + N \cdot \delta_n \tag{6}$$

and ECL is defined as

$$ECL = V_C - V_S - V_D. \tag{7}$$

A crucial quantity in ECL calculation is the discount factor δ. For the moment, we assume that discounting is done with the contractual interest rate z and discount factors are computed as $\delta_i = (1+z)^{-i}$. This is only approximately what IASB (2014) requires. We will comment on this assumption later in Section 3. With this choice of the discount factor (6) simplifies resulting in $V_C = N$ and

$$ECL = N - V_S - V_D. \tag{8}$$

In the next section, we will use (8) as a starting point and derive an expression from it that is using weighted losses in a similar form as the *ECL* formula used in practice (2).

2. Expected Lifetime Loss Based on Weighted Losses

We compute *ECL* for both versions of recovery rates (4) and (5) arriving at formulas similar but not identical to (2). We start with Case I.

Proposition 1 *When ECL is defined as $ECL = N - V_S - V_{D,I}$ we can transform the present values into a weighted sum of losses given as*

$$ECL = \sum_{i=1}^{n} \frac{1}{(1+z)^i} \cdot (p_i - p_{i-1}) \cdot (LGD_i + z) \cdot N. \tag{9}$$

Proof:

$$\begin{aligned} ECL &= N - V_S - V_{D,I} \\ &= N - \sum_{i=1}^{n} N \cdot z \cdot \delta_i \cdot q_i - N \cdot \delta_n \cdot q_n - \sum_{i=1}^{n} N \cdot R_i \cdot \delta_i \cdot (q_{i-1} - q_i) \\ &= N - \sum_{i=1}^{n} N \cdot z \cdot \delta_i - N \cdot \delta_n + \sum_{i=1}^{n} N \cdot z \cdot \delta_i \cdot p_i + N \cdot \delta_n \cdot p_n - \sum_{i=1}^{n} N \cdot R_i \cdot \delta_i \cdot (p_i - p_{i-1}) \end{aligned}$$

We have used $q_i = 1 - p_i$. The first part of the above expression $N - \sum_{i=1}^{n} N \cdot z \cdot \delta_i - N \cdot \delta_n$ is zero by construction because the discount rate is identical to the loan's interest rate.

For the second term, we find

$$\sum_{i=1}^{n} N \cdot z \cdot \delta_i \cdot p_i + N \cdot \delta_n \cdot p_n = \sum_{i=1}^{n} N \cdot \delta_{i-1} \cdot (p_i - p_{i-1}).$$

This expression can be proved by mathematical induction. For $n = 1$ we find

$$N \cdot z \cdot \delta_1 \cdot p_1 + N \cdot \delta_1 \cdot p_1 = N \cdot p_1 \cdot \delta_1 \cdot (1+z) = N \cdot p_1 = N \cdot \delta_0 \cdot (p_1 - p_0).$$

Note that $\delta_0 = 1$ and $p_0 = 0$. Now assume, the relation is correct for $n-1$ and show that under this assumption it is also correct for n:

$$\begin{aligned}
\sum_{i=1}^{n} N\cdot z\cdot \delta_i \cdot p_i + N\cdot \delta_n \cdot p_n &= \sum_{i=1}^{n-1} N\cdot z\cdot \delta_i \cdot p_i + N\cdot \delta_{n-1}\cdot p_{n-1} \\
&\quad - N\cdot \delta_{n-1}\cdot p_{n-1} + N\cdot z\cdot \delta_n\cdot p_n + N\cdot \delta_n\cdot p_n \\
&= \sum_{i=1}^{n-1} N\cdot \delta_{i-1}\cdot (p_i - p_{i-1}) - N\cdot \delta_{n-1}\cdot p_{n-1} + N\cdot \delta_{n-1}\cdot p_n \\
&= \sum_{i=1}^{n} N\cdot \delta_{i-1}\cdot (p_i - p_{i-1})
\end{aligned}$$

This allows us to finalize the calculation of the ECL formula using $R_i = 1 - LGD_i$:

$$\begin{aligned}
ECL &= \sum_{i=1}^{n} N\cdot \delta_{i-1}\cdot (p_i - p_{i-1}) - \sum_{i=1}^{n} N\cdot (1-LGD_i)\cdot \delta_i\cdot (p_i - p_{i-1}) \\
&= \sum_{i=1}^{n} N\cdot (p_i - p_{i-1})\cdot \left(\delta_{i-1} - \delta_i + LGD_i\cdot \delta_i\right) \\
&= \sum_{i=1}^{n} N\cdot (p_i - p_{i-1})\cdot \delta_i\cdot (1+z-1+LGD_i) \\
&= \sum_{i=1}^{n} \frac{1}{(1+z)^i}\cdot N\cdot (p_i - p_{i-1})\cdot (LGD_i + z). \quad \square
\end{aligned}$$

We find that the ECL formula is similar to (2) but not identical. Using identical risk parameters leads to a higher ECL in (9) compared to (2) due to the correction for the loss in interest.

Proposition 2 *When ECL is defined as $ECL = N - V_S - V_{D,II}$ we can transform the present values into a weighted sum of losses given as*

$$ECL = \sum_{i=1}^{n} \frac{1}{(1+z)^{i-1}}\cdot (p_i - p_{i-1})\cdot \overline{LGD}_i\cdot N. \tag{10}$$

ECL computed by (10) given identical risk parameters is smaller than ECL computed by (9).

Proof: Most steps for proving Proposition 1 can be reused for the proof of Proposition 2. The only difference is in the term computing the present value of liquidation proceeds (5) in case of a borrower default. Note, that $\delta_i \cdot (1+z) = \delta_{i-1}$. Using this and results from the proof of Proposition 1 we find

$$\begin{aligned} ECL &= N - V_S - V_{D,II} \\ &= \sum_{i=1}^{n} N \cdot \delta_{i-1} \cdot (p_i - p_{i-1}) - \sum_{i=1}^{n} N \cdot \overline{R}_i \cdot \delta_i \cdot (1+z) \cdot (q_{i-1} - q_i) \\ &= \sum_{i=1}^{n} N \cdot \delta_{i-1} \cdot (p_i - p_{i-1}) - \sum_{i=1}^{n} N \cdot (1 - \overline{LGD}_i) \cdot \delta_{i-1} \cdot (p_i - p_{i-1}) \\ &= \sum_{i=1}^{n} \delta_{i-1} \cdot (p_i - p_{i-1}) \cdot \overline{LGD}_i \cdot N \\ &= \sum_{i=1}^{n} \frac{1}{(1+z)^{i-1}} \cdot (p_i - p_{i-1}) \cdot \overline{LGD}_i \cdot N. \end{aligned}$$

If $LGD_i = \overline{LGD}_i$ the ECL in Proposition 2 is smaller. This can be easily seen by taking differences of the terms containing LGD_i:

$$\frac{LGD_i + z}{(1+z)^i} - \frac{LGD_i}{(1+z)^{i-1}} = \frac{LGD_i + z - LGD_i \cdot (1+z)}{(1+z)^i} = \frac{z \cdot (1 - LGD_i)}{(1+z)^i} \geq 0. \quad \square$$

In this section we have derived two versions of ECL formulas based on weighted loss per period from present values of cash flows. Note, that both versions result in higher ECL than the formula commonly used for lifetime ECL calculations under IFRS 9 in practice (2) if identical risk parameters are used for the calculation. Whether the formula in Proposition 1 or Proposition 2 is more appropriate depends on the way a bank is estimating LGD. If it is based on outstanding balance only, then (9) should be used. If accrued interest is included, then (10) is more accurate.

3. Generalizations

In this section two generalizations of the framework are discussed. The first is the inclusion of prepayments. Prepayments are reducing the expected lifetime of a loan and therefore reduce expected lifetime loss. Under IFRS 9 lifetime expected loss is only relevant for loans where a deterioration of credit quality was observed. One would expect that voluntary prepayments are not a big issue for these clients. However, in many loan markets one can empirically verify that also loans that recently moved into an arrears status show material prepayments. In these cases prepayments should be included in the framework to improve the accuracy of ECL. In the second part of this section the discount rate used for calculating ECL is discussed in more detail. In particular, the requirement in IFRS 9 of using an effective interest rate for discounting which could be different from the loan's contractual rate is reflected. A numerical example for illustration concludes this section.

3.1. Including Prepayment Probabilities into ECL

The are two ways how prepayment can be modeled and prepayment probabilities can be defined leading to different formulas and different *ECL*. Similar to the discussion on *LGD* modeling on Section 1, one has to pick the framework that is consistent with the determination of risk parameters to ensure consistent *ECL* calculation.

The first alternative is modeling prepayment conditional on survival, i.e. in each period i the quantity PR_i is the probability that a surviving borrower prepays. Denoting the random prepayment time by η, we have $PR_i = Pr(\eta = i|\tau \geq i)$. The default probability of a borrower conditional on neither default nor prepayment is $PD_i = Pr(\tau = i|\tau > i-1, \eta > i-1)$. In this setup, a borrower prepays with probability $(1-PD_i)\cdot PR_i$ and continues the loan with probability $(1-PD_i)\cdot(1-PR_i)$. The second alternative is modeling default and prepayment as competing risks. Here, in each period a borrower can either default with probability $PD_i = Pr(\tau = i|\tau > i-1, \eta > i-1)$, prepay with probability $\overline{PR}_i = Pr(\eta = i|\tau > i-1, \eta > i-1)$ or continue servicing the loan with probability $1-PD_i-\overline{PR}_i$. Both modeling alternatives lead to different formulas for present values of future cash flows.

Similar to a term-structure of default probabilities $p_i = 1-\prod_{j=1}^{i}(1-PD_i)$ a term-structure of prepayment probabilities $\pi_i = 1-\prod_{j=1}^{i}(1-PR_i)$ (or $\overline{\pi}_i = 1-\prod_{j=1}^{i}(1-\overline{PR}_i)$, respectively) can be estimated empirically using similar techniques as for PD models. Once this term-structure is available, it can be included in the present value formulas. In this section only the version using $V_{D,I}$ will be discussed. The result for $V_{D,II}$ can be derived easily from the result for $V_{D,I}$.

In the first step, (3) and (4) are generalized to include prepayment probabilities. Here, only full prepayments are considered. The generalization of V_S is given by

$$\hat{V}_S = \sum_{i=1}^{n} \delta_i \cdot q_i \cdot (z \cdot N \cdot (1-\pi_{i-1}) + N \cdot (\pi_i - \pi_{i-1})). \tag{11}$$

Here, $1-\pi_{i-1}$ is the probability that there was no prepayment until period $i-1$. Only if the loan is not prepaid it can still pay interest and, therefore, interest rate payments have to be weighted with this probability. The new term $N\cdot(\pi_i-\pi_{i-1})$ is the expected prepayment at time i, i.e. the outstanding balance weighted with the marginal prepayment probability in period i. Since this term is multiplied with q_i it represents the fraction of surviving borrowers that prepay in period i. Note that $\pi_n = 1$ since the outstanding balance has to be repaid at maturity.

For $V_{D,I}$ the extension is given as

$$\hat{V}_{D,I} = \sum_{i=1}^{n} N \cdot (1 - \pi_{i-1}) \cdot R_i \cdot \delta_i \cdot (q_{i-1} - q_i) \tag{12}$$

A loan can only default in period i if it has not yet been prepaid in a previous period. Therefore the expected recovery has to be weighted with $1 - \pi_{i-1}$.

Proposition 3 *When including prepayment probabilities PR into ECL calculation using (11) and (12) ECL can be computed as $N - \hat{V}_S - \hat{V}_{D,I}$ and the present values can be transformed into a sum of weighted losses given as*

$$ECL = \sum_{i=1}^{n} \frac{1}{(1+z)^i} \cdot (p_i - p_{i-1}) \cdot (LGD_i + z) \cdot N_i, \tag{13}$$

where N_i is defined as $N_i := N \cdot (1 - \pi_{i-1})$.

Proof:

Recall that *ECL* was defined as the difference of the present value of contractual cash flows and the expected present value of cash flows. First, it has to be verified that the present value of contractual cash flows is still equal to N. Note, that prepayment risk is not a loss risk and, therefore, prepayment probabilities have to be included in the present value calculation of contractual cash flows:

$$V_C = \sum_{i=1}^{n} \delta_i \cdot (z \cdot N \cdot (1 - \pi_{i-1}) + N \cdot (\pi_i - \pi_{i-1})) = \sum_{i=1}^{n} \delta_i \cdot (z \cdot N_i + N_i - N_{i+1})$$

Define $M_j = N_j - N_{j+1}$ and note that $N_{n+1} = 0$. We obtain

$$V_C = \sum_{i=1}^{n} \delta_i \cdot \left(z \cdot \sum_{j=i}^{n} M_j + M_i \right) = \sum_{j=1}^{n} \sum_{i=1}^{j} \left(\delta_i \cdot z \cdot M_j + \delta_j \cdot M_j \right) = \sum_{j=1}^{n} M_j = N_1 - N_{n+1} = N.$$

At the second equality sign a change in summation order was performed. Similar steps are applied to prove the main part of the proposition.

$$\begin{aligned} ECL = N - \hat{V}_S - \hat{V}_{D,I} = N - &\sum_{i=1}^{n} \delta_i \cdot q_i \left(z \cdot N \cdot (1 - \pi_{i-1}) + N \cdot (\pi_i - \pi_{i-1}) \right) \\ - &\sum_{i=1}^{n} N \cdot (1 - \pi_{i-1}) \cdot R_i \cdot \delta_i \cdot (q_{i-1} - q_i) \end{aligned}$$

$$
\begin{aligned}
&= N - \sum_{i=1}^{n} \delta_i \cdot q_i \left(z \cdot N_i + (N_i - N_{i+1})\right) - \sum_{i=1}^{n} N_i \cdot R_i \cdot \delta_i \cdot (q_{i-1} - q_i) \\
&= \sum_{j=1}^{n} M_j - \sum_{i=1}^{n} \delta_i \cdot q_i \left(z \cdot \sum_{j=i}^{n} M_j + M_i\right) - \sum_{i=1}^{n} \sum_{j=i}^{n} M_j \cdot R_i \cdot \delta_i \cdot (q_{i-1} - q_i) \\
&= \sum_{j=1}^{n} \left[M_j - \sum_{i=1}^{j} \delta_i \cdot q_i \cdot z \cdot M_j - \delta_j \cdot q_j \cdot M_j - \sum_{i=1}^{j} M_j \cdot R_i \cdot \delta_i \cdot (q_{i-1} - q_i)\right] \\
&= \sum_{j=1}^{n} \left[\sum_{i=1}^{j} \frac{1}{(1+z)^i} \cdot (p_i - p_{i-1}) \cdot (LGD_i + z) \cdot M_j\right] \\
&= \sum_{i=1}^{n} \left[\sum_{j=i}^{n} \frac{1}{(1+z)^i} \cdot (p_i - p_{i-1}) \cdot (LGD_i + z) \cdot M_j\right] \\
&= \sum_{i=1}^{n} \frac{1}{(1+z)^i} \cdot (p_i - p_{i-1}) \cdot (LGD_i + z) \cdot N_i. \quad \square
\end{aligned}
$$

If an LGD using the convention in $V_{D,II}$ is applied, an analogous result could be obtained carrying out exactly the same steps.

To derive the formulas for the second modeling alternative, define v_i as the probability that there is neither a default nor a prepayment in periods $j = 1, \ldots, i$. As shown in Xu (2016), it can be computed as $v_i = \prod_{j=1}^{i}(1 - PD_j - \overline{PR}_j)$. Under these modeling assumptions, the present value of future cash flows conditional on no default $\dot{V}_S$ is given as

$$
\dot{V}_S = \sum_{i=1}^{n} \delta_i \cdot N \left(z \cdot v_i + (1+z) \cdot v_{i-1} \cdot \overline{PR}_i\right). \tag{14}
$$

The first term under the sum represent the interest rate payment in case the borrower continues servicing the loan and the second term represents the payment of interest plus outstanding balance in case of a prepayment. Note that in the final period n the loan must be repaid if the borrower survives leading to $\overline{PR}_n = 1 - PD_n$ and $v_n = 0$.

For the present values of cash flows in case of default $\dot{V}_{D,I}$, we find

$$
\dot{V}_{D,I} = \sum_{i=1}^{n} N \cdot R_i \cdot \delta_i \cdot v_{i-1} \cdot PD_i. \tag{15}
$$

A borrower can only default in period i if he has neither prepaid not defaulted earlier which is captured in v_{i-1}.

Proposition 4 *When including prepayment probabilities $\overline{PR}$ into ECL calculation using (14) and (15) ECL can be computed as $N - \dot{V}_S - \dot{V}_{D,I}$ and the present values can be transformed into a sum of weighted losses given as*

$$ECL = \sum_{i=1}^{n} \frac{1}{(1+z)^i} \cdot (p_i - p_{i-1}) \cdot (LGD_i + z) \cdot \overline{N}_i, \tag{16}$$

where $\overline{N}_i$ is defined as $\overline{N}_i := N \cdot \frac{v_{i-1}}{q_{i-1}}$.

Proof:

The prepayment probabilities PR_i and $\overline{PR}_i$ are related by $\overline{PR}_i = PR_i \cdot (1 - PD_i)$. Using this equation allows a transformation of v_i:

$$v_i = \prod_{j=1}^{i}(1 - PD_j - \overline{PR}_j) = \prod_{j=1}^{i}(1 - PD_j - PR_i \cdot (1 - PD_j)) = \prod_{j=1}^{i}(1 - PD_i) \cdot (1 - PR_j)$$
$$= q_i \cdot (1 - \pi_i)$$

Using this to transform $\dot{V}_S$ yields

$$\begin{aligned}
\dot{V}_S &= \sum_{i=1}^{n} \delta_i \cdot N \left(z \cdot v_i + (1+z) \cdot v_{i-1} \cdot \overline{PR}_i\right) \\
&= \sum_{i=1}^{n} \delta_i \cdot N \left(z \cdot v_{i-1} \cdot (1 - PD_i - \overline{PR}_i) + (1+z) \cdot v_{i-1} \cdot \overline{PR}_i\right) \\
&= \sum_{i=1}^{n} \delta_i \cdot N \left(z \cdot v_{i-1} \cdot (1 - PD_i) + v_{i-1} \cdot \overline{PR}_i\right) \\
&= \sum_{i=1}^{n} \delta_i \cdot N \left(z \cdot v_{i-1} \cdot (1 - PD_i) - v_{i-1} \cdot (1 - PD_i - \overline{PR}_i - (1 - PD_i))\right) \\
&= \sum_{i=1}^{n} \delta_i \cdot N \left(z \cdot v_{i-1} \cdot (1 - PD_i) - v_i + v_{i-1} \cdot (1 - PD_i)\right) \\
&= \sum_{i=1}^{n} \delta_i \cdot N \cdot q_i \cdot \left(z \cdot \frac{v_{i-1}}{q_{i-1}} - \frac{v_i}{q_i} + \frac{v_{i-1}}{q_{i-1}}\right) \\
&= \sum_{i=1}^{n} \delta_i \cdot N \cdot q_i \cdot (z \cdot (1 - \pi_{i-1}) + \pi_i - \pi_{i-1}) = \hat{V}_S
\end{aligned}$$

A similar consideration shows $\dot{V}_{D,I} = \hat{V}_{D,I}$. Since the present values of cash flows are identical the same is true for *ECL*. Using the result of Proposition 3 leads us to the proof of this proposition:

$$N_i = N \cdot (1 - \pi_{i-1}) = N \cdot \frac{v_{i-1}}{q_{i-1}} = \overline{N}_i. \quad \square$$

3.2. Modifying the Discount Rate

One requirement of the IFRS 9 rules is the use of an effective interest rate for discounting, e.g. see B.5.5.44 in (IASB 2014). This means that the rate for discounting cash flows could be different from the contractual interest rate of a loan. This requires some adjustments to the *ECL* calculation formula of Section 1.

When the discount rate is different from the loan's interest rate, it is no longer true that the present value of contractual cash flows is equal to the loan's outstanding balance. For this present value, we find

$$\tilde{V}_C = \sum_{i=1}^{n} N \cdot z \cdot \tilde{\delta}_i + N \cdot \tilde{\delta}_n \tag{17}$$

where $\tilde{\delta}$ is the discount factor using the effective instead of the contractual interest rate. For V_S and $V_{D,I}$ similar adjustments have to be made resulting in

$$\tilde{V}_S = \sum_{i=1}^{n} N \cdot z \cdot \tilde{\delta}_i \cdot q_i + N \cdot \tilde{\delta}_n \cdot q_n \tag{18}$$

$$\tilde{V}_{D,I} = \sum_{i=1}^{n} N \cdot R_i \cdot \tilde{\delta}_i \cdot (q_{i-1} - q_i) \tag{19}$$

Lifetime expected loss is then computed as

$$ECL = \tilde{V}_C - \tilde{V}_S - \tilde{V}_{D,I}. \tag{20}$$

It is observed in practical applications that (2) is applied with an interest rate different from the loan's contractual interest rate. It was possible to find a link between present values and weighted losses as in (2) where the derivation has shown that (2) needs modifications to be consistent with present values when the discount rate and the contractual rate are equal. When they are different it is no longer possible to come up with a weighted losses formula consistent with present values. It was essential in deriving these formulas to have identical contractual and discount rates. From this we can conclude that taking B5.5.29 and B5.5.44 in IASB (2014) literally makes it impossible to use a weighted loss formula for computing *ECL*. To be consistent with the text present value formulas have to be used instead.

3.3. Numerical Example

To illustrate the differences between the present values and the weighted loss formulas, a 20-year fixed rate loan with a contractual rate of 5% is considered. The outstanding balance is normalized to 100. For the risk parameters, assume for Year 1 a default probability $PD = 1\%$ and compute p_i iteratively as $p_i = (1 - p_{i-1}) \cdot PD + p_{i-1}$. The term-structure of prepayment rates is built in a similar way starting from a 1Y prepayment rate $PR = 2\%$, and computing π from $\pi_i = (1 - \pi_{i-1}) \cdot PR + \pi_{i-1}$. For loss given default, a period-independent value of 40% is assumed. To illustrate the different ECL formulas we compute ECL with and without prepayment, i.e. we set PR to 0% in the first test example and to 2% in the second. Besides that, we use in one test set the contract rate for discounting while in the second test set we discount with an assumed effective interest rate of 4.80% to outline the differences.

In total, seven different ECL numbers are computed. For completeness, $ECL_{practice}$ following (2) is reported. As the reference case, $ECL_{wl,I}$ which uses weighted losses under Case I which was derived in (13) and $ECL_{pv,I}$ which is the corresponding ECL based on present values are used. Besides that, $ECL_{wl,II}$ computes the version of Proposition 3 with Case II for the recovery payments which is presented together with its present values counterpart $ECL_{pv,II}$. Finally, $\overline{ECL}_{wl,I}$ implements (16) from Proposition 4 together with its present values version $\overline{ECL}_{pv,I}$. Note that conceptually it is not sensible to use all these formulas for the same set of risk parameters since depending on the estimation of these parameters only one version is consistent. The main purpose of this illustration is to measure the potential error of using the wrong formula for ECL computation.

Scenario 1: $PR = 0\%$ and Discount Rate = Contract Rate

Scenario 2: $PR = 2\%$ and Discount Rate = Contract Rate

Scenario 3: $PR = 0\%$ and Discount Rate = Effective Rate

Scenario 4: $PR = 2\%$ and Discount Rate = Effective Rate

The results are displayed in Table 1 below.

ECL	Scenario 1	Scenario 2	Scenario 3	Scenario 4
$ECL_{practice}$	4.612	3.981	4.688	4.042
$ECL_{wl,I}$	5.188	4.479	5.274	4.547
$ECL_{pv,I}$	5.188	4.479	5.465	4.699
$ECL_{wl,II}$	4.842	4.180	4.913	4.236
$ECL_{pv,II}$	4.842	4.180	5.114	4.396
$\overline{ECL}_{wl,I}$	5.188	4.472	5.274	4.541
$\overline{ECL}_{pv,I}$	5.188	4.472	5.465	4.695

Table 1. Expected credit loss for a 20-year fixed rate loan with a contract rate of 5% under the four risk parameter and discount rate scenarios.

From the first two scenarios it can be seen that there is no difference between the weighted loss formula and the ECL based on present values since the weighted loss formulas are exact when the discount rate and the contractual rate are identical. From $ECL_{wl,II}$ and $ECL_{pv,II}$, respectively, we see that the LGD assumption has a non-negligible impact on ECL. If parts of accrued interest are recovered ECL is lower when identical LGD numbers are used both in $ECL_{wl,I}$ and $ECL_{wl,II}$. Furthermore, prepayment effects if present in a credit portfolio have a significant effect on ECL and cannot be ignored. However, if the inappropriate prepayment formula is erroneously applied the effect seems to be immaterial as the values of $ECL_{*,I}$ and $\overline{ECL}_{*,I}$ are very similar. Formula (2) in all cases leads to the lowest numbers.

In Scenarios 3 and 4 the effect of changing the discount rate from the contractual rate to an effective rate was illustrated. Since the derivation of the weighted loss formulas required in many places that the discount rate equals the contractual rate differences should be expected. We see in the examples that reducing the discount rate by 20 basis points results in ECL differences of about 5% between the weighted loss formulas and the present values framework for the given set of risk parameters. This is considerable and one should think about abandoning the weighted loss formulas when computing such a scenario.

4. Conclusion

In this article formulas for lifetime expected loss in the light of the IFRS 9 impairment rules have been analyzed. Starting from a simple formula (2) that seems to be wide-spread in practice, it was shown that this formula is inconsistent with differences in present values as prescribed in IASB (2014), B5.5.29, but needs corrections depending on the way LGD is modeled and estimated. An extension of the formula using weighted losses including prepayment probabilities was derived. Finally, the case where discount rate and contractual rate are different was outlined and it was shown that only present values lead to a correct answer from a conceptual viewpoint which was illustrated by a numerical example.

Overall, it was demonstrated that the application of weighted losses can be justified only when discount and contract rates are equal. Although this approach is more intuitive since it states the expected loss in every period and has a simple aggregation rule, it becomes problematic when deviations from its core assumptions are introduced. In these cases it is unclear how big the error compared to present values could become. A numerical example outlined that the differences can be significant. In the view of the author, when developing IFRS 9 similar steps should have been taken as in Basel Committee on Banking Supervision (2006) where for the calculation of risk weights a simple formula based on a theoretically sound model was prescribed and only the parameterization was left to the banks. This makes the implementation easier and gives less room for discussions and interpretations.

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