Capital allocation under the FRTB regime via marginal measures

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We present a way to compute additive marginal contributions for the six capital measures forming the capital computation under the new Basel III market risk regime, commonly named "Fundamental Review of the Trading Book regime" (FRTB, [9, 11]). Marginal contributions are most useful for the allocation of capital charges and risk weighted assets (RWA) to individual risk factors, financial instruments and/or portfolios. For most components we apply the Euler theorem.

1. Overview

As response to issues observed in the capitalization of banks during the Lehman crisis, the Basel Committee started out to fundamentally reform the computation of regulatory capital in a series of proposed regulations and consultations [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. While the process is still ongoing, a number of features have stabilized and the general framework can be assumed to be fixed. Consequently, certain legislations have started the process to implement the regulation into local law, cf. e.g. the European CRR2 draft [13].

In ongoing monitoring and reporting as well as allocation of capital costs, the impact of single portfolios, financial instruments / trades and/or certain risk factors presents a permanent challenge. This is especially evident when considering the enlarged complexity of the FRTB component risk measures, substantially increasing efforts on operative risk management side. Measures of the individual contribution of sub-portions of the full book are an efficient tool to facilitate such ongoing tasks.

While in practice, one or a mixture of marginal, incremental and standalone measures are used, marginal measures derived using the Euler method (also known as allocation by gradient) have been shown to be the only suitable definition of risk contributions both from the practical aspect of risk adjusted performance measurement [15] as well as a quite fundamental axiomatic approach [16]. This was confirmed in Reference [17] which, apart from a broad overview on the topic, also depicts the results of a long-term comparison of the various measures in a real-life application. Therefore, the application of marginal measures for the allocation of the FRTB risk charges suggests itself.

In the following we will briefly comprise the regulatory framework as well as provide an overview on the methods used to compute the marginals. Section 2 outlines the necessary quantities and methods

[*]The views, thoughts and opinions expressed in this paper are those of the author in his individual capacity and should not be attributed to UniCredit or to the author as representative or employee of UniCredit.

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with particular focus on the Euler theorem and the treatment of inhomogeneities. Subsequently, we then present the specific results for the individual FRTB components. To work in order of increasing complexity, we first address the IMA in Part I and continue to the SA in Part II.

Outline of the regulatory framework

The proposed framework consists of two approaches, a Standardized Approach (SA) and an Internal Models Approach. In turn, the SA is composed of three components:

- Sensitivity-based Approach (SBA): approximates a classical (partial-revaluation variance/covariance delta+) market risk model by applying prescribed shocks on sensitivities and aggregating using prescribed correlations; it is composed of three contributions: delta, curvature and vega risk
- Residual Risk Add-On (RRAO): notional-based add-on for risks not covered by the SBA and DRC components
- Default Risk Component (DRC): Jump-to-Default (JtD) sensitivity-based capital charge against default risks

Also, the IMA is composed of three components

- Internal Model Capital Charge (IMCC): classical market risk simulation over a stress period, however, applying an expected shortfall (ES) risk measure and, in order to restrict diversification and account for liquidity horizon, based on up to 90 profit/loss (PL) distributions
- Scenarios of Extreme Stress (SES): capital add-on for risk factors not eligible for inclusion into the IMCC component (non-modellable risk factors (NMRF)), computed as individual stress tests
- Default Risk Component (DRC): simulation of default events not included in the IMCC, based on a Value-at-Risk (VaR) risk measure

For most components it is prudent to compute the marginals using the Euler method as it is the methodologically correct way [16]. Additionally, it carries the advantage that it allows re-using intermediate results from the actual charge computation.

Merely for IMA DRC an alternative approach is used, leading to more stable marginals than obtained from the Euler method.

2. Methodology

For brevity we assume familiarity of the reader with the notion of scenarios, PL distributions and risk measures in general as well as sub-additivity, coherence, the Value-at-Risk and the expected shortfall in particular. Clarifying nomenclature, by trade we refer to a booking of a financial instrument into a specific portfolio. It is the smallest considered entity and, therefore, usually, the goal is to identify the marginal contributions of trades. Marginal contributions of overall financial instrument positions, portfolios, issuers or other levels of aggregation are obtained by addition of trade-level marginals. To bear flexibility, we will, in the following, refer to the smallest entities considered as elements.

2.1. The Euler theorem

The concept of the Euler allocation is based on the fact that a homogeneous function $f(x)$ of degree τ, i.e. for which

$$f(kx) = k^{\tau} f(x)$$

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holds, can be represented by its partial derivatives with regard to the coordinate(s) $x \in \mathbb{R}^N$ (thus, it is sometimes also referred to as allocation by gradient).

Namely, differentiating the homogeneity relation w.r.t. k yields

$$\sum_i^N \frac{\partial f(kx)}{\partial(kx_i)} \frac{\partial(kx_i)}{\partial k} = \tau k^{\tau-1} f(x)$$

and with $k = 1$ the Euler theorem follows as

$$f(x) = \sum_{i=1}^N \tau^{-1} x_i \frac{\partial f(x)}{\partial x_i}.$$

If $f(x)$ is e.g. a risk charge, then the N components of x are the risk contributors, i.e. elements, which jointly yield the overall charge and whose individual contributions are given by the addends. In this setting, the allocation of the risk charge is trivial.

Appendix A outlines the usage of the Euler theorem with a series of examples helpful for the application to the FRTB risk measures. For further details on the application in risk management refer to [1].

2.2. Inhomogeneous functions

For the cases considered here, inhomogeneities arise primarily from minimum, maximum, absolute value and Heaviside functions as part of the aggregation formulae. In order to apply the Euler theorem in a valid fashion, these inhomogeneities need to be addressed.

To this avail, one can make use of the fact that these functions are actually evaluated only once: during the computation of the relevant top level (standalone) risk charge. The decision, which argument of the function prevails is then fixed. When, in a second step, computing the marginals, these functions can be considered mere decision rules, the outcome of which has already been decided in the first step. Consequently, the marginal computation would always take the contribution from the same argument which prevailed during the computation of the top level charge.

If keeping track of these decisions when computing the top-level charge, max(x)/min(x) functions thus turn into deterministic operations for the computation of the marginals. The same is true for the absolute value $|x| = \text{abs}(x)$ which is equal to $\max(x, -x)$. The resulting function is then homogeneous and can be safely allocated using the Euler theorem.

In the following, we denote such operations, the outcome of which is determined/saved during the top-level aggregation and employed as deterministic operation/decision rule during the marginal computation, by an index which indicates the object to which the logic is related (e.g. the bucket, the computation for which the function is used for). We define it formally as

$$\max_b^f(\{x_j\}) \quad := \quad x_i : x_i^{\text{top-level}} \geq x_j^{\text{top-level}} \forall j \tag{1}$$

$$\min_b^f(\{x_j\}) \quad := \quad x_i : x_i^{\text{top-level}} \leq x_j^{\text{top-level}} \forall j \tag{2}$$

$$\text{abs}_b^f(x) \quad := \quad \max_b^f\{x, -x\} \tag{3}$$

$$\Theta_b^f(x) \quad := \quad \Theta_b^f\left(x^{\text{top-level}}\right) \tag{4}$$

where the indices f and b imply the use in the computation of some $f_b(x_j)$.

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Part I.
Internal Model Approach (IMA)

3. Internal Model Capital Charge (IMCC) = Expected Shortfall Component (IMA ES)

The IMCC is based on the coherent expected shortfall risk measure, consequently, one may expect that the capital allocation should be straightforward. Indeed, the marginal expected shortfall components present the building blocks of the allocation. However, the latter has to also account for the nonlinear aggregation across the individual expected shortfall computations.

Definition

The internal model capital charge $IMCC$ is given by [11, page 24] (cf. also [13, pages 171f], denominated as expected shortfall risk measure ES therein)

$$IMCC = \rho IMCC_T + (1-\rho)\sum IMCC_b \tag{5}$$

where the sum is over the broad risk-factor categories (BRC) $b = \{\mathrm{IR, FX, EQ, CR, CO}\}$. The expressions for $IMCC_b$ of each BRC and the fully diversified $IMCC_T$ (collectively referred to as «unconstrained expected shortfall measures» UES in [13]) are given by

$$IMCC_y = ES_{y,SR}\frac{ES_{y,CF}}{ES_{y,CR}} \tag{6}$$

where $y \in \{T\} \cup b$ and the index x of the partial expected shortfall contributions $ES_{y,x}$ (denominated as PES in [13]) indicates a specific risk factor set (R..reduced, F..full) and a specific time period (C..current, S..stress). The current regulation draft [13] provisions a flooring of the ratio ES_{CF}/ES_{CR} to 1. Since, in practice it is extremely unlikely that this floor will ever come into effect on a bank's top level, we omit this peculiarity.

Finally, each partial expected shortfall $ES_{y,x}$ is given in the form

$$ES_{y,x} = \sqrt{\sum s_h ES_{y,x,h}}$$

which mirrors, and therefore can be addressed by leveraging on, the example function X in Appendix A.2.

Homogeneity

We first confirm the homogeneity of (6)

$$\begin{aligned} IMCC_y(kw_i) &= ES_{y,SR}(kw_i)\frac{ES_{y,CF}(kw_i)}{ES_{y,CR}(kw_i)} \\ &\overset{(24)}{=} kES_{y,SR}(w_i)\frac{kES_{y,CF}(w_i)}{kES_{y,CR}(w_i)} \\ &= kIMCC(w_i) \end{aligned}$$

whereby the homogeneity of (5), which is only a sum of the former, directly follows

$$IMCC(kw_i) = kIMCC(w_i).$$

Both expressions are homogeneous of grade 1.

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Euler marginal

Due to the Euler theorem, the contributions $IMCC_y$ can be decomposed as

$$\begin{aligned} IMCC_y &= \sum_i 1^{-1} w_i \frac{\partial IMCC_y(w_i)}{\partial w_i} \\ &= \sum_i w_i \sum_x \frac{\partial IMCC_y}{\partial ES_{y,x}} \frac{\partial ES_{y,x}}{\partial w_i}, \end{aligned}$$

where

$$\frac{\partial IMCC_y}{\partial ES_{y,x}} = \begin{cases} \frac{ES_{y,CF}}{ES_{y,CR}} & x = SR \\ \frac{ES_{y,SR}}{ES_{y,CR}} & x = CF \\ -ES_{y,SR} \frac{ES_{y,CF}}{ES^2_{y,CR}} & x = CR \end{cases}$$

and, cf. Appendix A.2,

$$\frac{\partial ES_{y,x}}{\partial w_i} \overset{(25)}{=} \sum_h \frac{s_h ES_{y,x,h}}{w_i ES_{y,x}} mES_{y,x,h,i}$$

such that the decomposition is

$$IMCC_y = \sum_i \sum_x \sum_h \frac{\partial IMCC_y}{\partial ES_{y,x}} \frac{s_h ES_{y,x,h}}{ES_{y,x}} mES_{y,x,h,i}$$

and, thus, the marginal contribution is found as

$$mIMCC_{y,i} = \sum_x \sum_h \frac{\partial IMCC_y}{\partial ES_{y,x}} \frac{s_h ES_{y,x,h}}{ES_{y,x}} mES_{y,x,h,i}. \tag{7}$$

Due to its additive nature, the marginal of the full *IMCC* w.r.t. some element i is then given by

$$mIMCC_i = \rho\, mIMCC_{T,i} + (1-\rho) \sum_b mIMCC_{b,i}. \tag{8}$$

Discussion

While for capital allocation purposes $mIMCC_i$ is the relevant marginal, monitoring and analyses efforts may find the base-level marginal contribution $mES_{y,x,h,i}$ to a specific ES run quite helpful. Also the intermediate marginals provide quantitative insight into the regulatory charge. For example, the additive nature of marginals can be used to determine the contribution of a specific liquidity horizon by selecting one h and, instead, summing across all i in Eqs. (7) and (8).

With regard to the computational effort it is worth noting that the marginal is the product of a number of prefactors and the base-level marginals of the element under consideration. Since the prefactors are computed from existing top-level results and are static and equal for all elements i, while the base-level marginals have to be determined for each element separately, the actual effort lies with the latter. Still, since the PL strip does not need to be sorted, the computation of a base-level marginal actually requires less effort than the computation of a standalone figure. Consequently, the overall computational effort of computing marginal measures turns out to be much smaller than the effort to compute standalone figures for the same elements.

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4. Non-modellable Risk Factors (IMA NMRF)

Definition

The Scenarios of Extreme Stress (SES) capital charge for NMRF is determined by [9, page 65][1]

$$SES = \sqrt{\sum_r \mathbb{1}_C(r) SES_r^2} + \sum_r (1 - \mathbb{1}_C(r))\, SES_r \tag{9}$$

where r iterates over all NMRF and $\mathbb{1}_C(r)$ is the indicator function determining whether a NMRF is within the set C of NMRF associated with idiosyncratic credit spread risk. Only the latter, and possibly also idiosyncratic equity risk [11, page 10], may be aggregated using a zero correlation assumption, i.e. using a square root of sum of squares.

In the following we assume that the stress tests for the NMRF r are computed in a similar fashion as currently consulted in [14], namely, as PL valuations of a predetermined risk factor stress shock

$$SES_r = \sum_i PL_i(r \to r'). \tag{10}$$

In the – numerically extremely demanding and, thus, at the time of writing appearing very unlikely – case that SES_r are determined by full revaluation of all risk factor scenarios and a subsequent application of a risk measure on the obtained top-level PL distribution, this additional level of complexity needs to be accounted for as well in the computation of the marginals.

Euler marginal

The allocation is straightforward since the Euler allocation can be employed without effort. For NMRF not in C the aggregation is an addition of stress tests, the Euler allocation of which is trivially an addition as well. For NMRF in C the aggregation, the allocation of a square root of sum of squares is discussed in Appendix A.2 and can be applied here similar to the IMCC in Section 3.

Leveraging on Eq. (26) the marginal contribution of element i is then found as

$$mSES_i = \sum_r \left(\mathbb{1}_C(r) \frac{SES_r}{SES} + (1 - \mathbb{1}_C(r)) \right) SES_{r,i} \tag{11}$$

where the single addends are the marginal contributions $mSES_{r,i}$ of the element i to the single stress tests SES_r. Hence, the marginal contribution $mSES_r$ of a single stress test r to the overall SES charge can be found by summation across all individual contributions i instead of r.

Discussion

As for the IMCC, the marginal is simple sum of a static prefactor, computed from top-level measures, and a element-specific base-level marginal which in this case is trivially the standalone SES charge with respect to the NMRF under consideration.

For the credit risk factor aggregation the top-level SES_i charges for the various credit risk factors have to be available for the computation of the prefactor. It is prudent to thus save the $SES_{i,j}$ of the elements j until the SES_i are available and compute the marginals then (simple multiplication).

For non-credit risk factors the $mSES_{i,j}$ are given by the $SES_{i,j}$, no additional computation is necessary.

[1] Compared to the reference, the notation was slightly modified without any change to the meaning. In [13, page 187] the capital charge is denominated as stress scenario risk measure SS_t.

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Extension to multiple stress tests

Where a bank chooses to compute multiple stress tests for a certain risk factor (set), of which one is chosen according to some criteria, the outcome of these criteria needs to be saved on top level and applied as a decision rule in the computation of the marginals.

Consider, for example, the case that two risk factor shocks (upward/downward) are predetermined and the one with the larger loss (more negative PL) on top-level is chosen, i.e.

$$\begin{aligned} SES_r &= \min\left\{\sum_i SES_{r,1,i}, \sum_i SES_{r,2,i}\right\} \\ &= \min{}_r\left\{\sum_i SES_{r,1,i}, \sum_i SES_{r,2,i}\right\}, \end{aligned} \tag{12}$$

in Eq. (9) instead of (10), where we have already substituted the minimum function by a deterministic decision rule $\min_r$, which is fixed on top-level for each considered risk factor r. The marginal SES is then still given by Eq. (11) but with element-wise $SES_{r,i}$ replaced by

$$SES_{r,i} = \min{}_r\left\{SES_{r,1,i}, SES_{r,2,i}\right\}.$$

Clearly, this is not limited to only two shocks and also the selection/decision logic can be more complex. In this way, the marginal computation can also be carried out for more advanced approaches, e.g. the one proposed by the EBA [14, Art. 247].

5. Internal Default Risk Charge (IMA DRC)

The IMA DRC capital charge is based on a *VaR*, i.e. percentile, risk measure. Recalling the results of a prior investigation w.r.t. the allocation of the historical *VaR* [17], in such a case the allocation weights should be derived from an alternative risk measure.

In said investigation, it was found that, while the incremental *VaR* yields allocation weights with wrong convergence properties, the weights obtained from an *ES* on the same (or, for stability, a reshuffled[2]) *PL* strip show converge correctly to the analytic limit. In addition, the incremental *VaR* figures are not additive and require an ex-post rescaling which gives rise to frequently occurring, extremely high allocation weights. The marginal *ES* is additive per se and the *ES* weights can be used directly to allocate the *VaR*. Finally, it was found that the achieved stability increase by enlarging the number of included scenarios (by adjusting the *ES* confidence level) slows down and even inverses beyond the 12.5%/87.5% level.

The IMA DRC charge is measured as 99.9% percentile of the loss distribution. Assuming a normal distribution this corresponds to an *ES* with confidence level 99.738%[3]. Assuming that the final IMA

[2]In case of the IMA DRC, the number of scenarios is far greater than in the case of the historical (stressed) *VaR*. Therefore a reshuffling of the scenarios is not necessary.

[3]Assuming a normal distribution (WLOG transformed to zero mean) having arbitrary σ, we obtain from

$$ES = (1-\alpha)^{-1}\,(\sigma\sqrt{2\pi})^{-1}\int_{VaR_\alpha}^{\infty} t\exp\left(-t^2/2\sigma^2\right)dt = (1-\alpha)^{-1}\,\sigma/\sqrt{2\pi}\exp\left(-VaR_\alpha^2/2\sigma^2\right) \overset{!}{=} VaR_\beta$$

with $VaR_x = \sigma\Phi^{-1}$ that for a *VaR* confidence level of $\beta = 99.9\%$ an *ES* with confidence level of $\alpha = 99.738\%$ yields exactly the same result. Dependencies on the standard deviations cancel. While deviations from normality could be accounted for by adjusting the *ES* confidence level obtained above, we estimate the impact of such correction on the ultimately derived marginals to be negligible.

Alternatively, instead of using a fixed *ES* confidence level based on distribution assumptions, the confidence level could also be determined dynamically for each date from the observed PL distribution such that $ES(x) = VaR(99.9\%)$. However, the continuous change of the confidence level would increase the variability of the obtained marginals.

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DRC will use one million scenarios, a common choice for current IRC models, this means that the tail average is averaged from 2,620 scenarios which – in most cases – should well suffice for stable marginals.[4]

Employing the ES correspondence, the IMA DRC marginal for any element i can be computed as

$$m\text{IMA DRC}_i = \frac{\text{IMA DRC}_{\text{top-level}}}{ES_{\text{top-level}}} mES_i.$$

6. IMA Final Aggregation

The final capital charge for approved desks is given by aggregation

$$C_A = \max\left(IMCC_{t-1} + SES_{t-1}, m_c IMCC_{\text{avg}} + SES_{\text{avg}}\right) + \max\left(\text{IMA DRC}_{t-1}, \text{IMA DRC}_{\text{avg}}\right)$$

where the $IMCC/SES$ average is taken across the 60 last business days as opposed to the 12 last weeks for the IMA DRC [13, Article 325bb].

The marginal computation is straightforward. As in the previous cases, the max function is converted to a decision rule whether on top level the last day's value or the average are chosen for $IMCC/SES$ and IMA DRC, respectively. The average and the additions are fully linear, so that the marginal C_A for element i is given by

$$\begin{aligned} mC_{A,i} \quad = \quad & \max{}^{IMCC}\left(mIMCC_{i,t-1} + mSES_{i,t-1}, m_c\, mIMCC_{i,\text{avg}} + mSES_{i,\text{avg}}\right) + \\ & \max{}^{\text{IMA DRC}}\left(m\text{IMA DRC}_{i,t-1}, m\text{IMA DRC}_{i,\text{avg}}\right). \end{aligned}$$

Part II.
Standardized Approach

7. Sensitivity-Based Approach (SA SBA)

All SBA measures are computed threefold for three different correlation scenarios (low, mid, high). For each contribution, the highest of the three computed scenario charges contributes to the top-level SBA charge. For the purpose of brevity, in the following we do not differentiate the three cases. The computation of the marginals can either be performed in parallel to the computation of all three scenarios (and the relevant marginal set shown) or for just the relevant one ex post (corresponding to a decision rule for the maximum operation).

Here we present the marginals for the various SBA components. For a better understanding of the mechanism, the derivations of partial marginals (with increasing complexity) are given in the Appendix.

7.1. Delta Charge

For the delta charge the weighted sensitivities WS (= risk factor capital charges) are pre-aggregated to bucket capital charges K_b using

$$K_b = \sqrt{\max\left\{0, \sum_{k_b=1}^{n_b} WS_{k_b}^2 + \sum_{k_b=1}^{n_b} \sum_{l_b \neq k_b} \rho_{k_b l_b} WS_{k_b} WS_{l_b}\right\}} \tag{13}$$

[4]Alternatively, the percentile could be determined employing a Harrell-Davis estimator [18] instead of the usual one. This would allow for a rather stable quantile marginal, however, still more unstable than the ES marginals.

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with the exception of the «Others» bucket m of the credit and equity risk classes which is pre-aggregated using

$$K_m = \sum_{k=1}^{n_m} \left| WS_{k_m} \right|.$$

The delta capital charge is then obtained as

$$\begin{aligned} \text{Delta} &= \sqrt{\sum_{b=1}^{m-1} (K_b)^2 + \sum_{b=1}^{m-1} \sum_{c \neq b; c \neq m} \gamma_{bc} S_b S_c} + K_m \\ &=: \sqrt{Z(S_b)} + K_m \end{aligned}$$

where the quantity S_b is determined depending on the argument Z of the square root as

$$S_b = \begin{cases} S'_b & Z(S'_b) > 0 \text{ (A)} \\ \max\left\{\min\left\{\sum_{k=1}^{n_b} WS_{k_b}, K_b\right\}, -K_b\right\} & Z(S'_b) < 0 \text{ (B)} \end{cases} \tag{14}$$

where $S'_b = \sum_{k=1}^{n_b} WS_{k_b}$.

For our purposes, we rewrite the regulatory formula as

$$\begin{aligned} \text{Delta} &= \sqrt{Z(S_b)} + K_m \\ Z(S_b) &= \sum_{b=1}^{m-1} K_b^2 + \sum_{b=1}^{m-1} \sum_{c \neq b; c \neq m} \gamma_{bc} S_b S_c \\ K_b &= \sqrt{\max{}_b^K \left\{ 0, \sum_{k_b=1}^{n_b} WS_{k_b}^2 + \sum_{k_b=1}^{n_b} \sum_{l_b \neq k_b} \rho_{k_b l_b} WS_{k_b} WS_{l_b} \right\}} \\ K_m &= \sum_{k_m=1}^{n_m} \max{}_m^K \left\{ WS_{k_m}, -WS_{k_m} \right\} \\ S_b &= S'_b \Theta^Z \left(Z(S'_b) \right) + \max{}_b^S \left\{ \min{}_b^S \left\{ S'_b, K_b \right\}, -K_b \right\} \left(1 - \Theta^Z \left(Z(S'_b) \right) \right) \\ S'_b &= \sum_{k_b=1}^{n_b} WS_{k_b} \\ WS_{k_b} &= \sum_p w_{k_b,p} WS_{k_b,p} \end{aligned}$$

where sensitivities from elements (e.g. trades or portfolios) p with weights w are shown explicitly. All inhomogeneous components have been replaced by decision rules (being fixed at top level) so that the expression is homogeneous with grade 1.

The decomposition with regard to element j and risk factor i (belonging to bucket b_i) is given by

$$\begin{aligned} \text{Delta} &= \sum_{i,j} 1^{-1} w_{i,j} \frac{\partial \text{Delta}(w_{i,j})}{\partial w_{i,j}} \\ &= \sum_{i,j} w_{i,j} \left(1 - \delta_{b_i m} \right) \frac{\partial \text{Delta}}{\partial Z(S_b)} \left(\frac{\partial Z(S_b)}{\partial K_{b_i}} \frac{\partial K_{b_i}}{\partial w_{i,j}} + \frac{\partial Z(S_b)}{\partial S_{b_i}} \left(\frac{\partial S_{b_i}}{\partial S'_{b_i}} \frac{\partial S'_{b_i}}{\partial w_{i,j}} + \frac{\partial S_{b_i}}{\partial K_{b_i}} \frac{\partial K_{b_i}}{\partial w_{i,j}} \right) \right) \\ &\quad + w_{i,j} \delta_{b_i m} \frac{\partial \text{Delta}}{\partial K_m} \frac{\partial K_m}{\partial w_{i,j}} \end{aligned}$$

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where the derivatives are

$$\begin{aligned}
\frac{\partial \text{Delta}}{\partial Z(S_b)} &= \frac{1}{2\sqrt{Z(S_b)}} \\
\frac{\partial \text{Delta}}{\partial K_m} &= 1 \\
\frac{\partial Z(S_b)}{\partial K_{b_i}} &= 2K_{b_i} \\
\frac{\partial Z(S_b)}{\partial S_{b_i}} &= 2\sum_{c\neq b_i;c\neq m}\gamma_{b_i c}S_c \\
\frac{\partial S_{b_i}}{\partial S'_{b_i}} &= \Theta^Z\left(Z(S'_b)\right)+\max{}^S_{b_i}\left\{\min{}^S_{b_i}\{1,0\},0\right\}\left(1-\Theta^Z\left(Z(S'_b)\right)\right) \\
\frac{\partial S_{b_i}}{\partial K_{b_i}} &= \max{}^S_{b_i}\left\{\min{}^S_{b_i}\{0,1\},-1\right\}\left(1-\Theta^Z\left(Z(S'_b)\right)\right) \\
\frac{\partial K_{b_i}}{\partial w_{i,j}} &= \frac{1}{K_{b_i}}\left(\max{}^K_{b_i}\left\{0,\left(WS_i+\sum_{l_{b_i}\neq i}\rho_{i l_{b_i}}WS_{l_{b_i}}\right)\right\}\right)WS_{i,j} \\
\frac{\partial S'_{b_i}}{\partial w_{i,j}} &= WS_{i,j} \\
\frac{\partial K_m}{\partial w_{i,j}} &= \max{}^K_m\{1,-1\}\,WS_{i,j}
\end{aligned}$$

and thus the marginal (setting $w_{i,j}=1$)

$$m\text{Delta}_{i,j} = P_i\left(\text{Delta}, K_i, WS_i\right)WS_{i,j}$$

with

$$\begin{aligned}
P_i = {} & \left(1-\delta_{b_i m}\right)\frac{1}{\text{Delta}-K_m}\Bigg\{ \\
& \max{}^K_{b_i}\left\{0,\left(WS_i+\textstyle\sum_{l_{b_i}\neq i}\rho_{i l_{b_i}}WS_{l_{b_i}}\right)\right\} \\
& +\sum_{c\neq b_i;c\neq m}\gamma_{b_i c}S_c\Bigg(\\
& \quad \Theta^Z\left(Z(S'_b)\right)+\left(1-\Theta^Z\left(Z(S'_b)\right)\right)\Big[\\
& \qquad \max{}^S_{b_i}\left\{\min{}^S_{b_i}\{1,0\},0\right\} \\
& \qquad +\max{}^S_{b_i}\left\{\min{}^S_{b_i}\{0,1\},-1\right\}\tfrac{1}{K_{b_i}}\left(\max{}^K_{b_i}\left\{0,\left(WS_i+\textstyle\sum_{l_{b_i}\neq i}\rho_{i l_{b_i}}WS_{l_{b_i}}\right)\right\}\right)\Big]\Bigg)\Bigg\} \\
& +\delta_{b_i m}\max{}^K_m(1,-1)
\end{aligned}$$

It is notable that, similar to the IMA marginals, the computation of the Delta marginals is a mere multiplication of the element- and risk-factor-level $WS_{i,j}$ with an applicable prefactor. Since the prefactors P_i are based on the top-level K_b and WS_i aggregates, they are identical for all $WS_{i,j}$ on the same risk factor i and only a limited number of prefactors needs to be computed.

For validation, when applying for the FX risk factor class ($K_b=S_b=WS_i$, $Z(S'_b)>0$, no other bucket m, no $\max_{K_b}$ flooring necessary)

$$m\text{Delta}_{i,j}=\frac{1}{\text{Delta}}\left\{WS_i+\sum_{c\neq b_i}\gamma_{b_i c}S_c\right\}WS_{i,j}$$

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expression (28) as derived in Appendix B is retrieved.

7.2. Vega Charge

Vega sensitivities are aggregated using the formula as delta sensitivities; only a different correlation has to be employed.

7.3. Curvature Charge

For the curvature charge the individual curvature risk charges CVR are pre-aggregated to bucket risk charges K_b using

$$K_b = \sqrt{\max\left\{0, \sum_{k=1}^{n_b} \max(\mathrm{CVR}_{k_b}, 0)^2 + \sum_{k_b=1}^{n_b} \sum_{l_b \neq k_b} \rho^2_{k_b l_b} \mathrm{CVR}_{k_b} \mathrm{CVR}_{l_b} \psi_{k_b l_b}(\mathrm{CVR}_{k_b}, \mathrm{CVR}_{l_b})\right\}}$$

where

$$\psi(x,y) \quad = \quad \begin{cases} 1 & \text{otherwise} \\ 0 & x<0 \cap y<0 \end{cases}.$$

The curvature charge is then calculated using

$$\mathrm{Curvature} = \sqrt{\sum_{b=1}^{m-1} K_b^2 + \sum_{b=1}^{m-1} \sum_{c \neq b; c \neq m} \gamma^2_{bc} S_b S_c \psi(S_b, S_c)} + K_m$$

where $S_b = \sum_{k=1}^{n_b} \mathrm{CVR}_{k_b}$ and $K_m = \sum_{k=1}^{n_m} \left|\mathrm{CVR}_{k_m}\right|$.[5] As in the Delta case, there is a fallback to $S_b = \max_{S_b}\left\{\min_{S_b}\left\{S'_b, K_b\right\}, -K_b\right\}$ in case the argument of the square root is negative.

The expressions are largely equivalent to the Delta case with the exception of the added function ψ. In order to ensure homogeneity, as in the other cases, this two-dimensional step function needs to be replaced by a decision rules fixed at top level.

Equal to the Delta case, we rewrite the most common regulatory workflow in precise terms, replace the max/min/abs/Θ functions as well as said ψ by decision rules and add the weights $w_{k_b,p}$ to the sensitivity contributions of elements p and risk factor k_b:

$$\begin{aligned}
\mathrm{Curvature} &= \sqrt{Z(S_b)} + K_m \\
Z(S_b) &= \sum_{b=1}^{m-1} K_b^2 + \sum_{b=1}^{m-1} \sum_{c \neq b; c \neq m} \gamma^2_{bc} S_b S_c \psi^Z_{bc} \\
K_b &= \sqrt{\max{}^K_b \left\{0, \sum_{k_b=1}^{n_b} \max{}^K_{k_b} (\mathrm{CVR}_{k_b}, 0)^2 + \sum_{k_b=1}^{n_b} \sum_{l_b \neq k_b} \rho^2_{k_b l_b} \mathrm{CVR}_{k_b} \mathrm{CVR}_{l_b} \psi^K_{k_b l_b}\right\}} \\
K_m &= \sum_{k_m=1}^{n_m} \max{}^K_m \left\{\mathrm{CVR}_{k_m}, -\mathrm{CVR}_{k_m}\right\} \\
S_b &= S'_b \Theta^Z\left(Z(S'_b)\right) + \max{}^S_b \left\{\min{}^S_b \left\{S'_b, K_b\right\}, -K_b\right\}\left(1 - \Theta^Z\left(Z(S'_b)\right)\right) \\
S'_b &= \sum_{k_b=1}^{n_b} \mathrm{CVR}_{k_b}
\end{aligned}$$

[5] W.r.t. the “Other” buckets in the Curvature, the regulation is unfortunately not very precise and certain assumptions need to be made - which we did in a rather conservative fashion, mirroring the Delta workflow: aggregate the absolute values and allow no diversification with the remaining buckets.

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$$\mathrm{CVR}_{k_b} = \sum_p w_{k_b,p}\mathrm{CVR}_{k_b,p}$$

so that the decomposition with regard to element j and risk factor i (belonging to bucket b_i) can be given in a formal way as

$$\begin{aligned}
\mathrm{Curvature} &= \sum_{i,j} 1^{-1} w_{i,j} \frac{\partial \mathrm{Curvature}(w_{i,j})}{\partial w_{i,j}} \\
&= \sum_{i,j} w_{i,j}\left(1-\delta_{b_i m}\right) \frac{\partial \mathrm{Curvature}}{\partial Z(S_b)} \left(\frac{\partial Z(S_b)}{\partial K_{b_i}} \frac{\partial K_{b_i}}{\partial w_{i,j}} + \frac{\partial Z(S_b)}{\partial S_{b_i}} \left(\frac{\partial S_{b_i}}{\partial S'_{b_i}} \frac{\partial S'_{b_i}}{\partial w_{i,j}} + \frac{\partial S_{b_i}}{\partial K_{b_i}} \frac{\partial K_{b_i}}{\partial w_{i,j}} \right)\right) \\
&\quad + w_{i,j}\delta_{b_i m} \frac{\partial \mathrm{Curvature}}{\partial K_m} \frac{\partial K_m}{\partial w_{i,j}}
\end{aligned}$$

with the derivatives

$$\begin{aligned}
\frac{\partial \mathrm{Curvature}}{\partial Z(S_b)} &= \frac{1}{2\sqrt{Z(S_b)}} \\
\frac{\partial \mathrm{Curvature}}{\partial K_m} &= 1 \\
\frac{\partial Z(S_b)}{\partial K_{b_i}} &= 2K_{b_i} \\
\frac{\partial Z(S_b)}{\partial S_{b_i}} &= 2 \sum_{c\neq b_i; c\neq m} \gamma_{b_i c} S_c \psi^Z_{b_i c} \\
\frac{\partial S_{b_i}}{\partial S'_{b_i}} &= \Theta^Z\left(Z(S'_b)\right) + \max^S_{b_i}\left\{\min^S_{b_i}\{1,0\},0\right\}\left(1-\Theta^Z\left(Z(S'_b)\right)\right) \\
\frac{\partial S_{b_i}}{\partial K_{b_i}} &= \max^S_{b_i}\left\{\min^S_{b_i}\{0,1\},-1\right\}\left(1-\Theta^Z\left(Z(S'_b)\right)\right) \\
\frac{\partial K_{b_i}}{\partial w_{i,j}} &= \frac{1}{K_{b_i}}\left(\max^K_{b_i}\left\{0,\left(\mathrm{CVR}_i + \sum_{l_{b_i}\neq i} \rho_{i l_{b_i}} \mathrm{CVR}_{l_{b_i}} \psi^K_{i l_{b_i}}\right)\right\}\right)\mathrm{CVR}_{i,j} \\
\frac{\partial S'_{b_i}}{\partial w_{i,j}} &= \mathrm{CVR}_{i,j} \\
\frac{\partial K_m}{\partial w_{i,j}} &= \max^K_m\{1,-1\}\,\mathrm{CVR}_{i,j}
\end{aligned}$$

Again, a (solely risk factor i-dependent) prefactor can be extracted and the marginal computed as a simple product of $WS_{i,j}$ and the corresponding prefactor:

$$m\mathrm{Curvature}_{i,j} = P_i \mathrm{CVR}_{i,j}$$

where

$$\begin{aligned}
P_i = &\left(1-\delta_{b_i m}\right)\frac{1}{\mathrm{Curvature}-K_m}\Bigg\{ \\
&\max^K_{b_i}\left\{0,\left(WS_i + \textstyle\sum_{l_{b_i}\neq i} \rho_{i l_{b_i}} WS_{l_{b_i}} \psi^K_{i l_{b_i}}\right)\right\} \\
&+ \sum_{c\neq b_i; c\neq m} \gamma_{b_i c} S_c \psi^Z_{b_i c}\Bigg(\\
&\quad \Theta^Z\left(Z(S'_b)\right) + \left(1-\Theta^Z\left(Z(S'_b)\right)\right)\Bigg[
\end{aligned}$$

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$$
\begin{aligned}
&\max{}^S_{b_i}\left\{\min{}^S_{b_i}\{1,0\},0\right\} \\
&\quad+\max{}^S_{b_i}\left\{\min{}^S_{b_i}\{0,1\},-1\right\}\frac{1}{K_{b_i}}\left(\max{}^K_{b_i}\left\{0,\left(\mathrm{CVR}_i+\sum_{l_{b_i}\neq i}\rho_{i l_{b_i}}\mathrm{CVR}_{l_{b_i}}\psi^K_{i l_{b_i}}\right)\right\}\right)\Bigg]\Bigg)\Bigg\} \\
&+\delta_{b_i m}\max{}^K_m(1,-1)
\end{aligned}
$$

8. Residual Risk Add-On (SA RRAO)

Opposed to common conception, the RRAO is not purely additive. This is due to the absolute value of the sign-sensitive notional being taken on top level.

If i is the index of the RRAO-relevant instruments and RW_i and N_i are their assigned risk weights and notionals, respectively, and t_i the index of trades on instrument i, the RRAO is given as

$$
\begin{aligned}
RRAO &= \sum_i \left| RW_i \sum_{t_i} N_{t_i} \right| \\
&= \sum_i \max{}_i \left\{ RW_i \sum_{t_i} N_{t_i}, -RW_i \sum_{t_i} N_{t_i} \right\} \\
&= \sum_{i,t_i} \max{}_i \{1,-1\}\, RW_i N_{t_i}
\end{aligned}
$$

where, as usual, the absolute value is replaced by a decision rule fixed on top level.

Consequently, the RRAO marginal of any trade t_i on instrument i is given as

$$
mRRAO_{t_i} = \max{}_i \{1,-1\}\, RW_i N_{t_i}
$$

where $\max{}_i\{1,-1\}$ remains as indicator of the contribution of instrument i to the overall add-on.

9. Standardized Default Risk Charge (SA DRC)

Also for the default risk charge component of the standardized approach, the aggregation formulas are homogeneous as long as the max/min functions are treated as in the above cases. As a consequence, Euler allocation can be used.

9.1. Non-securitisations

Basis of the SA DRC computation is the (maturity- and LH-)weighted jump-to-default $\mathrm{WJTD}_{g_b q r}$ which is given per instrument q, issuer/obligor g_b (being part of bucket b) and per position in portfolio r.

In a first aggregation step the WJTD are aggregated depending on the seniority of each instrument's debt

$$
\begin{aligned}
\mathrm{WJTD}^{\mathrm{sn}}_{g_b} &= \sum_q \sum_r w_{g_b,q,r}\mathrm{WJTD}_{g_b,q,r}\cdot \mathbb{1}_{\{\mathrm{sen}_i=\mathrm{sn}\}}, \\
\mathrm{WJTD}^{\mathrm{ns}}_{g_b} &= \sum_q \sum_r w_{g_b,q,r}\mathrm{WJTD}_{g_b,q,r}\cdot \mathbb{1}_{\{\mathrm{sen}_i=\mathrm{ns}\}}
\end{aligned}
\tag{15}
$$

where "sn" refers to senior, "ns" to non-senior issues and we have already inserted the weights $w_{g,q,r}$ for the decomposition (ultimately being set to 1).

In a second step net long/short WJTD by issuer across seniority are computed in a conservative aggregation, where short positions are only recognized if they refer to (i.e. protect against default of) the same or lower seniority than the long positions.

$$
\mathrm{netWJTD}^{\mathrm{long}}_{g_b} = \Theta(\mathrm{WJTD}^{\mathrm{sn}}_{g_b})\Theta(\mathrm{WJTD}^{\mathrm{ns}}_{g_b})\left\{\mathrm{WJTD}^{\mathrm{sn}}_{g_b}+\mathrm{WJTD}^{\mathrm{ns}}_{g_b}\right\}
$$

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$$
\begin{aligned}
& +\Theta(\mathrm{WJTD}^{\mathrm{sn}}_{g_b})\big(1-\Theta(\mathrm{WJTD}^{\mathrm{ns}}_{g_b})\big)\max\Big\{0, \mathrm{WJTD}^{\mathrm{sn}}_{g_b}+\mathrm{WJTD}^{\mathrm{ns}}_{g_b}\Big\} \qquad (16)\\
& +\big(1-\Theta(\mathrm{WJTD}^{\mathrm{sn}}_{g_b})\big)\Theta(\mathrm{WJTD}^{\mathrm{ns}}_{g_b})\Big\{\mathrm{WJTD}^{\mathrm{ns}}_{g_b}\Big\}\\
\mathrm{netWJTD}^{\mathrm{short}}_{g_b} = & \big(1-\Theta(\mathrm{WJTD}^{\mathrm{sn}}_{g_b})\big)\big(1-\Theta(\mathrm{WJTD}^{\mathrm{ns}}_{g_b})\big)\Big\{\mathrm{WJTD}^{\mathrm{sn}}_{g_b}+\mathrm{WJTD}^{\mathrm{ns}}_{g_b}\Big\}\\
& +\Theta(\mathrm{WJTD}^{\mathrm{sn}}_{g_b})\big(1-\Theta(\mathrm{WJTD}^{\mathrm{ns}}_{g_b})\big)\min\Big\{0, \mathrm{WJTD}^{\mathrm{sn}}_{g_b}+\mathrm{WJTD}^{\mathrm{ns}}_{g_b}\Big\}\\
& +\big(1-\Theta(\mathrm{WJTD}^{\mathrm{sn}}_{g_b})\big)\Theta(\mathrm{WJTD}^{\mathrm{ns}}_{g_b})\Big\{\mathrm{WJTD}^{\mathrm{sn}}_{g_b}\Big\}
\end{aligned}
$$

In the third step, determining the capital requirement by bucket cr_b, the net long/short WJTD by issuer are aggregated across issuers and a rating-specific risk weight is applied

$$
cr_b = \max\left\{0, \sum_{g_b} RW_{g_b}\,\mathrm{netWJTD}^{\mathrm{long}}_{g_b} - \mathrm{WtS}_b RW_{g_b}\left|\mathrm{netWJTD}^{\mathrm{short}}_{g_b}\right|\right\} \qquad (17)
$$

permitting a certain bucket-specific amount of diversification given by

$$
\mathrm{WtS}_b = \frac{\sum_{g_b}\mathrm{netWJTD}^{\mathrm{long}}_{g_b}}{\sum_{g_b}\mathrm{netWJTD}^{\mathrm{long}}_{g_b} + \left|\mathrm{netWJTD}^{\mathrm{short}}_{g_b}\right|} \qquad (18)
$$

i.e. the ratio of the bucket's overall long exposure to its gross long and short exposure.

Finally, the total capital requirement for non-securitisations is given as sum across all buckets

$$
tcr_{\text{non-sec}} = \sum_b cr_b. \qquad (19)
$$

For the computation of the marginal we need to replace the Θ functions in Eqs. (16), the max/min function in Eqs. (16) and (17) and the absolute values in Eqs. (17) and (18) by top-level decision rules $\Theta^{\mathrm{sn}}_{g_b}$, $\Theta^{\mathrm{ns}}_{g_b}$, $\max^{\mathrm{netWJTD}}_{g_b}/\min^{\mathrm{netWJTD}}_{g_b}$, $\max^{cr}_{b}$ and $\max^{cr}_{g_b}\Big\{\mathrm{netWJTD}^{\mathrm{short}}_{g_b}, -\mathrm{netWJTD}^{\mathrm{short}}_{g_b}\Big\} = \max^{\mathrm{WtS}}_{g_b}$ $\Big\{\mathrm{netWJTD}^{\mathrm{short}}_{g_b}, -\mathrm{netWJTD}^{\mathrm{short}}_{g_b}\Big\}$, respectively.

After the replacements, $tcr_{\text{non-sec}}$ is homogeneous with grade 1 since a factor k applied to all weights can be pulled through all terms to finally give an overall factor k.

To identify the marginal we decompose the total capital requirement in the sense of the Euler theorem formally as

$$
\begin{aligned}
tcr_{\text{non-sec}} &= \sum_{j,i,p} 1^{-1} w_{j,i,p}\frac{\partial tcr_{\text{non-sec}}}{\partial w_{i,j}}\\
&= \sum_{j,i,p} w_{j,i,p}\frac{\partial tcr_{\text{non-sec}}}{\partial cr_{b_j}}\Bigg\{\underbrace{\left[\frac{\partial cr_{b_j}}{\partial \mathrm{netWJTD}^{\mathrm{long}}_{j}} + \frac{\partial cr_{b_j}}{\partial \mathrm{WtS}_{b_j}}\frac{\partial \mathrm{WtS}_{b_j}}{\partial \mathrm{netWJTD}^{\mathrm{long}}_{j}}\right]}_{A_{b_j,j}}\\
&\qquad \left(\frac{\partial \mathrm{netWJTD}^{\mathrm{long}}_{j}}{\partial \mathrm{WJTD}^{\mathrm{sn}}_{j}}\frac{\partial \mathrm{WJTD}^{\mathrm{sn}}_{j}}{\partial w_{j,i,p}} + \frac{\partial \mathrm{netWJTD}^{\mathrm{long}}_{j}}{\partial \mathrm{WJTD}^{\mathrm{ns}}_{j}}\frac{\partial \mathrm{WJTD}^{\mathrm{ns}}_{j}}{\partial w_{j,i,p}}\right)\\
&\qquad + \underbrace{\left[\frac{\partial cr_{b_j}}{\partial \mathrm{netWJTD}^{\mathrm{short}}_{j}} + \frac{\partial cr_{b_j}}{\partial \mathrm{WtS}_{b_j}}\frac{\partial \mathrm{WtS}_{b_j}}{\partial \mathrm{netWJTD}^{\mathrm{short}}_{j}}\right]}_{B_{b_j,j}}
\end{aligned}
$$

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$$\left(\frac{\partial \mathrm{netWJTD}_j^{\mathrm{short}}}{\partial \mathrm{WJTD}_j^{\mathrm{sn}}}\frac{\partial \mathrm{WJTD}_j^{\mathrm{sn}}}{\partial w_{j,i,p}}+\frac{\partial \mathrm{netWJTD}_j^{\mathrm{short}}}{\partial \mathrm{WJTD}_j^{\mathrm{ns}}}\frac{\partial \mathrm{WJTD}_j^{\mathrm{ns}}}{\partial w_{j,i,p}}\right)\Bigg\}$$

where we make use of the fact that risk factors j strictly contribute only to the capital charge contribution cr_{b_j} of that bucket b_j, which they are assigned to.

The terms are

$$\begin{aligned}
\frac{\partial tcr_{\text{non-sec}}}{\partial cr_{b_j}} &= 1\\
\frac{\partial cr_{b_j}}{\partial \mathrm{netWJTD}_j^{\mathrm{long}}} &= RW_j \max{}_{b_j}^{cr}\{0,1\}\\
\frac{\partial cr_{b_j}}{\partial \mathrm{netWJTD}_j^{\mathrm{short}}} &= -\mathrm{WtS}_{b_j} RW_j \max{}_j^{cr}\{+1,-1\} \max{}_{b_j}^{cr}\{0,1\}\\
\frac{\partial cr_{b_j}}{\partial \mathrm{WtS}_{b_j}} &= -\left(\sum_{g_{b_j}} RW_{g_{b_j}} \mathrm{netWJTD}_{g_{b_j}}^{\mathrm{short}} \max{}_{g_{b_j}}^{cr}\{+1,-1\}\right) \max{}_{b_j}^{cr}\{0,1\}\\
\frac{\partial \mathrm{WtS}_{b_j}}{\partial \mathrm{netWJTD}_j^{\mathrm{long}}} &= \frac{\mathrm{WtS}_{b_j}-\mathrm{WtS}_{b_j}^2}{\sum_{g_{b_j}} \mathrm{netWJTD}_{g_{b_j}}^{\mathrm{long}}}\\
\frac{\partial \mathrm{WtS}_{b_j}}{\partial \mathrm{netWJTD}_j^{\mathrm{short}}} &= -\frac{\mathrm{WtS}_{b_j}^2}{\sum_{g_{b_j}} \mathrm{netWJTD}_{g_{b_j}}^{\mathrm{long}}} \max{}_j^{\mathrm{WtS}}\{+1,-1\}\\
\frac{\partial \mathrm{netWJTD}_j^{\mathrm{long}}}{\partial \mathrm{WJTD}_j^{\mathrm{sn}}} &= \Theta_j^{\mathrm{sn}}\Theta_j^{\mathrm{ns}}+\Theta_j^{\mathrm{sn}}\left(1-\Theta_j^{\mathrm{ns}}\right) \max{}_j^{\mathrm{netWJTD}}\left\{0,1\right\}\\
\frac{\partial \mathrm{netWJTD}_j^{\mathrm{long}}}{\partial \mathrm{WJTD}_j^{\mathrm{ns}}} &= \Theta_j^{\mathrm{ns}}+\Theta_j^{\mathrm{sn}}\left(1-\Theta_j^{\mathrm{ns}}\right) \max{}_j^{\mathrm{netWJTD}}\left\{0,1\right\}\\
\frac{\partial \mathrm{netWJTD}_j^{\mathrm{short}}}{\partial \mathrm{WJTD}_j^{\mathrm{sn}}} &= \left(1-\Theta_j^{\mathrm{sn}}\right)+\Theta_j^{\mathrm{sn}}\left(1-\Theta_j^{\mathrm{ns}}\right) \min{}_j^{\mathrm{netWJTD}}\left\{0,1\right\}\\
\frac{\partial \mathrm{netWJTD}_j^{\mathrm{short}}}{\partial \mathrm{WJTD}_j^{\mathrm{ns}}} &= \left(1-\Theta_j^{\mathrm{sn}}\right)\left(1-\Theta_j^{\mathrm{ns}}\right)+\Theta_j^{\mathrm{sn}}\left(1-\Theta_j^{\mathrm{ns}}\right) \min{}_j^{\mathrm{netWJTD}}\left\{0,1\right\}\\
\frac{\partial \mathrm{WJTD}_j^{\mathrm{sn}}}{\partial w_{j,i,p}} &= \mathrm{WJTD}_{j,i,p}\cdot \mathbb{1}_{\{\mathrm{sen}_i=\mathrm{sn}\}}\\
\frac{\partial \mathrm{WJTD}_j^{\mathrm{ns}}}{\partial w_{j,i,p}} &= \mathrm{WJTD}_{j,i,p}\cdot \mathbb{1}_{\{\mathrm{sen}_i=\mathrm{ns}\}}.
\end{aligned}$$

The marginal contribution of instrument i w.r.t. issuer/obligor j and the position in portfolio r is thus given by

$$\begin{aligned}
mtcr_{j,i,p}^{\text{non-sec}} &= \begin{cases} P_j^{\mathrm{sn}}\,\mathrm{WJTD}_{j,i,p} & \mathrm{sen}_i=\mathrm{sn}\\ P_j^{\mathrm{ns}}\,\mathrm{WJTD}_{j,i,p} & \mathrm{sen}_i=\mathrm{ns}\end{cases}\\
P_j^{\mathrm{sn}} &= A_{b_j,j}\left(\Theta_j^{\mathrm{sn}}\Theta_j^{\mathrm{ns}}+\Theta_j^{\mathrm{sn}}\left(1-\Theta_j^{\mathrm{ns}}\right) \max{}_j^{\mathrm{netWJTD}}\left\{0,1\right\}\right)\\
&\quad +B_{b_j,j}\left(\left(1-\Theta_j^{\mathrm{sn}}\right)+\Theta_j^{\mathrm{sn}}\left(1-\Theta_j^{\mathrm{ns}}\right) \min{}_j^{\mathrm{netWJTD}}\left\{0,1\right\}\right)
\end{aligned}$$

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$$
\begin{aligned}
P_j^{\text{ns}} &= A_{b_j,j}\left(\Theta_j^{\text{ns}} + \Theta_j^{\text{sn}}\left(1-\Theta_j^{\text{ns}}\right) \max{}_j^{\text{netWJTD}}\left\{0,1\right\}\right) \\
&\quad + B_{b_j,j}\left(1-\Theta_j^{\text{sn}}\right)\left(1-\Theta_j^{\text{ns}}\right) + \Theta_j^{\text{sn}}\left(1-\Theta_j^{\text{ns}}\right) \min{}_j^{\text{netWJTD}}\left\{0,1\right\}.
\end{aligned}
$$

The prefactors can be computed once, for each issuer/obligor j and each seniority sn/ns, and applied subsequently for any allocation to instruments, portfolios or other hierarchy levels.

9.2. Securitisations

The default risk for securitisations is very similar to the non-securitisation component. The difference concerning the computation of the WJTD (no application of the LGD), a mere input for the computation of the marginals, is not relevant. However, the computation of the netWJTD is more restrictive in that only exactly the same tranches may be offset. In this way a consideration by seniority is not necessary for securitisations. Let g_b, therefore, here, be a distinct tranche of a distinct pool being part of securitisation bucket b, then

$$
\text{WJTD}_{g_b} = \sum_q \sum_r w_{g_b,q,r} \text{WJTD}_{g_b,q,r}, \tag{20}
$$

$$
\begin{aligned}
\text{netWJTD}_{g_b}^{\text{long}} &= \max\left\{0, \text{WJTD}_{g_b}\right\} \\
\text{netWJTD}_{g_b}^{\text{short}} &= \min\left\{0, \text{WJTD}_{g_b}\right\}
\end{aligned} \tag{21}
$$

Note that the Basel [9, Art. 160] permits to offset tranche replications (i.e. where a collection of tranches of a pool replicates another tranche of the same pool). To include, such combinations need to be saved in the computation of the full charge and the marginals adjusted by including a respective decision rule.

The computation of cr_b, WtS_b and, ultimately, tcr^{sec} mirrors the non-securitisation workflow. Therefore, following the same steps as therein, we can assume homogeneity and decompose as follows

$$
\begin{aligned}
tcr_{\text{sec}} &= \sum_{j,i,p} 1^{-1} w_{j,i,p} \frac{\partial tcr_{\text{sec}}}{\partial w_{i,j}} \\
&= \sum_{j,i,p} w_{j,i,p} \frac{\partial tcr_{\text{sec}}}{\partial cr_{b_j}} \Bigg\{ \\
&\qquad \underbrace{\left[\frac{\partial cr_{b_j}}{\partial \text{netWJTD}_j^{\text{long}}} + \frac{\partial cr_{b_j}}{\partial \text{WtS}_{b_j}} \frac{\partial \text{WtS}_{b_j}}{\partial \text{netWJTD}_j^{\text{long}}}\right]}_{A_{b_j,j}} \left(\frac{\partial \text{netWJTD}_j^{\text{long}}}{\partial \text{WJTD}_j} \frac{\partial \text{WJTD}_j}{\partial w_{j,i,p}}\right) \\
&\qquad + \underbrace{\left[\frac{\partial cr_{b_j}}{\partial \text{netWJTD}_j^{\text{short}}} + \frac{\partial cr_{b_j}}{\partial \text{WtS}_{b_j}} \frac{\partial \text{WtS}_{b_j}}{\partial \text{netWJTD}_j^{\text{short}}}\right]}_{B_{b_j,j}} \left(\frac{\partial \text{netWJTD}_j^{\text{short}}}{\partial \text{WJTD}_j} \frac{\partial \text{WJTD}_j}{\partial w_{j,i,p}}\right) \Bigg\}.
\end{aligned}
$$

The expressions $A_{b_j,j}$ and $B_{b_j,j}$ as well as the (trivial) derivative $\partial tcr_{\text{sec}}/\partial cr_{b_j}$ remain identical with the replacement of non-sec with sec. The remaining derivatives simplify

$$
\frac{\partial \text{netWJTD}_j^{\text{long}}}{\partial \text{WJTD}_j} = \max{}_j^{\text{netWJTD}}\left\{0,1\right\}
$$

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$$
\begin{aligned}
\frac{\partial \mathrm{netWJTD}_j^{\mathrm{short}}}{\partial \mathrm{WJTD}_j} &= \min{}_j^{\mathrm{netWJTD}}\left\{0,1\right\} \\
\frac{\partial \mathrm{WJTD}_j}{\partial w_{j,i,p}} &= \mathrm{WJTD}_{j,i,p} \\
\frac{\partial \mathrm{WJTD}_j}{\partial w_{j,i,p}} &= \mathrm{WJTD}_{j,i,p}
\end{aligned}
$$

so that the marginal contribution of instrument i w.r.t. distinct tranche j (of a distinct pool) and the position in portfolio r is given by

$$
\begin{aligned}
mtcr_{j,i,p}^{\mathrm{sec}} &= P_j\,\mathrm{WJTD}_{j,i,p} \\
P_j &= A_{b_j,j}\max{}_j^{\mathrm{netWJTD}}\left\{0,1\right\} + B_{b_j,j}\min{}_j^{\mathrm{netWJTD}}\left\{0,1\right\},
\end{aligned}
$$

where the prefactor is again to be computed only once for each relevant tranche.

10. Conclusion

In the preceding chapters we formulated marginal measures for all components of the new market risk capitalization framework. For the vast majority, these are based on the Euler allocation. The marginals are found to have a very simple structure as product of a common prefactor and the standalone contribution of that element of which the marginal is being sought. This reduces computational cost considerably in contrast to e.g. an incremental measure.

A corner stone of the marginal derivation is the replacement of inhomogeneous functions with homogeneous decision rules. These are fixed at the computation's top level and the fixed decision is then valid for all levels of the marginal computation.

For the Internal Model Default Risk Charge, which is based on a Value-at-Risk-based at a high confidence level, an alternative approach is proposed which allows for stable marginals even in this challenging setting.

At the time of writing, the FRTB regulation is not fully fixed, neither are the respective implementations into national law. Where regulatory requirements change, adjustments to the marginals become necessary. Where such changes do not fundamentally change the charges, the demonstrated concepts and tools should allow for a straightforward derivation also of changed regulatory expressions.

Part III. Appendix

A. The Euler theorem - Examples

A.1. Example 1: Expected Shortfall

As first example we first consider the vanilla expected shortfall $ES = \sum_s e_s PL_s / \sum_s e_s$ where s indexes the scenarios, $e_s = 1_{\Phi^{-1}(s)<\mathrm{confidence\,level}}$ is the tail indicator function (we assume that PL is a gain distribution, i.e. losses carry a negative sign; the indicator is determined on top level) and $\Phi(s)$ is the cumulative distribution function which yields the percentile of scenario s.

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In order to apply the Euler principle, we split the overall $PL = \sum_i w_i PL_i$ into its component contributions PL_i, formally weighted by factors w_i (could e.g. be interpreted as the component notional values but in case no preference is desired can be set to 1), such that

$$ES = \frac{\sum_s e_s \sum_i w_i PL_{s,i}}{\sum_s e_s}. \tag{22}$$

We directly see that homogeneity of degree 1 (linear homogeneity)

$$ES(kwPL) = kES(wPL) \tag{23}$$

holds.

From the Euler theorem we obtain the decomposition

$$\begin{aligned} ES &= \sum_i 1^{-1} w_i \frac{\partial ES(w_i)}{\partial w_i} \\ &= \sum_i w_i \frac{\sum_s e_s PL_{s,i}}{\sum_s e_s} \\ &= \sum_i mES_i. \end{aligned}$$

Where, in the last step, the addends of the sum are identified as the marginal contributions, i.e.

$$mES_i := \frac{\sum_s w_i e_s PL_{s,i}}{\sum_s e_s}.$$

The marginal of component i is the tail average across the component's PL contribution to the overall tail scenarios.

While, this decomposition could clearly have been inferred from Eq. (22) directly, this straightforward example reveals much about the mode of action of the Euler principle.

A.2. Example 2: Square-Root Aggregated Expected Shortfall

As second example, we consider the square-root of sum of squares, as appearing in the IMCC aggregation,

$$X = \sqrt{\sum_h s_h ES_h^2},$$

where the ES_h are e.g. the partial expected shortfall results for various liquidity horizons h. The scaling factor s_h depends only on the liquidity horizon h and can, for our purposes, be regarded as a static factor.

While the square is homogeneous of degree 2, the square root of sum of squares is homogeneous of degree 1:

$$\begin{aligned} X(kw) &= \sqrt{\sum s_h ES_h^2(kw)} \\ &\overset{(23)}{=} \sqrt{\sum k^2 s_h ES_h(w)} \\ &= kX(w) \end{aligned} \tag{24}$$

Thus, the Euler theorem can be applied in a straightforward fashion

$$X = \sum_i 1^{-1} w_i \frac{\partial X(w_i)}{\partial w_i}$$

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$$
\begin{aligned}
&= \sum_i w_i \frac{\partial X}{\partial(\sum_h s_h ES_h^2)} \frac{\partial(\sum_h s_h ES_h^2)}{\partial ES_h} \frac{\partial ES_h}{\partial w_i} \\
&= \sum_i w_i \cdot \frac{1}{2} \left(\sum_h s_h ES_h^2 \right)^{-1/2} \cdot \sum_h s_h 2 ES_h \cdot \frac{mES_{h,i}}{w_i} \\
&= \sum_i \sum_h \frac{s_h ES_h}{X} mES_{h,i} \qquad (25)
\end{aligned}
$$

such that the contribution from component i can be written as

$$
mX_i := \sum_h \frac{s_h ES_h}{X} mES_{h,i}
$$

or even more granular as

$$
mX_{h,i} := \frac{s_h ES_h}{X} mES_{h,i}. \qquad (26)
$$

The marginal contribution of a single liquidity horizon can be found by summation across all components i

$$
mX_h = \frac{s_h ES_h}{X} \sum_i mES_{h,i} = \frac{s_h ES_h^2}{X}.
$$

The result is not surprising: a simple rule of proportion where each liquidity horizon is weighted by its contribution to the square root. It underlines the purposefulness of the Euler theorem, that, for simple and clearly laid out problems, its outcome is not only numerically consistent but equally simple and mirroring the mathematically intuitive result. For more conclusive arguments on the correct convergence of Euler marginals we refer to [17].

B. Derivation of SBA marginals

In order to better outline the mechanism, in the following, we first derive partial marginals and increasing the complexity step by step. At first we consider the FX asset class and go from marginals per risk factor to marginals per risk factor and element. In a second step we address the other asset classes, where we also introduce the treatment of the "Other" bucket.

B.1. FX

We first consider FX as here each bucket K_b represents only one exchange rate, i.e. the bucket $K_b = WS_{k_b} = S'_b =: WS_b$ (there is no sum since there is only one WS per currency bucket).

Case (A) Assuming case (A) in (14) we thus have

$$
\text{Delta} = \sqrt{\sum_b (w_b WS_b)^2 + \sum_b \sum_{c \neq b} \gamma_{bc} w_b WS_b w_c WS_c}.
$$

Indeed, the expression is homogeneous of grade 1

$$
\begin{aligned}
\text{Delta}(kw) &= \sqrt{\sum_b (k w_b WS_b)^2 + \sum_b \sum_{c \neq b} \gamma_{bc} k w_b WS_b k w_c WS_c} \\
&= k \, \text{Delta}(w)
\end{aligned}
$$

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which allows to write

$$
\begin{aligned}
\text{Delta} &= \sum_i 1^{-1} w_i \frac{\partial \text{Delta}(w_i)}{\partial w_i} \\
&= \sum_i \frac{w_i}{2\,\text{Delta}} \left(2 w_i WS_i^2 + \sum_{c \neq i} \gamma_{ic} WS_i w_c WS_c + \sum_{b \neq i} \gamma_{bi} w_b WS_b WS_i \right) \\
&= \sum_i \frac{w_i}{\text{Delta}} \left(w_i WS_i^2 + \sum_{c \neq i} \gamma_{ic} WS_i w_c WS_c \right).
\end{aligned}
$$

Setting the w_i to unity we find the marginal contribution of currency i as

$$
m\text{Delta}_i = \frac{1}{\text{Delta}} \left(WS_i^2 + \sum_{c \neq i} \gamma_{ic} WS_i WS_c \right) \tag{27}
$$

which impresses by simplicity.

It is worth noting one peculiarity: one may intuitively reason that a hedging contribution ($WS < 0$) may lead to issues when computing the marginal from the square root expression without sign protection in the second part of the argument. One might suspect that the argument of the square root in the expression for the marginal could become negative. However, this is not the case. As observed before, the aggregation function-specific scaling is contained in a prefactor while the representation of the fundamental quantity to be scaled follows as simple product from the inner derivative which does not contain any ambiguities such as a square root of a negative quantity. This is a feature of the homogeneity.

Case (B) The fallback case (B) is selected in Eq. (14) if the argument of the square root in case (A) would be negative. For FX this case does however not occur[6].

Multi-instrument/multi-portfolio marginals We now consider the case where the weighted sensitivities stem from multiple origins (e.g. multiple instruments/trades or multiple portfolios). Since the weighted sensitivities are computed by simple addition, this is straightforward. Let p, respectively q, index the elements of the lowest aggregation level, e.g. instruments, and j the single element for which the marginal is sought, then

$$
\text{Delta} = \sqrt{\sum_b (\sum_p w_{b,p} WS_{b,p})(\sum_q w_{b,q} WS_{b,q}) + \sum_b \sum_{c \neq b} \gamma_{bc} \left(\sum_p w_{b,p} WS_{b,p} \right) \left(\sum_q w_{c,q} WS_{c,q} \right)}
$$

and

$$
\begin{aligned}
\text{Delta} &= \sum_i 1^{-1} w_{i,j} \frac{\partial \text{Delta}(w_{i,j})}{\partial w_{i,j}} \\
&= \sum_i \frac{w_{i,j}}{2\,\text{Delta}} \Bigg(\left(\sum_p w_{i,p} WS_{i,p} \right) WS_{i,j} + WS_{i,j} \left(\sum_q w_{i,q} WS_{i,q} \right) + \\
&\qquad \sum_{c \neq i} \gamma_{ic} WS_{i,j} \left(\sum_q w_{c,q} WS_{c,q} \right) + \sum_{b \neq i} \gamma_{bi} \left(\sum_p w_{b,p} WS_{b,p} \right) WS_{i,j} \Bigg) \\
&= \sum_i \frac{w_{i,j}}{\text{Delta}} \left(WS_{i,j} \left(\sum_p w_{i,p} WS_{i,p} \right) + \sum_{c \neq i} \gamma_{ic} WS_{i,j} \left(\sum_p w_{c,p} WS_{c,p} \right) \right)
\end{aligned}
$$

[6] It can be shown that the term can only turn negative for $\gamma > 1$.

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$$= \sum_i \frac{w_{i,j}}{\text{Delta}} \left(WS_{i,j}\, w_i\, WS_i + \sum_{c \neq i} \gamma_{ic}\, WS_{i,j}\, w_c\, WS_c \right),$$

so that the marginal delta of instrument j and currency i is given as

$$\begin{aligned} m\text{Delta}_{i,j} &= \frac{1}{\text{Delta}} \left(WS_{i,j}\, WS_i + \sum_{c \neq i} \gamma_{ic}\, WS_{i,j}\, WS_c \right) \\ &= \frac{1}{\text{Delta}} \left(WS_i + \sum_{c \neq i} \gamma_{ic}\, WS_c \right) WS_{i,j}. \end{aligned} \tag{28}$$

The derivation is fully analogous to the previous case. Since the result is a product of *WS* and prefactor, the same prefactor is obtained and the *WS* is decomposed into its multiple components.

B.2. IR and other asset classes

For IR and the other asset classes both aggregation steps need to be performed, i.e. the weighted sensitivities of currency b have to be aggregated to bucket level (Eq. (13)) before the K_b are aggregated to the Delta charge as in the FX case.Let p, respective q, index the elements of the lowest aggregation level, e.g. individual trades, and j the single element for which the marginal is sought, then

$$K_b(w) = \sqrt{\max{}_b \left\{ 0, \sum_{k_b=1}^{n_b} \left(\sum_p w_{k_b,p} WS_{k_b,p} \right)^2 + \sum_{k_b=1}^{n_b} \sum_{l_b \neq k_b} \rho_{k_b l_b} \left(\sum_p w_{k_b,p} WS_{k_b,p} \right) \left(\sum_q w_{k_b,q} WS_{l_b,q} \right) \right\}}.$$

The decomposition is given by

$$\begin{aligned} K_b &= \sum_i 1^{-1} w_{i,j} \frac{\partial K_b(w_{i,j})}{\partial w_{i,j}} \\ &= \sum_i \frac{w_i}{2\,K_b} \left(\max{}_b \left\{ 0, 2 WS_{i,j} \left(\sum_p w_{i,p} WS_{i,p} \right) + 2 \sum_{l_b \neq i} \rho_{i l_b} WS_{i,j} \left(\sum_p w_{l_b,p} WS_{l_b,p} \right) \right\} \right) \\ &= \sum_i \frac{w_i}{K_b} \left(\max{}_b \left\{ 0,\ w_i\, WS_{i,j}\, WS_i + \sum_{l_b \neq i} \rho_{i l_b} WS_{i,j}\, w_{l_b}\, WS_{l_b} \right\} \right). \end{aligned}$$

and thus the marginal found as

$$\begin{aligned} mK_b &= \frac{1}{K_b} \left(\max{}_b \left\{ 0,\ w_i\, WS_{i,j}\, WS_i + \sum_{l_b \neq i} \rho_{i l_b} WS_{i,j}\, w_{l_b}\, WS_{l_b} \right\} \right) \\ &= \begin{cases} \frac{1}{K_b} \left(WS_i + \sum_{l_b \neq i} \rho_{i l_b} WS_{l_b} \right) WS_{i,j} & \text{nonzero top} - \text{level contribution} \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

"Other" buckets The delta charge of the other classes is equivalent to the computation for IR with the only difference being that for CSR and EQ an «Other» bucket m also contributes, the K_b of which is given by

$$K_m = \sum_{k=1}^{n_m} \left| WS_{k_m} \right| = \sum_{k=1}^{n_m} \max \left\{ WS_{k_m}, -WS_{k_m} \right\}$$

where the maximum function is again translated to a decision rule $\max_m$ which is determined on risk factor k_m level. The marginal of a weighted sensitivity $WS_{k_m,j}$ of a risk factor k_m from an element j within the «Other» bucket is then given by

$$mK_m = \max{}_m \{1, -1\}\, WS_{k_m,j}$$

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$$= \begin{cases} WS_{k_m,j} & WS_{k_m} \geq 0 \\ -WS_{k_m,j} & WS_{k_m} < 0 \end{cases}$$

Note that the weighted sensitivity has no influence on the max decision rule and can, thus, be extracted from it.

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