Stress Testing and Economic Scenario Generation

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Scenarios and Risk Management

The course of **economy** is subject to crises. There is a need to assess their impact on **value** and **risk** of assets and liabilities.
Need For Real World Scenarios

- RW scenarios represent our estimate on the probabilities of future events.
- RW scenarios are used for risk modeling, need a good fit to extreme events.
- Modeling company strategy for each scenario – difficult problem.
Need For Risk Neutral Scenarios

- RN scenarios provide market consistent valuation of complex instruments/contracts
- RN scenarios need to focus on the expected values
Need For Stress Testing

Balance Sheet Start
- Cash
- Loans
- Trading Book
- Intangible assets
- Deposits
- ST Liability
- LT Liability
- Equity

Balance Sheet End
- Cash
- Loans
- Trading Book
- Intangible assets
- Deposits
- ST Liability
- LT Liability
- Equity

\[ Y = F(X) \]

X ~ r.v. scenarios
F ~ contracts
Y ~ result

Stress Testing Questions:
- What if the real X is different from our model? What is the impact on Y?
- Are there any scenarios which are dangerous for the whole industry?
Real World Scenario Generator

- RW stylized facts
- GARCH and equity index model
- Risk free interest rate model
- Corporate spread model
- Other models
- Dependency modeling
**Stylized Facts for RW Scenarios**

*Stylized fact* summarizes a statistical analysis or an economic hypothesis.

<table>
<thead>
<tr>
<th>Stylized Fact</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>No arbitrage</td>
<td>No (simple) arbitrage opportunities: interest rate parity, positive forward rates,…</td>
</tr>
<tr>
<td>No autocorrelation</td>
<td>Autocorrelations of marketable risk factor returns are insignificant.</td>
</tr>
<tr>
<td>Mean reversion</td>
<td>Interest rate, inflation and credit cycle exhibit mean reversion property.</td>
</tr>
<tr>
<td>Volatility clustering</td>
<td>High volatility events tend to cluster in time, e.g. equity indices, FX rates.</td>
</tr>
<tr>
<td>Heavy tails</td>
<td>Tails of observed return data deviate from normal or lognormal behavior.</td>
</tr>
<tr>
<td>Asymmetric tails</td>
<td>Negative returns often exhibit fatter tails than positive ones.</td>
</tr>
<tr>
<td>Tail dependence</td>
<td>Higher dependence under stressed market conditions.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

...
Stylized Fact: Mean Reversion

Mean Reversion of Real Interest Rate Based on USD 10Y Treasury and US CPI

Real IR
Sample Mean
Stylized Fact: Volatility Clustering

Volatility Clusters: MSCI US annual moving average volatilty
Stylized Fact: Tails

**Heavy Tails**

CDF of Monthly Returns of MSCI UK (1970-2010)

**Asymmetric Tails**

Stylized Fact: Tail Dependence

Overview of Scenario Generation

- Standards **econometric** models are used when available.
- **Historical** data, **market** expectations (e.g. forward rates), and **expert** judgment are used for calibration.
- **Quality** of scenarios is checked against stylized facts and expert judgment.
Modeling Equity Indices

GARCH(1,1) - a standard model for equity index.

- Modeling the returns, \( R_t = \log(S_t) - \log(S_{t-1}) \), as follows:
  \[
  R_t = \mu_t + \sigma_t \varepsilon_t
  \]
  return = expectation + volatility × stochastic residual

- Modeling volatility \( \sigma_t \) using auto-regression:
  \[
  \sigma_t^2 = a_0 + a_1 r_{t-1}^2 + b_1 \sigma_{t-1}^2 \quad \text{where} \quad r_t = R_t - \mu_t
  \]

- Using market historical average or expectation for \( \mu_t \).
GARCH and Volatility Clustering

GARCH volatility plot compared to white noise volatility plot

- GARCH
- White Noise
GARCH and Tails

GARCH can produce heavy and asymmetric tails.
Equity Index Dependencies

• Index 1:
  \[ R_t^1 = \mu_t^1 + \sigma_t^1 \varepsilon_t^1 \]

• Index 2:
  \[ R_t^2 = \mu_t^2 + \sigma_t^2 \varepsilon_t^2 \]

• T-copula between error terms \( \varepsilon_t^1 \) and \( \varepsilon_t^2 \) gives a dependency model for indices.
Modeling Risk-Free Yield Curve

- Need to model $i_{T,t}$ - interest rates at time $t$ with time $T$ to maturity.

- New difficulty: $i_{T,t}$ must be carefully constructed, otherwise – arbitrage.

Example:

If a scenario gives $i_{T=9,t=1} = 8\%$ and $i_{T=10,t=1} = 2\%$ => arbitrage.
Modeling Risk-Free Yield Curve

- Use **forward** interest rates: \( f_{T,t} = T \log(1 + i_{T,t}) - (T - 1) \log(1 + i_{T-1,t}) \)
- \( f_{T,t} > 0 \) keeps the yield curve arbitrage-free
- Think of \( f_{T,t} \) as a curve which evolves as \( t \) changes
- Write the curves in a good basis (say Legendre polynomials)
  \[
  f_{T,t} = c_0(t)P_0(T) + c_1(t)P_1(T) + c_2(t)P_2(T) + c_3(t)P_3(T) + \text{small noise term}
  \]
- The coefficients \((c_0(t), c_1(t), c_2(t), c_3(t))\) are modeled using GARCH(1,1)
Modeling Risk-Free Yield Curve

Each scenario generates a sequence of curves for $t=1,2,3,...$
Risk-free Yield Curve Simulation

Example: three scenarios of yield curve evolution
Need to model credit spreads for each credit rating, maturity, country, industry.

Difficulties:
- inconsistent credit spreads for different maturities may lead to arbitrage
- credit spread for AAA should be smaller than for BBB
Corporate Spreads

- Define **credit cycle** = weighted average of spreads for all ratings and maturities
- Model credit cycle using GARCH
- Model each credit spread based on credit cycle
ESG Variables

• **Market Risk**
  - Risk-free yield curves
  - Equity indices
  - Foreign exchange rates
  - Corporate yields/spreads
  - Hedge fund indices
  - Private equity indices
  - Real estate indices
  - MBS, ABS indices

• **Credit Risk**
  - Credit cycle
  - Probability of default
  - Migration matrix

• **Macro Variables**
  - GDP
  - Inflation
  - Unemployment
  - Real disposable income
  - Current account balance
Dependency Modeling

• For simple dependencies, copula on error terms is sufficient.

• Conditions like no-arbitrage require more complex dependency models.

• Complex dependencies are often modeled with the help of derived variables:
  – Modeling forward rates to model yield;
  – Modeling credit cycle to model credit spreads and migration probabilities.
Risk Neutral Scenarios

- Fundamental theorem of asset pricing
- Key building blocks for risk neutral scenarios
- Building risk neutral scenarios for equity Index
Need For Risk Neutral Scenarios

RN scenarios provide **market consistent valuation** of complex instruments/contracts
RN scenarios need to focus on the **expected values**
Fundamental Theorem of Asset Pricing

No arbitrage $\Leftrightarrow$ existence of risk neutral measure $Q$.

In risk neutral measure $Q$ (after discounting):

- Scenarios form a martingale.
- **Price** of any contingent claim $C$ is $E_Q[C]$.
- Any self-financing trading strategy has the same expected return.
Main Building Blocks of Risk Neutral Scenarios

**Implied volatility surface**
- Implied volatility values available only for liquid options
- **Moneyness** dimension gives volatility smile
- For illiquid markets or non-traded assets, models or judgment are used

**Martingale property**
- Martingale conditions apply to all simulated time points for all investment types over all holding periods, in principle → large set of martingale conditions
- In practice: priority order – martingale conditions/tests for the most relevant investment strategies

**Correlation between different risk factors**
- Correlations cannot be derived from derivative markets
- RN can use RW correlations model
- Many correlations are not important for valuation
Index Model: Volatility Surface

- Let \( C(S_0,K,T) \) be value of a European option with strike \( K \) and expiration \( T \).
- Instead of option prices, industry prefers to work with volatilities \( \sigma(S_0,K,T) \) implied by B-S.
- Using volatility surface, one can compute implied distribution of the index at any time \( t \).
RN Scenarios for an Index

• Given volatility surface, compute implied index distribution for each time t.
• Select a number of simple trading strategies.
• Run an optimization algorithm that modifies values of RW scenarios to
  – Match implied distributions as close as possible.
  – Satisfy the restriction: each trading strategy should have the same return (after discounting)
ESG Variables

• **Market Risk**
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  - Foreign exchange rates
  - Corporate yields/spreads
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  - Private equity indices
  - Real estate indices
  - MBS, ABS indices

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Stress Testing

- Stress test example
- Extending regulatory scenarios
- Generating regulatory scenarios
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Forward vs Reverse Stress Test

FORWARD

- Macro stress
- Extension to portfolio relevant risk factors
- P&L and BS impact

REVERSE

- Provide story line
- Identify stress scenarios leading to stress loss
- Portfolio stress loss
Stress Test Example 1

Assuming >= 5% GDP drop in 1 year, what is the expected impact on the credit portfolio namely migration probabilities?

- Keep only the RW scenarios with GDP drop >= %5.
- Take the average of the migration probabilities for the selected RW scenarios.
Simple Stress Test Example 1

Migration and default probabilities for the stress scenario:

<table>
<thead>
<tr>
<th>Initial Rating</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA Sev Adverse</td>
<td>78.20%</td>
<td>5.75%</td>
<td>0.64%</td>
<td>0.13%</td>
</tr>
<tr>
<td>Baseline</td>
<td>94.19%</td>
<td>1.44%</td>
<td>0.14%</td>
<td>0.03%</td>
</tr>
<tr>
<td>AA Sev Adverse</td>
<td>0.06%</td>
<td>67.65%</td>
<td>2.63%</td>
<td>0.34%</td>
</tr>
<tr>
<td>Baseline</td>
<td>0.27%</td>
<td>88.55%</td>
<td>0.62%</td>
<td>0.08%</td>
</tr>
<tr>
<td>A Sev Adverse</td>
<td>0.01%</td>
<td>0.05%</td>
<td>68.71%</td>
<td>2.30%</td>
</tr>
<tr>
<td>Baseline</td>
<td>0.06%</td>
<td>0.21%</td>
<td>85.56%</td>
<td>0.54%</td>
</tr>
<tr>
<td>BBB Sev Adverse</td>
<td>0.01%</td>
<td>0.01%</td>
<td>0.09%</td>
<td>68.52%</td>
</tr>
<tr>
<td>Baseline</td>
<td>0.02%</td>
<td>0.03%</td>
<td>0.38%</td>
<td>83.37%</td>
</tr>
</tbody>
</table>
Simple Stress Test Example 2

What is the expected value of MSCI UK given that MSCI US <= index value 80?

- Keep only the RW scenarios with MSCI US <= 80.
- Take the average of the MSCI UK for the selected RW scenarios.
ESG and Forward Stress Testing

1. Stress condition via quadrant
2. Select scenarios satisfying stress condition
3. Compute P&L and BS average impact
Picking Stress Test Conditions

- Companies submit scenarios with large losses according to their models.
- Group similar scenarios (clustering algorithm).
- For each scenario group, identify main factors and define quadrants.

Provide story line and quadrants for the groups
Identify stress scenarios groups
Collect portfolio stress loss for the industry
Creating a culture of risk awareness®

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