KVA and MVA: Capital Valuation Adjustment and Margin Valuation Adjustment

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Contents

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Introducing KVA

- What is KVA?
  - KVA = Capital Valuation Adjustment
  - "XVA" name for lifetime cost of capital
- How is it calculated?
  - Hurdle rate / $\mathbb{P}$ measure
  - Replication (Risk-neutral) - (Green, Kenyon, and Dennis 2014)
- How does it relate to other XVAs?
  - FVA - possible overlap with funding costs.
  - CVA - capital benefit for CVA hedging.

Current Industry Approaches to KVA

- To date limited quantitative research on the topic (in pricing context):
  - (Kenyon and Green 2013)
  - (Kenyon and Green 2014a; Kenyon and Green 2014b)
  - (Green, Kenyon, and Dennis 2014; Green and Kenyon 2014; Green 2015)
  - (Elouerkhaoui 2014; Elouerkhaoui 2015)
  - (Prampolini 2014)
  - (Hannah 2014)
  - (Albanese, Caenazzo, and Iabichino 2015)
- Pricing models
  - Economic Capital
  - $\mathbb{P}$ measure
  - Hurdle rate on Profit
- Management Metrics
  - Return on Capital measures

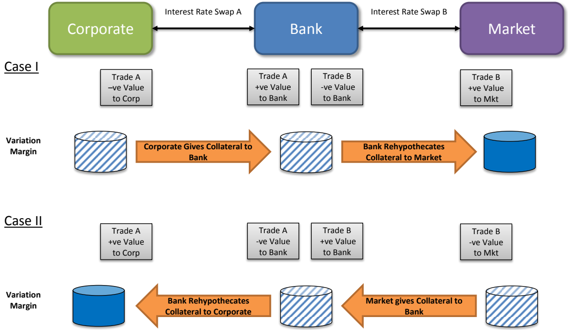
Introducing MVA

- What is MVA?
  - MVA = (Initial) Margin Valuation Adjustment
  - "XVA" name for funding cost of initial margin
  - Applies to CCP Margin and Bilateral Margin (BCBS-317 2015; BCBS-261 2013; BCBS-242 2013; BCBS-226 2012)
- How is it calculated?
  - Replication model
  - Integral over expected initial margin profile.
- How does it relate to other XVAs?
  - No overlap.
  - There are trade-offs vs other XVAs.

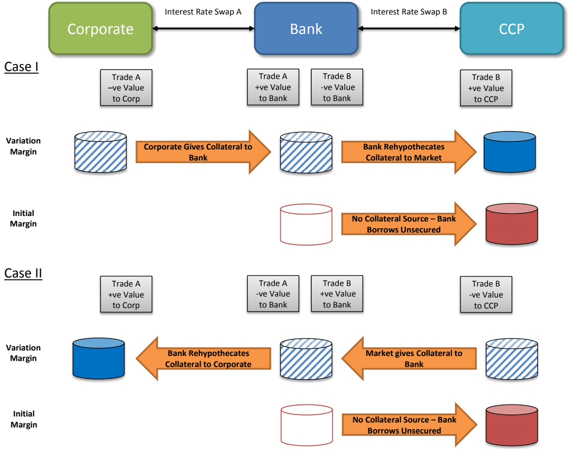
MVA and Initial Margin

- To see how MVA arises consider the following examples
  1. A bank trades a derivative with a corporate under a CSA and hedges by trading the opposite transaction with the interbank market under an identical CSA
  2. A bank trades a derivative with a corporate under a CSA and hedges with an interbank counterparty but clears the trade through a CCP

Case 1: Back-to-Back Trade CSA



Case 2: Back-to-Back Trade with Clearing



Funding cost of Margin = MVA

- Intuitively initial margin must be funded as in most cases rehypothecation of the initial margin is not allowed.
- Leads to a new valuation adjustment: Margin Valuation Adjustment
- Valuation of derivatives under collateralisation (VM) is now well established - (Piterbarg 2010; Piterbarg 2012; Piterbarg 2013)
- How do we model the funding cost of margin?

XVA by Replication I

- Extend (Burgard and Kjaer 2013) to include capital and initial margin (Green and Kenyon 2015; Green, Kenyon, and Dennis 2014).
- The dynamics of the underlying assets are

$$dS = \mu_s S dt + \sigma_s S dW \tag{1}$$
$$dP_C = r_C P_C dt - P_C dJ_C \tag{2}$$
$$dP_i = r_i P_i dt - (1 - R_i) P_i dJ_B \quad \text{for} \quad i \in \{1, 2\} \tag{3}$$

- On default of the issuer, B, and the counterparty, C, the value the derivative takes is

$$\hat{V}(t, S, 1, 0) = g_B(M_B, X) \tag{4}$$
$$\hat{V}(t, S, 0, 1) = g_C(M_C, X). \tag{5}$$

XVA by Replication II

- The g's allow flexibility around the value of the derivative after default.
- Usual close-out assumptions including initial margin posted by B and C, I_B and I_C respectively:

$$\begin{aligned} g_B =& (V - X + I_B)^+ + R_B(V - X + I_B)^- + X - I_B \\ g_C =& R_C(V - X - I_C)^+ + (V - X - I_C)^- + X + I_C, \end{aligned} \tag{6}$$

- where $x^+ = \max\{x, 0\}$ and $x^- = \min\{x, 0\}$ and the initial margin is segregated.
- We assume the funding condition:

$$\hat{V} - X + I_B + \alpha_1 P_1 + \alpha_2 P_2 - \phi K = 0, \tag{7}$$

- where ϕK represents the potential use of capital for funding.
- IM I_B is funded through the issuance of bonds (and equity if $\phi \neq 0$)

XVA by Replication III

- There is no I_C in equation (7) as we have assumed that initial margin cannot be rehypothecated.
- The growth in the cash account positions (prior to rebalancing) are

$$d\bar{\beta}_S = \delta(\gamma_S - q_S) S dt \tag{8}$$
$$d\bar{\beta}_C = -\alpha_C q_C P_C dt \tag{9}$$
$$d\bar{\beta}_X = -r_X X dt \tag{10}$$
$$d\bar{\beta}_K = -\gamma_K(t) K dt \tag{11}$$
$$d\bar{\beta}_I = r_{I_B} I_B dt - r_{I_C} I_C dt, \tag{12}$$

- where an additional cash account is now included for any return on initial margin.
- The growth in cash account associated with capital $d\bar{\beta}_K$ represents the payment of a dividend yield γ_K on the capital K borrowed from shareholders

XVA by Replication IV

- Itô's lemma on the derivative portfolio gives

$$d\hat{V} = \frac{\partial \hat{V}}{\partial t} dt + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 \hat{V}}{\partial S^2} dt + \frac{\partial \hat{V}}{\partial S} dS + \Delta \hat{V}_B dJ_B + \Delta \hat{V}_C dJ_C. \quad (13)$$

- Assuming the hedging portfolio, Π, is self-financing, we have

$$\begin{aligned} d\Pi =& \delta dS + \delta(\gamma_S - q_S) S dt + \alpha_1 dP_1 + \alpha_2 dP_2 + \alpha_C dP_C \\ & - \alpha_C q_C P_C dt - r_X X dt - \gamma_K K dt + r_{I_B} I_B dt - r_{I_C} I_C dt. \end{aligned} \quad (14)$$

XVA by Replication V

- Adding the derivative and hedging portfolio together gives,

$$d\hat{V} + d\Pi = \left[\frac{\partial \hat{V}}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 \hat{V}}{\partial S^2} + \delta(\gamma_S - q_S)S + \alpha_1 r_1 P_1 + \alpha_2 r_2 P_2 \right.$$
$$\left. + \alpha_C r_C P_C - \alpha_C q_C P_C - r_X X - \gamma_K K + r_{I_B} I_B - r_{I_C} I_C \right] dt \tag{15}$$
$$+ \epsilon_h dJ_B + \left[\delta + \frac{\partial \hat{V}}{\partial S}\right] dS + \left[g_C - \hat{V} - \alpha_C P_C\right] dJ_C,$$

- where

$$\begin{aligned} \epsilon_h &= \left[\Delta \hat{V}_B - (P - P_D)\right] \\ &= g_B - X + P_D - \phi K \end{aligned} \tag{16}$$

XVA by Replication VI

- is the hedging error on issuer default.
- Assuming replication of the derivative by the hedging portfolio, except on issuer default, gives,

$$d\hat{V} + d\Pi = 0, \tag{17}$$

- and eliminating the remaining sources of risk,

$$\delta = -\frac{\partial \hat{V}}{\partial S} \tag{18}$$

$$\alpha_C P_C = g_C - \hat{V}, \tag{19}$$

XVA by Replication VII

- leads to the PDE

$$
\begin{aligned}
0 =& \frac{\partial \hat{V}}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 \hat{V}}{\partial S^2} - (\gamma_S - q_S) S \frac{\partial \hat{V}}{\partial S} - (r + \lambda_B + \lambda_C)\hat{V} \\
& + g_C \lambda_C + g_B \lambda_B - \epsilon_h \lambda_B - s_X X - \gamma_K K + r\phi K + s_{I_B} I_B - r_{I_C} I_C \\
& \hat{V}(T, S) = H(S).
\end{aligned} \tag{20}
$$

- where the following have been used:
  - funding equation (7)
  - yield of the issued bond, $r_i = r + (1 - R_i)\lambda_B$
  - definition of ϵ_h in equation (16)
- to give,

$$
\alpha_1 r_1 P_1 + \alpha_2 r_2 P_2 = rX - rI_B - (r + \lambda_B)\hat{V} - \lambda_B(\epsilon_h - g_B) + r\phi K. \tag{21}
$$

- Note this paper assumes zero bond-CDS basis throughout.

XVA by Replication VIII

- Writing, $\hat{V}$, as the sum

$$\hat{V} = V + U \tag{22}$$

- recognising that V satisfies the Black-Scholes PDE,

$$\begin{aligned}\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - (\gamma_S - q_S) S \frac{\partial V}{\partial S} - rV =& 0 \\ V(T, S) =& 0,\end{aligned} \tag{23}$$

- gives a PDE for the valuation adjustment, U,

$$\begin{aligned}&\frac{\partial U}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 U}{\partial S^2} - (\gamma_S - q_S) S \frac{\partial U}{\partial S} - (r + \lambda_B + \lambda_C) U = \\ &\quad V\lambda_C - g_C\lambda_C + V\lambda_B - g_B\lambda_B + \epsilon_h\lambda_B + s_X X - s_{I_B} I_B + \gamma_K K - r\phi K \\ &\quad U(T, S) = 0\end{aligned} \tag{24}$$

XVA by Replication IX

- Applying Feynman-Kac gives,

$$U = \mathrm{CVA} + \mathrm{DVA} + \mathrm{FCA} + \mathrm{COLVA} + \mathrm{KVA}, \tag{25}$$

- where

$$\begin{aligned}\mathrm{CVA} = & -\int_t^T \lambda_C(u) e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))ds} \\ & \times \mathbb{E}_t \left[V(u) - g_C(V(u), X(u)) \right] du \end{aligned} \tag{26}$$

$$\begin{aligned}\mathrm{DVA} = & -\int_t^T \lambda_B(u) e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))ds} \\ & \times \mathbb{E}_t \left[V(u) - g_B(V(u), X(u)) \right] du \end{aligned} \tag{27}$$

$$\mathrm{FCA} = -\int_t^T \lambda_B(u) e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))ds} \mathbb{E}_t \left[\epsilon_{h_0}(u) \right] du \tag{28}$$

XVA by Replication X

$$
\begin{aligned}
\text{COLVA} = &- \int_t^T s_X(u) e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))ds} \mathbb{E}_t \left[X(u) \right] du \\
&+ \int_t^T s_{I_B}(u) e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))ds} \mathbb{E}_t \left[I_B(u) \right] du \\
&- \int_t^T r_{I_C}(u) e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))ds} \mathbb{E}_t \left[I_C(u) \right] du \qquad (29) \\
\text{KVA} = &- \int_t^T e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))ds} \\
&\times \mathbb{E}_t \left[(\gamma_K(u) - r(u)\phi) K(u) + \lambda_B \epsilon_{h_K}(u) \right] du. \qquad (30)
\end{aligned}
$$

- The COLVA term now contains
  - adjustment for the posted initial margin
  - any return which must be paid on received initial margin.
- KVA - Capital Valuation Adjustment appears as an integral over the capital

XVA by Replication XI

- The FCA term contains the margin funding costs as we will now demonstrate.
- Consider regular close-out + funding strategy = semi-replication with no shortfall on own-default (Burgard and Kjaer 2013).
- 2 issued bonds,
  - zero recovery bond, P_1, used to fund U
  - bond with recovery $R_2 = R_B$ +a hedge ratio given (7).
- Hence we have,

$$\alpha_1 P_1 = - U \tag{31}$$

$$\alpha_2 P_2 = - (V - \phi K - X + I_B). \tag{32}$$

- Hedge error, ϵ_h, is given by

$$\begin{aligned} \epsilon_h = & g_B + I_B - X - \phi K + R_B \alpha_2 P_2 \\ = & (1 - R_B)\left[(V - X + I_B)^+ - \phi K\right] \end{aligned} \tag{33}$$

XVA by Replication XII

- Hence we obtain the following for the valuation adjustment,

$$U = \mathrm{CVA} + \mathrm{FVA} + \mathrm{COLVA} + \mathrm{KVA} + \mathrm{MVA}, \tag{34}$$

where

$$\begin{aligned}\mathrm{CVA} = & -(1-R_C)\int_t^T \lambda_C(u) e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))ds} \\ & \times \mathbb{E}_t\left[(V(u)-X(u)-I_C(u))^+\right] du \end{aligned} \tag{35}$$

$$\mathrm{FVA} = -\int_t^T s_F(u) e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))ds} \mathbb{E}_t\left[(V(u)-X(u))\right] du \tag{36}$$

$$\begin{aligned}\mathrm{COLVA} = & -\int_t^T s_X(u) e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))ds} \mathbb{E}_t\left[X(u)\right] du \\ & -\int_t^T r_{I_C}(u) e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))ds} \mathbb{E}_t\left[I_C(u)\right] du \end{aligned} \tag{37}$$

XVA by Replication XIII

$$\mathrm{KVA} = -\int_t^T e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))ds} \mathbb{E}_t \left[K(u)(\gamma_K(u) - r_B(u)\phi) \right] du \tag{38}$$

$$\mathrm{MVA} = -\int_t^T (s_F(u) - s_{I_B}(u)) e^{-\int_t^u (r(s)+\lambda_B(s)+\lambda_C(s))ds} \mathbb{E}_t \left[I_B(u) \right] du, \tag{39}$$

- where
  - $s_F(u) = (1 - R_B)\lambda_B(u)$
  - $(V - X + I_B)^+ + (V - X + I_B)^- = (V - X + I_B)$
- As expected, the MVA takes the form of an integral over the expected initial margin profile.
- We have grouped the change to the COLVA term with MVA as both are determined by an integral over the initial margin profile.
- In the one-sided IM case (CCP) the COLVA integral over I_C is zero.

KVA Examples I

- Here are some example results to allow the impact of KVA to be assessed and compared to existing valuation adjustments.
- Valuation adjustments were calculated using numeric integration of the XVA equations.
- We choose to calculate the case of *semi-replication with no shortfall at own default* as described above.

KVA Examples II

- Assuming no initial margin the XVA becomes with regular bilateral closeouts

$$\mathrm{CVA} = -(1-R_C)\int_t^T \lambda_C(u) e^{-\int_t^u (\lambda_B(s)+\lambda_C(s))ds} \mathbb{E}_t\left[e^{-\int_t^u r(s)ds}(V(u))^+\right] du \tag{40}$$

$$\mathrm{DVA} = -(1-R_B)\int_t^T \lambda_B(u) e^{-\int_t^u (\lambda_B(s)+\lambda_C(s))ds} \mathbb{E}_t\left[e^{-\int_t^u r(s)}ds(V(u))^-\right] du \tag{41}$$

$$\mathrm{FCA} = -(1-R_B)\int_t^T \lambda_B(u) e^{-\int_t^u (\lambda_B(s)+\lambda_C(s))ds} \mathbb{E}_t\left[e^{-\int_t^u r(s)ds}(V(u))^+\right] du \tag{42}$$

$$\mathrm{KVA} = -\int_t^T e^{-\int_t^u (\lambda_B(s)+\lambda_C(s))ds} \mathbb{E}_t\left[e^{-\int_t^u r(s)ds} K(u)(\gamma_K(u) - r_B(u)\phi)\right] du, \tag{43}$$

KVA Examples III

- Since we consider an interest rate swap, interest rates are now assumed to be stochastic so rates now appear inside expectations (The derivation follows the same steps as for derivatives based on stocks.)

Setup

- Market Risk: standardized approach with current exposure method for EAD
- CCR: standardized approach with external ratings
- CVA: large numbers of counterparties approximation. The use of the standardized approaches avoids the complexity and bespoke nature of internal model methods.
- Issuer holds the minimum capital ratio requirement of 10.5% (including minimum capital and capital buffer requirements) and that the issuer cost of capital is 10%.

Trade

- Single 10 year GBP interest rate swap with semi-annual payment schedules in both directions.
- Fixed rate on the swap is 2.7% ensuring the unadjusted value is zero at trade inception.
- Issuer spread is flat 100bp across all maturities and the issuer recovery rate is assumed to be 40%.

Counterparties

Counterparty Rating	bp	Standardized Risk Weight
AAA	30	20%
A	75	50%
BB	250	100%
CCC	750	150%

- Counterparty recovery rate 40%.

XVA on Single IR Swap - Pay Fixed

						KVA				
ϕ	**Swap**	**Rating**	**CVA**	**DVA**	**FCA**	**MR**	**CCR**	**CVA**	**Total**	**IR01**
0	Pay	AAA	-4	39	-14	-262	-3	-9	-253	9.5
0	Pay	A	-10	38	-14	-256	-8	-10	-259	9.6
0	Pay	BB	-31	33	-12	-234	-14	-22	-279	10.0
0	Pay	CCC	-68	24	-9	-185	-16	-87	-342	11.3
1	Pay	AAA	-4	39	-14	-184	-2	-6	-170	9.5
1	Pay	A	-10	38	-14	-180	-4	-7	-176	9.6
1	Pay	BB	-31	33	-12	-166	-7	-16	-198	9.9
1	Pay	CCC	-68	24	-9	-134	-8	-63	-260	11.0

- Setting aside the Market Risk component of the capital we see that KVA from CCR and CVA terms gives an adjustment of similar magnitude to the existing CVA, DVA and FCA terms, demonstrating that KVA is a significant contributor to the price of the derivative.
- Market risk is assumed to be unhedged and so this KVA component is relatively large compared to the CCR and CVA terms.
- Under the standardized approach to market risk the capital requirement on a ten year transaction of this type is scaled according to a 60 bp move in rates.

- When pricing derivatives it is no longer sufficient to look at the impact of just the new trade, the impact of the trade and all hedging transactions should be considered.
- The hedge trades will themselves create additional capital requirements, although they may also mitigate other capital requirements.
- Consider a ten year interest rate swap traded with a corporate client on an unsecured basis. This trade has market risk, counterparty credit risk and CVA capital requirements associated with it.
- To hedge the market risk the trading desk enters another ten year swap with a market counterparty on a collateralised basis.
- This hedge trade generates a small amount of counterparty credit risk and CVA capital but reduces the market risk capita to zero as it is back-to-back with the client trade.

XVA on Interest Rate Swap with Back-to-Back Hedge - Pay Fixed

						KVA				
ϕ	**Swap**	**Rating**	**CVA**	**DVA**	**FCA**	**MR**	**CCR**	**CVA**	**Total**	**IR01**
0	Pay	AAA	-4	39	-14	0	-3	-9	9	0.6
0	Pay	A	-10	38	-14	0	-8	-10	-3	0.7
0	Pay	BB	-31	33	-12	0	-14	-22	-45	1.1
0	Pay	CCC	-68	24	-9	0	-16	-87	-156	2.4
1	Pay	AAA	-4	39	-14	0	-2	-6	13	0.6
1	Pay	A	-10	38	-14	0	-4	-7	3	0.7
1	Pay	BB	-31	33	-12	0	-7	-16	-32	1.0
1	Pay	CCC	-68	24	-9	0	-8	-63	-125	2.1

- KVA itself, like CVA and FVA, has market risk sensitivities.
- The CCR term, for example, is clearly driven by the EAD and hence by the exposure to the counterparty. Capital requirements go up as exposures rise irrespective to any impact on credit quality.
- Reducing the IR01 of the trade and adjustments to zero yields the following results
- However, Market Risk Capital is no longer zero as trade and hedge are not back-to-back.

XVA on Interest Rate Swap - $IR01 = 0$ - Pay Fixed

						KVA					
ϕ	**Swap**	**Rating**	**CVA**	**DVA**	**FCA**	**MR**	**CCR**	**CVA**	**Total**	**IR01**	**Hedge Change (%)**
0	Pay	AAA	-4	39	-14	-17	-4	-12	-13	0	7
0	Pay	A	-10	38	-14	-20	-11	-13	-30	0	8
0	Pay	BB	-31	33	-12	-28	-20	-31	-88	0	12
0	Pay	CCC	-68	24	-9	-45	-22	-127	-249	0	24
1	Pay	AAA	-4	39	-14	-12	-3	-8	-1	0	6
1	Pay	A	-10	38	-14	-13	-5	-9	-14	0	7
1	Pay	BB	-31	33	-12	-18	-9	-23	-59	0	11
1	Pay	CCC	-68	24	-9	-29	-12	-92	-187	0	21

Impact of KVA

- KVA model = risk-neutral approach to pricing the cost of capital.
- Clear that KVA really applies at Portfolio Level
- 'Hedging' KVA will involve optimization / iteration as hedge trades will *change* KVA
- Portfolio Level Effects - Leverage Ratio
- KVA has other impacts:
  - 'Exit Prices'
  - Bank resolution planning - large portfolios = large leverage ratio impact

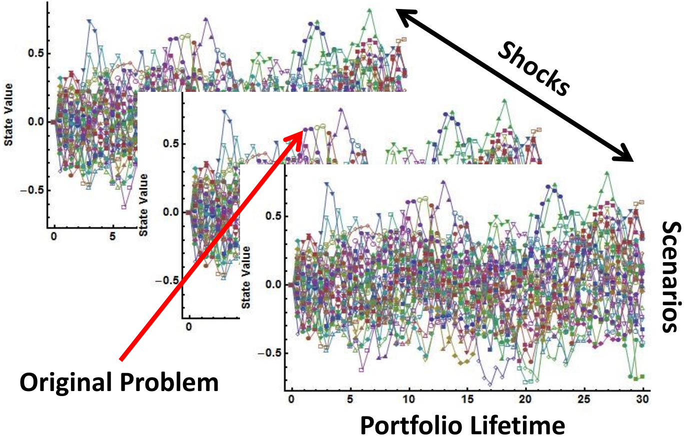
Scaling the Problem

- 10^4 trades per CCP Netting set
- 10^3 Monte Carlo paths
- 10^2 Observation points
- $10^2 - 10^3$ Scenarios for VAR

$$10^4 \times 10^3 \times 10^2 \times 10^3 = 10^{12}$$

valuations...

Problem Scale — Second Look



VAR and the Risk-Neutral Measure

- VAR is most commonly calculated using a historical simulation approach and hence the VAR scenarios that are generated lie in the real world measure.
- From equation (39) it is clear that to proceed we need to apply these shocks inside a risk-neutral Monte Carlo simulation.
- Here we choose to assume that the VAR shocks are exogenously supplied and that they do not change during the lifetime of the portfolio.
- Allowing the shocks to change inside the risk-neutral Monte Carlo would mean combining the $\mathbb{P}$ and $\mathbb{Q}$ measures.

Longstaff-Schwartz Augmented Compression 1/2

- To make VAR and CVAR calculations efficient the revaluation of the portfolio needs to be very fast.
- Longstaff-Schwartz regression functions - simply a polynomial in the explanatory variables $O_i(\omega, t_k)$ and hence very fast.
- Approximate value of the portfolio in each scenarios using regression functions with the explanatory variables calculated using the shocked rates,

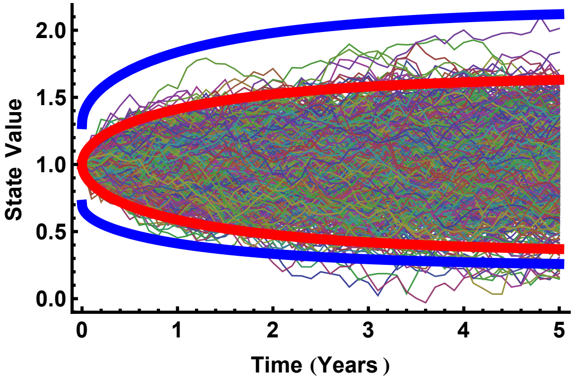
$$V_q^{\text{I}} \approx \bar{F}(\alpha_m, O_i(\bar{y}_q^a, t_k), t_k). \tag{44}$$

- We apply the Longstaff-Schwartz approach to all derivative products from vanilla linear products to more complex exotic structures.
- Resulting regressions is also a compression technique.
- A single polynomial is used to replace the entire portfolio valuation.
- *Longstaff-Schwartz Augmented Compression* provides a portfolio valuation cost that is constant and independent of portfolio size.
- Of course the regression phase of the calculation will itself be a function of the number of cash flows in the portfolio but our results show that the computational costs is independent of portfolio size in practice.

Longstaff-Schwartz Augmented Compression 2/2

- Longstaff-Schwartz, in its original form, requires augmentation for the VAR and CVAR calculations because:
  - At $t = 0$ portfolio NPV has exactly 1 value, so regression is impossible.
  - For $t > 0$ the state region explored by the state factor dynamics is much smaller than the region explored by VAR shocks.
- Consider with a simple example where our model is driven by an Orstein-Uhlenbeck process (i.e. mean reverting), $dx = \eta(\mu - x)dt + \sigma dW, \ x(0) = x0$, where W is the driving Weiner process.
- Slide 1 shows the analysis of the state space with 1024 paths.
- It is clear that the 1024 paths shown do not cover the state space required by VAR calculation when the VAR shocks can give a shift of up to 30% on a relative basis.
- This magnitude of relative VAR shock was found in the 5-year time series used in the numerical examples presented below.

Analysis of state space for an Orstein-Uhlenbeck process



Figure

Early Start Monte Carlo

There are two augmentation methods that we can apply, *early start Monte Carlo* and *shocked state augmentation*.

- Early Start Monte Carlo - starts the Monte Carlo simulation earlier than today so that enough Monte Carlo paths are present in the region required to obtain accurate regression results for VAR shocks.
- (Wang and Caflish 2009) suggested the use of early start Monte Carlo in order to obtain sensitivities.
- The advantage of the early start Monte Carlo is that it preserves path-continuity.
  - This is needed for portfolios which contain American or Bermudan style exercises.
  - For such products a continuation value must be compared with an exercise value to obtain the correct valuation during the backward induction step.
- Given the portfolios we will consider below contain only vanilla instruments we will not apply the early-start approach in this paper.

Shocked-State Augmentation 1/5

- When portfolios do not contain American or Bermudan style exercise we can use a simpler method to calculate regression functions giving portfolio values.
  - Typically the case for CCPs.
- We call this approach Shocked-State Augmentation.
- Simpler because it does not have to preserve path-continuity of prices, as no backwards-induction step is required.
- The regression at each stopping date is independent of of all other regressions $\Rightarrow$ parallel computation.
- Objective is to have portfolio regressions that are accurate over the range of the state space relevant for calculation of VAR.

Shocked-State Augmentation 2/5

- The dimensionality of the state space *for VAR* at any stopping date on any path is given by the dimensionality of a VAR shock
- One VAR shock, for example, for a single interest rate may be described by 18 numbers giving relative movements of the zero yield curve at different tenors, and hence be 18-dimensional.
- The driving dimensionality is defined by the space explored by the VAR shocks.
    - Usually this will be larger than the dimensionality of the simulation model.
    - This high-dimensionality must be explored by augmenting the state space.
- At each stopping date the portfolio price is calculated as the sum of the component trades on each path.
- If there are m paths then this gives m values to fit.
- We can chose anything for the state variables (swaps and annuities in the example), and have as many as we like.
- We fit a regression connecting the portfolio value to the state variable values. Usually $m << n$ where n is the dimensionality of a VAR shock so the problem is over determined.
- We use a least-squares fit, so larger fitting errors are relatively highly penalized, under the assumption that these are more likely to occur with more extreme scenarios.

Shocked-State Augmentation 3/5

We follow a parsimonious state augmentation strategy, that is complete at t=0. We assume that there are more simulation paths than VAR shocks.

> **Shocked-State Augmentation:** *apply one VAR shock at each stopping date, on each path.*

This strategy is parsimonious because we do not require any extra simulation paths, and because we use the same number of computations as for a usual simulation (apart from computing the effect of the shocks on the simulated data of course).

Shocked-State Augmentation 4/5

- Strategy is complete at t=0 in that we are certain to cover the full range required by VAR (as we have assumed more simulation paths than VAR shocks).
- Automatic as we use the VAR shocks themselves to expand the state space.
- Hence certain that all VAR computations will be within the range over which the regressions were calibrated.
- Shocked-State Augmentation is the most parsimonious strategy in that it uses one VAR shock on each stopping date per simulation path.
  - In this version of Shocked-State Augmentation we pick the VAR shocks sequentially for each path at each stopping date, So at t=1, say, path 1 uses shock 1, path 2 uses shock 2, etc.
  - As we have more paths than shocks we will use some shocks multiple times.

Shocked-State Augmentation 5/5

- Not interested in average effects of shocks — VAR is an extreme result of the shocks on the portfolio.
- Shocks cover a range of sizes (up to say 30% relative) it is not obvious which direction in the n-dimensional space (defined by VAR shock dimensionality) will have the biggest effect on the portfolio.
- Equally this is why we cannot simply pick the largest component of each of the VAR shocks and use this to expand the state space.
- Although a shock defined as the maximum component of each shock would be large, we cannot say whether it is in the direction which changes the portfolio the most in n-dimensional space, for that magnitude of shock.
- In Shocked-State Augmentation the shocks are applied exactly as they would be for computing VAR.
    - In our experiments interest rates VAR shocks are multiplicative shocks on zero yield curve tenor points.
    - They are applied in Shocked-State Augmentation just as they would be for VAR to create a new market data state (at each particular stopping date on each path).

MVA: Numerical Examples

- We calculate MVA on a series of portfolios of US Dollar interest rate swaps.
- We also calculate the FVA that would apply to the same portfolio if it were unsecured in order to provide a reference calculation to assess the impact of the MVA.
- We assume the use of the following initial margin methodology,
  - 99% one-sided VAR;
  - 10-day overlapping moves;
  - 5-year window including a period of significant stress. Our portfolio consists of IRS so a suitable period starts January 2007.
  - The 5-year window means that there were 1294 shocks.
- Each VAR shock was a change to the zero yield curve.
  - Each VAR shock defined at 18 maturities: 0, 0.5 ,1 ,2 ,3 ,4 ,5 ,6 ,7 ,8 , 9, 10, 11, 12, 15, 20, 25, 30 years.
  - Shocks are relative changes to zero yields, so given a zero yield r at T and a relative shock s the resulting discount factor is: $e^{-rT(1+s)}$.
  - Linear interpolation in yield between shock maturities.

- Test portfolios have n swaps with maturities ranging up to 30 years, and each swap has the following properties,
  - n swaps with maturity $i \times \frac{30}{n}$ where $i = 1, \dots, n$
  - notional = USD100M $\times(0.5 + x)$ where $x \sim$U(0,1)
  - strike = $K \times (y + x)$ where $x \sim$U(0,1), $K = 2.5\%$. $y = 1$ usually, or $y = 1.455$ for the special case where we balance positive and negative exposures.
  - gearing = $(0.5 + x)$ where $x \sim$U(0,1)
  - P[payer] = $\{90\%, 50\%, 10\%\}$
- All swaps have standard market conventions for the USD market. For $n = 1000$ the expected exposure profile of the portfolio is illustrated in Figure 2. We use $n = \{50,\ 100,\ 1000,\ 10000\}$ in our examples.
- The parameters MVA parameters used in the examples are as follows:
  - $\lambda_C = 0$ i.e. we assume that the issuer is facing a risk-free counterparty.
  - $\lambda_B = 167bp$
  - $R_B = 40\%$
  - $s_I = 0$

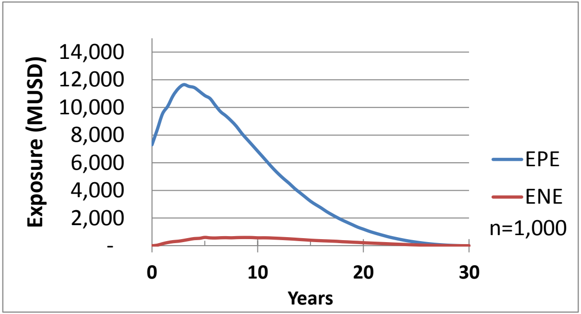
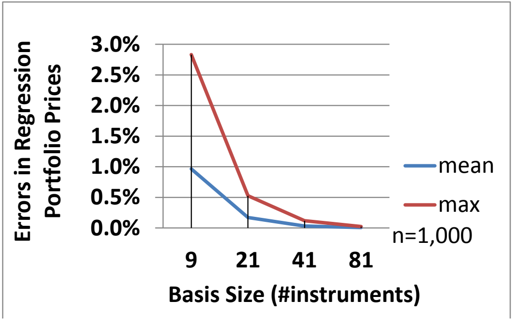


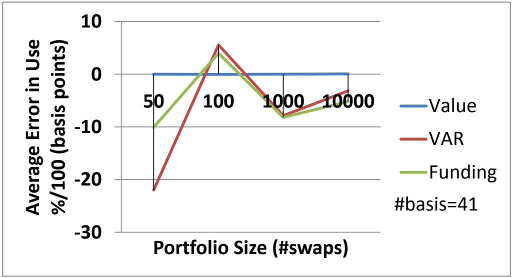
Figure: The expected positive exposure (EPE, blue) and expected negative exposure (ENE, red) when $n = 1000$ and P[payer] $= 90\%$.

Accuracy of Regression



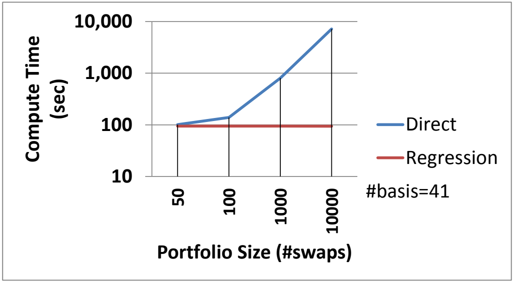
- Very good with just a few basis functions.
- Fewer basis functions are used at later times (because some of them have matured)

Accuracy of VAR and ES



- Better than pricing accuracy because of averaging effects — despite the extreme nature of VAR and ES.

Speed



- ES and VAR calculations take most of the time (more for larger problems)
- Regression is a constant-time algorithm
- Speedup increases with larger problem size, reaching x100 for medium-sized swap portfolio (10,000 swaps).

MVA Costs

Portfolio P[payer]	FVA bps of notional	MVA bps of notional
90%	115	53
50%	0	2
10%	-113	56

Table: FVA and MVA costs relative to each other for the three example portfolios. Here FVA = FCA + DVA is a symmetric approach to FVA costs and benefits.

Table 1 shows the MVA cost for the three portfolios and compares this to the FVA. The MVA is close to 50% of the cost of the FVA on the unsecured portfolio, and hence a significant valuation adjustment.

Clearing: XVA vs MVA

- The impact of clearing a portfolio of trades can be clearly seen now through XVA
- Cleared trades involve initial margin so MVA will be non-zero
- The impact on the other XVAs will depend on the state prior to clearing
- If no IM - C&FVA will likely reduce
- KVA
  - Risk weight of CCP lower
  - But leverage ratio impact less clear
- Key Issue: Clearing decision involves optimization over XVA
- Key Issue: CCP 'value' is the reference value for margin *not* the economic value

KVA, MVA and Accounting

- Do KVA and MVA have accounting impact?
- FVA now a part of accounting practice - 14 major banks now have FVA reserves
  - FVA not in current Accounting Standards
  - FVA included on basis it is in the 'exit price'
- Not clear what will happen to other valuation adjustments but see (Kenyon and Kenyon 2015).

Conclusions

- Introduced KVA & MVA
- Derived a risk-neutral model for KVA & MVA
- Considered the numerical / market impact of KVA
- Provided a numerical scheme to allow calculation of MVA
- Assessed impact of MVA

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Thanks for your attention — questions?

Forthcoming later this year: