Counterparty Valuation Adjustments

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Need for CVA

We live in an increasingly risky world.

- Bank failures.
- Global recession.
- Libor rates far from the "risk free" rate.

A year ago, swaps traded without counterparty risk being taken into account. Today, the counterparty could be a low quality bank.

Size

Despite the financial crisis, the OTC derivatives markets continue to be a big part of the market, and interest rate derivatives are the largest part of the OTC derivatives market.

- OTC derivatives notional outstanding
  - $547 trillion in December 2008
  - 70% in interest rate derivatives
  - $605 trillion in June 2009
- OTC derivatives gross market value
  - June 2008 — $20 trillion
  - December 2008 — $32 trillion — up 60%
- IRD gross market value
  - June 2008 — $9 trillion
  - December 2008 — $18 trillion — doubled in 6 months.
Demand

There is much demand for managing counterparty risk

- Accounting standards — FASB 157 (now "Topic 820, Section 10") and IAS 39 — credit risk must be taken into account.
- Regulators.
- Risk managers.

The IASB even issued a request for comment on counterparty risk calculation methodologies.

Counterparty risk

Counterparty risk — the exposure to loss due to a specific counterparty failing to meet contractual obligations, i.e. defaulting.

Bond counterparty risk

Example — Investor is long a bond.

- Counterparty is issuer.
- Issuer defaults — investor loses bond cashflows, but gets recovery on the bond.
- Recovery is a percentage of the principal of the bond.
- Default causes loss of interest payments, and early (but partial) return of principal.
- Could be an improvement if bond is trading at a sufficient discount.

Example — Investor is short a bond.

- No counterparty risk.

Swap counterparty risk

Example — Investor enters into a swap.

- Counterparty is the entity with which the swap was transacted.
- Investor is simultaneously long one leg of the swap, and short the other leg.
- Counterparty defaults — investor loses swap cashflows, but gets recovery on the swap.
- Recovery is a percentage of the market value of the swap.
- If swap value is positive then there’s a loss.
- If swap value is negative, then there is no loss.
**Counterparty Valuation Adjustments**

**Risk modifications**

Netting agreements:
- Optional part of the ISDA master agreement.
- In the event of default, recovery is on the net market value of all contracts covered by the agreement.

No netting agreement:
- Investor owns two 5 year 5% swaps to counterparty — one is pay fixed, the other is receive fixed.
- No net market exposure — the two positions cancel out.
- Substantial counterparty exposure:
  - Get recovery on the market value of the positive swap
  - Still owe full value on the market value of the negative swap

With netting agreements, exposure at any given time is on the net market value of all securities covered. Risk is reduced.

**Collateralization**

- ISDA credit support annex (CSA).
- At the end of the period (day, week, etc), if swap value exceeds a threshold, collateral must be posted.
- Exposure is to the threshold plus the movement of the market value over the period.

With collateralization, counterparty risk is pretty minimal.

**Counterparty risk — long only vs long/short**

Counterparty risk calculations are far more complicated for instruments that are a combination of long/short positions than for long only instruments.

- In a long only instrument, (like a bond position), counterparty risk can be judged by using models that can incorporate a discount curve shift.
- In a long/short instrument, (like a swap position), the instrument can potentially be an asset or a liability. When it’s an asset, default results in a loss. When it’s a liability, default results in no change.

How does the counterparty exposure and the risk of default impact the value of the security?

- The Credit Valuation Adjustment (CVA) is the cost of the potential loss.
- Risk free price - CVA = price of risky security.
Bond counterparty risk

If a bond with a coupon of \( C \) pays \( f \) times per year at times \( t_i \), with maturity \( t_n \), the value of the bond is:

\[
\sum_{i=1}^{n} \frac{C}{f} D(t_i) S(t_i) + 100D(t_n) S(t_n) + \int_{t_n}^{t} 100RD(t) P(t) dt,
\]

- \( P(t) \) — default probability density function.
- \( S(t) = 1 - \int_{0}^{t} P(s) ds \) — survival probability for time \( t \) (the probability of no default before time \( t \)).
- \( R \) — bond recovery rate.
- \( D(t) \) — risk free discount factor for time \( t \).

This is the standard CDS model applied to a fixed coupon bond. Note that it assumes independence of rates and default.

Equivalent par curve example

On the Bloomberg terminal, we do this calculation in YASN - the structured notes calculation screen.

Equivalent par curve

To get a feel for the impact of credit spreads on bond values, we can compute the par curve for the risky bond. For \( t_i = i/f \), and for each \( n \), solve for \( C(t_n) \) such that

\[
100 = \sum_{i=1}^{n} \frac{C(t_i)}{f} D(t_i) S(t_i) + 100D(t_n) S(t_n) + \int_{t_n}^{t} 100RD(t) P(t) dt,
\]

Then \( C(t_n) \) is the implied par curve — the coupons that the issuer with this CDS spread curve would theoretically use to issue debt at par.

Recovery rate impact

Flat curves and equal recovery rates give an equivalent par curve roughly equal to the swap curve shifted by the CDS spreads. Otherwise, there can be significant differences, as we see here for a flat 100bp CDS curve and a flat 3% swap curve.
Par curves vs default probabilities

Once the risky par curve is computed, one is tempted to strip it and use it for discounting.

Good points:
- This is simple, straight forward, and in line with common practices.
- This will properly price par bonds back to par.
- If the bond recovery rate is zero, this properly prices all bonds!

In the zero recovery rate case, this makes the risky discount factors

$$D(t_i)S(t_i),$$

so the risky spot rate curve $\bar{R}$ is given by

$$\bar{R}(t_i) = -\log(D(t_i)S(t_i))/t_i = R(t_i) - \log(S(t_i))/t_i,$$

where $R$ is the risk free rate. So, the survival probabilities add a spread of $-\log(S(t_i))/t_i$ (the average hazard rate) to the risk free rate. This spread is roughly the CDS spread, adjusted by the CDS recovery rate.

Par curves and OAS

If the spread curve is flat, it roughly amounts to a shift of the par curve, which is roughly adding an OAS.

Using the risky par curve in such a calculation is an improvement, in that it factors in the shape of the CDS spread curve.

Advanced risky bond calculations

Once we have a risky par curve, we can use it to apply risky spreads on top of a risk free interest rate derivatives models to analyze risky bonds with embedded options.
Counterparty risk in swaps — Characteristics

Consider a five year swap in a flat interest rate environment. There is volatility dependent risk:
- Zero volatility:
  - Swap is always nearly zero market value.
  - Minimal default risk.
- High volatility:
  - Swap value in future can be substantial.
  - Potentially substantial loss upon default.

There is curve dependent risk:
- In a steep interest rate environment, the swap is expected to be heavily off the money for the duration of its life.
- Far more counterparty risk.

Valuation

Let $V(t)$ be the value of the risk-free swap at time $t$, and let $R$ be the recovery rate on the underlying swap. If the counterparty defaults at time $t$, the payoff for holding the swap is:
- If $V(t) > 0$ we get $R \times V(t)$.
- If $V(t) < 0$ we still owe $V(t)$.

The above payoff is $R \max(V(t), 0) + \min(V(t), 0)$

The loss at default time is:
$$ (1 - R) \max(V(t), 0) $$

Since this is $(1 - R)$ times the payoff of the swaption maturing at time $t$, if we assume independence of rates and default, then the current value of the loss conditional on default at time $t$ is:
$$ (1 - R)S(t) $$

where $S(t)$ is the current value of a swaption to enter into the remainder of the swap at time $t$. Summing over the possible default times weighted by the default probabilities gives us the CVA adjustment (the difference between the value of the swap to a riskless counterparty and the risky counterparty) as:
$$ CVA = \int (1 - R)S(t)P(t)dt $$

where $P$ is the default probability density.

CVA subtleties

CVA valuation can be a little subtle. The swaption to enter into the tail of the swap is slightly different from the usual swaption to enter into a swap.

Swap cashflows:
CVA subtleties

When exercising a swaption to enter into a swap with an odd first coupon, the first coupon is adjusted — the floating leg references a shortened index, and the fixed leg only accrues over the remainder of the period.

However, in the event of default at that time, the full cashflows are lost.

Floating Cashflows

Fixed Cashflows

Exercise

Time

Forward swap rates

Consider the pay fixed swap with fixed rate \( F \) that the holder of a swaption would receive on exercise (at time \( t \)).

Let the underlying floating (fixed) leg pay at times \( t_i (t'_i) \). Then the time \( t \) value of the swap is

\[
S(t) = \sum L(t, t_i, t_{i+1}) Z(t, t_{i+1}) \alpha_i - \sum F \alpha'_i Z(t, t'_i),
\]

Since \( t \leq t_1 \),

\[
L(t, t_i, t_{i+1}) = (1/\alpha''_i)(Z(t, t_i)/Z(t, t_{i+1}) - 1)
\]

(with Libor accrual factor \( \alpha''_i \)), so if we assume \( \alpha_i = \alpha''_i \), then \( S(t) \) can be written in terms of \( Z \) as:

\[
S(t) = Z(t, t_1) - Z(t, t_n) - \sum F \alpha'_i Z(t, t'_i),
\]

This gives us the standard expression for the time \( t \) value of a forward start swap.

\[L(w, x, y)\] is the time \( w \) forward Libor rate setting at time \( x \) and paying at time \( y \), \( Z(x, y) \) be the time \( x \) price of a zero coupon bond paying at time \( y \), and \( \alpha_i \) are the accrual fractions for floating (fixed) payment periods \( t_i \) to \( t_{i+1} \) (\( t'_i \) to \( t'_{i+1} \)), respectively.

Tail swap rate

The tail of the existing swap is slightly different. Its cashflows and accruals are the same as for the forward swap, except for the first period, where the reset date \( (t_1) \) is earlier than the valuation time \( t \), and the accrual factors correspond to the full period rather than the partial period. The time \( t \) value of the first floating cashflow is

\[
\frac{Z(t, t_2)}{Z(t_1, t_2)} - Z(t, t_2),
\]

so the value of the tail is

\[
S(t) = \frac{Z(t, t_2)}{Z(t_1, t_2)} - Z(t, t_2) + \sum L(t, t_i, t_{i+1}) Z(t, t_{i+1}) \alpha_i
\]

\[
-F \alpha'_i Z(t, t_1) - \sum F \alpha'_i Z(t, t'_i)
\]

\[
= \frac{Z(t, t_2)}{Z(t_1, t_2)} - Z(t, t_n) - F \alpha'_i Z(t, t_1) - \sum F \alpha'_i Z(t, t'_i).
\]
Forward swaps and tail swaps compared

The difference between the values of the two is

\[ \bar{S}(t) - S(t) = Z(t, t_2) \frac{Z(t_1, t_2)}{Z(t_1, t_2)} - Z(t, t_1) - F(\alpha_1' - \alpha_1')Z(t, t_1). \]

This is the adjustment that needs to be made to convert the swaption payoff to the payoff of the option on the tail swap.

Counterparty risk in swaps — CVA<Go>

On the Bloomberg, the CVA function does this calculation.

Visualizing counterparty risk

The calculation and risks can be better visualized by exposure graphs over time:

Forward swap vs tail

Under the equivalent martingale measure with respect to numeraire \( N \), the time zero value of the forward start swaption is

\[ N(0)E_0[\max(S(t), 0)/N(t)] \]

Instead of this, we need the value of the option on the tail of the swap. Concentrating on the payoff, if

\[ D = \bar{S}(t) - S(t) = \frac{Z(t, t_2)}{Z(\bar{t}_1, t_2)} - \frac{Z(t, t_1)}{Z(\bar{t}_1, t_2)} - \frac{F(\alpha_1' - \alpha_1')Z(t, t_1)}{Z(t, t_1)} \]

we have

\[ \max(\bar{S}(t), 0) = \max(S(t) + D, 0) = \max(S(t), -D) + D \]

So the option to enter into the tail of the swap is the same as the option to enter into the swaption with an associated fee of \( S(t) - \bar{S}(t) \) with an additional cash component of \( \bar{S}(t) - S(t) \). One could apply a convexity adjustment technique to work this out, but it replacing \( Z(t, t_1) \) and \( Z(t, \bar{t}_1) \) with their corresponding time zero forward values is a reasonable approximation and it makes these adjustments fixed values and thus easily handled.
CCDS

Contingent credit default swaps are a variant of credit default swaps. With both CDS and CCDS, the buyer of insurance pays the spread until default or maturity (whichever comes first), and receives \((1 - R)\) of something at default time.

What makes the two contracts substantially different is what exactly is being made whole.

- **CDS** — Receives \((1 - R)\) on the principal of a reference security.
- **CCDS** — Receives \((1 - R)\) on the value of a reference security.

With a CDS contract, the only uncertainty regarding the former’s payout given default is the recovery rate, because recovery is on a fixed principal amount.

With a CCDS contract, there is uncertainty on the value on which recovery is applied as well.

CCDS and CVA

Consider a CCDS contract on an interest rate swap.

- The payoff of the floating leg of the CCDS contract is the loss given default for the interest rate swap.
- The value of the floating leg is then the CVA value.
- The fixed leg of the CCDS contract must then be the annuitized (to default time) value of the CVA.

The CCDS contract is thus the perfect hedge for the counterparty exposure on the interest rate swap, except that

- CCDS contracts are thought to be expensive forms in which to buy this protection.
- CCDS contracts have counterparty risk too!

Any hedge that would be used for the counterparty risk in a swap can be adjusted to serve as the replicating strategy for the CCDS contract (except for the counterparty exposure of the CCDS).

Hedging issues

Hedging counterparty risk can be difficult. The natural hedge:

- Take on a position in CDS that neutralizes exposure to credit moves.
- This locks in the value of the CVA.

But,

- Dynamically hedging with CDS can be expensive.
- Where can you get a risk free swap?
- What about the CVA of the CDS for that matter?
- How do you neutralize credit exposure and default exposure at the same time?

Portfolio counterparty risk

In the presence of netting agreements, the counterparty exposure must be computed on the portfolio of securities covered by the netting agreement.

- Loss given default is no longer the sum of the losses in the individual positions.
- Loss given default is \((1 - R)\) of the payoff of a call on the underlying portfolio.
Calculating portfolio counterparty risk

Calculation options:

- Feed call options on the portfolio to your defaultable interest rate derivatives valuation system.
- Compute value of appropriate call options on the portfolio via your interest rate derivatives valuation system, and proceed as above.
- Make some assumptions and get formulas.

Brigo and Massimo take the latter approach.

- Net all of the interest rate swaps covered by a given netting agreement.
- The floating “leg” of the portfolio is now some sort of amortizing floating leg with time dependent leverage.
- The fixed “leg” is now some sort of amortizing step coupon fixed leg.

Difficulties ensue because the leverage could be positive or negative (i.e. - at some times the aggregate can be a payer swap while at other times it can be a receiver swap).

Portfolio woes

Consider a portfolio consisting of an at the money 5 year payer swap with fixed rate $F_1$, and an at the money 3 year receiver swap with fixed rate $F_2$.

- Credit exposure 1 year out is to the difference between the value of the two tails being positive.
- Similar to the difference between the 4 year swap rate and the 2 year swap rate.

The portfolio behaves roughly like a spread between two rates, so the credit exposure is like a spread option, and thus, difficult to price.

Accounting considerations

Accounting for counterparty risk is required by accounting standards FASB 157 (US) and IAS 39 (Europe).

FASB 157, Appendix B, Paragraph 5:

- B5. Risk-averse market participants generally seek compensation for bearing the uncertainty inherent in the cash flows of an asset or liability (risk premium). A fair value measurement should include a risk premium reflecting the amount market participants would demand because of the risk (uncertainty) in the cash flows.
Accounting considerations

Accounting rules are currently being updated and globalized. Topic 820, Section 10 replaces FASB 157, but it strengthens the position that credit risk must be accounted for:

- 55-8. A fair value measurement should include a risk premium reflecting the amount market participants would demand because of the risk (uncertainty) in the cash flows. Otherwise, the measurement would not faithfully represent fair value. In some cases, determining the appropriate risk premium might be difficult. However, the degree of difficulty alone is not a sufficient basis on which to exclude a risk adjustment.

While accurate valuation of the embedded default risk is preferred, a number of alternative approaches have traditionally been accepted.

Shifty calculations

One alternative is the discount shift method. The CVA is calculated by shifting the discount curve by the credit spread (like in the bond approach).

When all of the swaptions are in the money, this is the zero volatility version of the CVA. For a 5 year 5% receiver swap, ignoring volatility can lead to errors of 15% to 20%.

Table: 5% 5 year receiver swap, 10 million notional, with market data yielding swap rate of 3.16%

<table>
<thead>
<tr>
<th>CDS rate</th>
<th>CVA</th>
<th>Discount Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>17,610</td>
<td>15,227</td>
</tr>
<tr>
<td>200</td>
<td>35,751</td>
<td>29,970</td>
</tr>
<tr>
<td>300</td>
<td>52,950</td>
<td>44,249</td>
</tr>
<tr>
<td>400</td>
<td>69,181</td>
<td>58,085</td>
</tr>
</tbody>
</table>

The errors from the shift approximation are illustrated by the results on an at the money swap.

Table: At the money 5 year receiver swap, 10 million notional, with market data

<table>
<thead>
<tr>
<th>CDS rate</th>
<th>CVA</th>
<th>Discount Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>11,852</td>
<td>7,348</td>
</tr>
<tr>
<td>200</td>
<td>22,870</td>
<td>14,316</td>
</tr>
<tr>
<td>300</td>
<td>33,108</td>
<td>20,923</td>
</tr>
<tr>
<td>400</td>
<td>42,620</td>
<td>27,189</td>
</tr>
</tbody>
</table>

In general:

- Errors grow as relevant swaptions go out of the money.
- Large errors when swaptions are close to the money (so that volatility plays a large part).
- Cannot be used for out of the money swaps — CVA is negative.
- Same typically holds for pay fixed swaps.

Shifty calculations

To address the problem of swaps that are a liability, the discount method is modified.

- Positions of positive value:
  - Treat as assets.
  - Reduce value according to counterparty risk (by discount curve shift).
- Positions of negative value:
  - Treat as liabilities.
  - Increase value (reduce liability) according to investor’s credit!

Nicely symmetric and based on the theory that the money is owed to the counterparty and only a fraction of it will be paid if the investor defaults.

But adds problems of bilateral CVA.
Net current exposure

Another approach — the current net exposure method.

- Net current exposure = value at immediate risk of default to investor = max(current market value, 0).
- Cost of the risk of immediate default = cost of insuring against default via CDS on counterparty’s default.
- CDS notional = net current exposure.
- CDS maturity = some measure of life of deal (maturity or duration).
- CVA = Cost of insurance = cost of fixed leg of CDS contract ≈ the cost of the CDS spread applied over the life of the CDS contract.

The appeal of the net current exposure method lies largely in its ease of implementation.

- If you can value the positions and get the CDS spreads, you can easily compute the CVA.
- Easily extended to portfolios and to take netting agreements into account.
- Easily extended to take collateralization into account — reduce the current net exposure to the threshold level (the level beyond which the position must be collateralized).
- Easily extended into a type of bilateral CVA — do as above if position is positive, and do the reverse (impact on counterparty of one’s own default) if negative.

Net current exposure

The negative of this approach comes from its inaccuracy. Consider the CDS position needed to insure the portfolio to maturity.

- Forward value of the portfolio is not constant.
- Improve hedge by using a portfolio of CDS so that the forward market values of the portfolio are matched.
- This CDS portfolio hedges against default risk assuming zero interest rate risk.
- Current net exposure method is a rough approximation of this with just one CDS contract.

As such, there are a number of disadvantages:

- Zero volatility approach, so similar problems to the discount curve shift approach — neglects value from interest rate volatility, which can be substantial.
- Even less accurate, in that only one CDS contract is used.
- Method tends to be unstable — value is proportional to market value, which has much higher volatility than the true CVA.

Bilateral CVA

In accounting circles, one often finds support for bilateral CVA calculations.

- Unilateral CVA — value of contract taking into account default of counterparty. What we’ve discussed up until now.
- Bilateral CVA — value of contract taking into account both default of counterparty and default of investor.

Bilateral CVA is a complicated calculation.

- Need to know relationship between default of counterparty and default of investor.
- Often approximated as difference in unilateral CVA of investor and counterparty.
- Approximation is true bilateral CVA assuming the probability of both parties defaulting before contract maturity is zero.
- Can lead to significant error with high default correlation.
Bilateral CVA

Bilateral CVA is often looked upon favorably.

- Reduces CVA charge.
- Liabilities behave more like bond liabilities - a drop in credit of the investor will potentially improve the balance sheet.

It also has some drawbacks.

- If investor’s credit is worse than counterparties, bilateral CVA increases value of the derivative above the risk free value!
- All derivatives (even assets) increase in value when credit rating drops.
- Prices derivative without an associated hedge.

Because of these issues, accounting boards have been lobbied to reject bilateral CVA as an acceptable approach. We can hope that these issues will be addressed as IASB, FASB and other accounting standards boards work together on global convergence of accounting standards.

Summary

- It’s important to factor credit risk into valuations.
- Netting agreements and collateralization must be accounted for.
- The prices of risky bonds can be calculated based on CDS spreads.
- Bond calculations roughly correspond to just shifting the discount curve by an appropriate amount.
- Swap credit risk is more complicated because swaps can be assets or liabilities, depending on rates.
- For swaps, the discount curve shift is sometimes used, but it is not very accurate.
- The net current exposure method is used as well, but has similar shortcomings.
- Swap credit risk can be calculated using swaptions and CDS rates.
- The CVA is the value of the default leg in a CCDS on the underlying swap.
- Portfolio CVA calculations are not so easy...

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