Pros and Cons of Different CLO Models

Every firm that holds a portfolio of illiquid collateralized loan obligations (CLOs) must figure out the best way to price these structured products. Both stochastic and static default models can be used to value CLOs, but what are the respective strengths and weaknesses of these approaches?

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Thanks to a lack of liquidity in the secondary markets, many companies have been left holding a portfolio of structured products, including collateralized loan obligations (CLOs). Decreased liquidity, combined with recent accounting rule changes, has prompted a number of companies to develop models to help them determine the fair value of these securities.

These models are highly dependent on inputs, which can be quite difficult to determine, as many are not readily observable in the market and are prospective in nature. Additionally, there are a number of modeling techniques available for valuing these securities, and choices must be made based on the strengths and weaknesses of each model, familiarity with the credit behavior of the collateral and the overall sophistication of the valuation group.

This article will demonstrate how CLO securities can be priced using both static and stochastic modeling approaches, contrast the results of these two common CLO valuation methodologies and discuss their respective limitations.

CLO Market Overview

A CLO is a special purpose vehicle (SPV) that issues debt and equity commonly collateralized by bank loans. Unlike mortgage-backed securities (MBS), a collateral manager actively manages the collateral pool in a CLO during what is referred to as the revolving period, typically ranging from five to seven years, after which the principal paydown of the liability structure begins.

To value CLO securities, market participants commonly use a discounted cash flow approach. Given the potential for diversity in the collateral pool, varying collateral contractual cash flow terms, complex credit and prepayment forecasting (as well as structures that revolve), this valuation approach can be complex. Further, the model also needs to account for deal-specific payment rules, excess interest tests, subordination tests and credit trigger events dynamically.

The more common method for modeling these transactions is a static approach where a single best estimate of collateral cash flows, prepayments and defaults is made. An alternative method is to model the cash flows for these securities using a stochastic default model that involves Monte Carlo simulations that generate multiple default scenarios by modeling the default behavior of the underlying collateral.

CLO Structures

As mentioned earlier, a collateral manager actively manages a CLO’s collateral pool by purchasing and selling loans as permitted in the deal’s indenture. Collateral loans may consist of term loans, revolving credit agreements, acquisition or equipment lines, bridge loans, second-lien loans and covenant-lite loans. Some CLO structures allow the collateral manager to...
invest in other CLO securities, resulting in the notations “CLO Squared” securities.

CLO securities have varying capital structures, and generally feature debt that is tranched using seniority related to receiving interest and principal payments and the distribution of losses. A typical CLO structure may have super senior, senior, mezzanine, and subordinate bonds, as shown in Figure 1 (right). Each CLO features payment rules that specify how collateral interest and principal cash flows are distributed to the securities. Additionally, CLOs have a revolving period during which principal cash flows from the collateral are reinvested rather than paid out to the securities. During this reinvestment period, the collateral manager purchases new bank loans or other eligible investments. The CLO structure in Figure 1 contains a pro rata split for the super senior bonds. All collateral principal is first paid to the super senior bonds, followed by the senior, mezzanine, and subordinate bonds, forming the appropriately termed “waterfall.” The credit ratings and ratings of these securities commonly parallel this waterfall, respectively running from high to low ratings and from short to long durations.

Collateral losses of interest or principal are typically first absorbed by excess spread, then by the lowest rated bond and then by more senior bonds, in reverse order of payment seniority. Collateral losses can potentially reach the senior bonds, depending on the loss levels incurred by the collateral pool. In addition, CLO structures typically have interest and principal credit enhancement coverage tests to further protect senior bonds from losses. When any of these tests fail below predetermined levels, collateral interest and principal cash flows are commonly redirected to the most senior securities. The defaulting bond’s interest and principal cash flows are allocated to the various liabilities making up the capital structure and the fee/expense requirements. The resulting projection of cash flows for each security is then discounted using a risk adjusted discount rate to determine fair value. This valuation process is summarized in Figure 2 (below).

Figure 1: Cash Flow Structure of a Typical CLO

<table>
<thead>
<tr>
<th>CF from Interest</th>
<th>CF from Principal</th>
<th>CF from Prepayment</th>
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<tbody>
<tr>
<td>Step #1: Inputs &amp; Assumptions</td>
<td>Step #2: Collateral Cash Flows</td>
<td>Step #3: Bond Cash Flows</td>
</tr>
<tr>
<td>Collateral cash flows are generated by making assumptions on prepayments, defaults, amortization, downgrades, and upgrades.</td>
<td>Collateral cash flows are paid to the bonds according to the contractual cash flow structure after paying CLO fees and expenses.</td>
<td>The net present value of interest cash flows is taken. For stochastic pricing, an average of the simulations is taken.</td>
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Below, we provide a more thorough explanation for each of these important steps.

Step #1: Inputs & Assumptions. Initially the model needs to be set up to account for all deal-specific inputs. The interest rate term structure needs to be estimated to project interest yield for the floating rate loans and securities, as well as the applicable reinvestment income. All fees and expenses must be accounted for because they reduce available collateral cash flows. Also, valuation dates, current security balances and collateral amortization terms need to be updated to reflect amortization.

Step #2: Collateral Cash Flows. The model should attempt to amortize each loan according to its contractual terms, but adjust the amortization for projected voluntary prepayments, defaults, discount recoveries, interest, and reinvestment income. Per-payments and defaults are usually specified as constant prepayment rate (CPR) curves and constant default rate (CDR) curves, respectively. These curves are a vector of percentages that indicate the percentage of current collateral expected to prepay or default in that period. The percentages are specified as a yearly amount and must be converted to a periodic equivalent below being applied to the collateral balance. The recovery assumption determines the amount of principal recovered from defaulted loans and the timing of those recoveries. The interest rate must also make additional assumptions on the type of collateral that will be purchased during the revolving period.

Step #3: Bond Cash Flows. The model needs to allocate accurately the projected collateral cash flows to the liabilities based on the specified payment rules, trigger events and fees/expenses, as detailed in the deal indenture. Structures vary from one CLO to the next, resulting in differences in trancheing and payment rules.

Step #4: Bond Analytics. Once security cash flows are projected as a result of analytics, they can be calculated such as duration, weighted-average life, convexity, yield, discount margin and (ultimately) fair value. To calculate the appropriate fair value or net present value, an appropriate discount rate/rate of return must be selected. A discount rate should account for the risk-free rate, credit risk, model risk and liquidity premium. Monte Carlo simulated values, the average of all prices is taken.

Default Modeling. Default and recovery in a CLO valuation can be modeled as a constant default rate (CDR) or modeled stochastically using advanced mathematical tools. In the case of static default (CDR) modeling, a certain percentage of the outstanding collateral balance is assumed to default in each period. The default assumption is specified as a vector of yearly default rates called a CDR vector, which can be easily converted into a desired periodic rate.

However, defaulting a portion of a loan is not consistent with real-world behavior, so a default is either entirely or not at all (a binary event). Stochastic default modeling tries to incorporate this reality by modeling the approximate time that a loan defaults.

Simulation is the process by which one can generate possible outcomes of a random event. For example, if one wants to simulate throwing a die, one would first need the probabilities of obtaining the various possible outcomes one through six. One could then simulate the outcomes using a random number generator that produces one of the six numbers for each roll, such that the probabilities are maintained, i.e., after a number of rolls, on average one expects to get each number one-sixth of the time. Similarly, given a probability of default curve that specifies probabilities of a loan defaulting over time, one could simulate the time to default using a random number generator, such that the probability distribution is maintained.

In order to simulate the time to default for each loan, the probability of default (P) for each loan is determined (1) a default curve for each loan, (2) a correlation structure; and (3) a process for generating correlated time to default for each loan. Most of the stochastic default modeling techniques discussed in this article can be found in a key paper authored by David X. Li.

Since the credit crisis began to unfold, there have been a number of criticisms hurled at this stochastic model. Some have even blamed the model as the cause of the credit crisis. However, though this method has serious limitations, it has, until recently, been the most widely used model for pricing CDOs, and it is therefore worth examining the construction and shortcomings of this model.

Probability of Default Curves (Credit Curves). The probability of a default curve is one of the key inputs into the default model. It specifies the cumulative probability of default over time for a loan. It is general industry practice to vary the probability of default based on the underlying loan credit quality. The expectation would be that higher quality loans would have a lower probability of default over time.

Default curves can be derived using a number of methods. For example, Moody’s has a table of historic probability of default curves based on credit ratings. Alternatively, default curves can be generated from market information, such as bond spreads or credit-default swap (CDS) spreads.

One can also adjust historic probability curves using market covenants, such as the North American Loan Credit
A hypothetical loan portfolio is used to demonstrate the differences between static and Monte Carlo valuations. The hypothetical portfolio consists of 10 B2-rated loans with varying maturity dates, equal balances, equal coupon spreads, an assumed recovery rate of 50 percent and an assumed prepayment rate of 10 percent CPR.

The CLO structure consists of five CLO bonds with a parity ratio of 110 percent. Figure 1 summarizes the CLO structure used in this example. We further assumed that the correlation between any two loans is 0.75, and used Moody’s historic default probability curve for B2-rated loans.

For static modeling, we examined the impact of different CDR assumptions on the CLO securities. We assumed a constant CDR rate for the life of the transaction and examined the effect of increasing the CDR on security losses and resulting prices.

Figure 4 illustrates the impact of different CDRs on CLO bond losses. It is noticeable that as the CDR increases, losses are absorbed by the lowest bond first before working their way up the capital structure. Note that the bonds do not experience losses immediately as a result of overcollateralization (i.e., a 110 percent parity ratio) and excess spread. The subordinate bond experiences its first losses when the CDR exceeds 8 percent annually.

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The remainder of the valuation process is similar to the static approach. This process results in a one-price outcome from a number of possible outcomes. In order to value the security, one would have to repeat the process a number of times to derive a distribution of possible prices for the security. The steps in the Monte Carlo simulation are summarized below:

1. Generate random numbers from a correlated standard normal variable $Z = MVN(0, \Sigma)$ using a random number generator.
2. Map $Z$ to a cumulative probability $t = \Phi^{-1}(Z)$ for each of the N assets.
3. Map the probability to an appropriate time-until-default as $t = F^{-1}(t)$ for each of the N assets.
4. Project cash flows for loans and securities using assumptions and in accordance with waterfall rules.
5. Obtain a price for the security based on projected cash flows and an appropriate discount rate.
6. Repeat above steps multiple times to generate a distribution of prices.

Figure 5 shows how lower CDR impacts the pricing of the various bonds. Prices were calculated by matching a discount margin to each bond’s accruing spread. As CDRs increase, triggers are breached leading to cash flows being redirected from subordinate bonds to senior bonds. This figure also highlights that even at a 100 percent CDR, the super senior and senior bond will retain some value, which is due to cash flows from recoveries.
For Monte Carlo simulation, we applied Monte Carlo analysis to our hypothetical deal, using 100 default curve iterations. The distribution of losses impacting each bond is shown in Figure 7 (below). We can see that the super senior bond rarely faces severe losses in the majority of the 100 scenarios. The expected loss is the average of the 100 scenarios.

As with the static CDR approach, bond experience losses according to seniority. The key difference is that stochastic modeling creates various loss scenarios while the static default modeling creates only one outcome.

Figure 8 (right) shows the pricing distribution for the stochastic valuation approach. As with the static CDR pricing example earlier, discount margins were matched with accrual spreads. The final bond price is the average of the 100 pricing outcomes. Again, it is noticeable that triggers protect the bonds higher up in the capital structure and that recoveries also cause super senior bonds to retain value even in severe loss scenarios.

A benefit of using a stochastic pricing approach is that it presents a picture of possible prices at different loss severity outcomes. A static approach, where we are using the best estimate CDR curve to forecasts future collateral losses, would present a picture of possible prices at different loss severity scenarios.

The stochastic and static default models both fail to address complex counterparty loan features, such as loan conversion options, refinancing options, facility extension options, potential interest rate increases in the event of delinquencies and other drivers of collateral behavior. These features have always required a more robust default modeling approach to credit analysis or more robust default modeling coefficients developed through rigorous regression analysis utilizing full cycle data sets.

The stochastic model has also proven ineffective in hedging, because it has been unable to capture large movements in price consistently. The rating downgrade of General Motors’ debt in May 2010 is one example on the ineffectiveness of delta hedging using this model.

Model Limitations
The copula-driven stochastic default model described in this article makes certain simplifying assumptions regarding the behavior of the underlying loans. It assumes that any two loans in the portfolio have a fixed correlation coefficient between them. However, default correlations—which in essence measure how two companies are likely to fail at the same time—are time varying in nature. Additionally, the assumption of a Gaussian Copula structure assigns low probabilities to extreme outcomes. Over the past few years, market participants have moved toward a single-factor stochastic default model that assigns a single correlation number between all assets in the collateral pool, partly because this model made it easy to back out an implied correlation from market prices found in, for example, CDS trades (which further translated into a simplified explanation of risk and pricing). This fueled the acceptance of single-factor copula model as a market standard, it allowed more people to trade these securities and it made structuring new CLO transactions— including complex instruments like CLO/CDO squares—convenient. However, as can be implied from the epic failures of the credit crisis, people did not fully consider or understand the limitations of this model completely.

As a consequence of an extended period of benign credit performance and vast amounts of global liquidity, CLO credit loss models were calibrated to an overwhelming amount of positive economic data, resulting in low credit loss expectations and lower probabilities of extreme events. Subsequently, capital markets rewarded CLO collateral pool diversification accomplished through geography and industries (and even through various cash backed issuers), resulting in thinly capitalized CLO structures and overvalued securities.

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Closing Thoughts
In spite of the limitations associated with the models described in this article, they are still commonly used for CLO valuations. Using a static CDR curve for CLO valuations is more commonly used by market participants for fair value accounting purposes, due to its more simplistic approach and easier interpretation of results. Research is usually available to calibrate market inputs such as losses and prepayment rates.

The stochastic modeling approach requires additional effort in setting up the model and determining the appropriate probability of default curves, correlation matrices and discounting spreads. Stochastic models are more effective when one can regularly calculate calibrated prices to market prices. Stochastic modeling thus provides in reasonable valuations when assumptions are constructed appropriately based on contemporaneous market consensus on credit, careful monitoring of both the collateral pool and CLO manager’s performances, corroboration (using quality industry research) and price comparisons (using relevant trade proxies from both the secondary and primary markets).

One advantage of stochastic modeling is the consideration for distribution of outcomes. Stochastic modeling makes it possible to analyze the distribution of possible prices, as opposed to a static valuation that produces a single outcome. Unlike simpler instruments such as interest-rate swaps, CLOs have complicated structures that include dynamic rules for allocating cash and losses to the bonds, which can have a significant impact on both duration and pricing.

For example, a slight increase in losses could trip a trigger, thereby cutting off cash flows to a particular bond or reducing the life of another bond. A static valuation with a single outcome would not capture these nuances, but a stochastic model would be able to capture a distribution of possible loss outcomes and the implications that loan loss scenarios would have on the respective bonds in the capital structure.

There is a popular saying in statistics, “Essentially, all models are wrong, but some are useful.” This should be kept in mind when interpreting results from any model. However, when faced with the task of choosing a model, one has to weigh complexity and cost, and select the most appropriate model for the application at hand while being aware of the limitations a model presents to the user. That said, investors in structured collateralized products like CLOs can and should supplement credit loss modeling with sound, fundamental credit analysis and careful input from a collateral manager.

FOOTNOTES
4. A standard normal distribution is a mean zero and variance of one.
7. George E.P. Box, a famous British statistician, is thought to be the original source of this saying.

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