Evolution of Portfolio Construction Methodologies

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‘The Quantitative Revolution and the Crisis: How Have Quantitative Financial Models Been Used and Misused?’

‘Those who are responsible, and who gain the greatest rewards from their use, point to the models as justification for risk-taking that on occasion far exceeds what judgment and experience would indicate.’

‘The power of financial innovations and of quantitative models in particular, is unprecedented. In wise hands they enable a deeper, more farseeing grasp of how complex realities will evolve. The poor use or misuse of quantitative models can have catastrophic results.’
Purpose

- Disabuse below notion

DOGBERT THE FINANCIAL ADVISOR

YOU SHOULD INVEST ALL OF YOUR MONEY IN DISEASED LIVESTOCK.

IT WOULD BE UNWISE TO INVEST IN JUST ONE SICK COW, BUT IF YOU AGGREGATE A BUNCH OF THEM TOGETHER, THE RISK GOES AWAY.

IT'S CALLED MATH.

SUDDENLY I FEEL ALL SAVVY.

- Focus on the Intuition
  - Math-lite, Understanding-heavy
Agenda

- The Portfolio Optimization Process
- Mean Variance Optimization and Beyond
  - Mean Variance – Key Issues
  - Improvisations
- Black-Litterman and Beyond
  - Intuition & Drawbacks
- Risk Based Portfolio Construction
  - Equal Risk Contribution, Minimum Variance & Risk Parity
Section 1
THE PORTFOLIO OPTIMIZATION PROCESS
The Portfolio Optimization Process

- **1950s, Nobel laureate Harry Markowitz**
  - Modern Portfolio Theory

- **Step 1 – Specify Assets**
  - Effective for Strategic Asset Allocation

- **Step 2 – Model forward-looking assumptions**
  - Estimate Required Inputs: Expected Returns, Risk & Correlations

- **Step 3 – Specify Optimization Algorithm**
  - Risk-Return problem: Mean-Variance, Mean-Downside Risk, Excess Return-Tracking Error
Traditional Mean Variance Optimization Issues

- **Asset Class Returns Normally Distributed**
  - ‘Real world’ Non-Normality
  - Asymmetric Return Distributions
  - Ignoring Skewness and Kurtosis may create riskier portfolios
  - *Alternative – Fat-Tailed Distributions*

- **Linear Co-Variation of Asset class Returns**
  - Assumes relationship consistent across entire spectrum of Return Distribution
  - Inadequate during times of Financial Stress
  - *Alternative – Scenario or Simulation based approach*
Traditional Mean Variance Optimization Issues

- Volatility as the sole measure of Risk
  - Does not differentiate between gains & losses
  - *Alternative* – Incorporate Downside Risk Measures e.g. Downside Deviation, CVaR

- Single-period Optimization Model
  - Arithmetic Expected Return used as measure of Reward
  - *Alternative* – Use Geometric Expected Return

- Asset Class returns – Tame or Wild Randomness
  - Lognormal distribution assumes Tame Randomness
  - *Alternative* – Use Parametric Distributions that can accommodate Wild Randomness
Bootstrapping

- **Joint behavior of Assets**
  - Parametric return distributions based on Model assumptions
  - Robust estimation of highly skewed and fat-tailed distribution is difficult
  - Multiple peaks, Non-linear Co-Variation not captured

- **Bootstrapping**
  - Replacement based Sampling method
  - Asset class relationships preserved
  - Assign probabilities for particular sub-periods
Bootstrapping

Kernel Density Estimate: $\alpha$
Simple Non-parametric Bootstrap

Kernel Density Estimate: $r_{\text{min}}$
Simple Non-parametric Bootstrap

Positive tail

Negative tail

Positive tail

Negative tail
Section 2
MEAN VARIANCE OPTIMIZATION AND BEYOND
The Modern Portfolio Theory

- Optimal set of portfolio weights

  - Select \( \{w_i\} \) so as to minimize:
    \[
    \sigma_p^2 = [w_1^2 \sigma_1^2 + \ldots + w_n^2 \sigma_n^2] + [2w_1 w_2 \sigma_1 \sigma_2 \rho_{1,2} + \ldots + 2w_{n-1} w_n \sigma_{n-1} \sigma_n \rho_{n-1,n}]
    \]
    subject to:  
    (i) \( E(R_p) = R^* \)
    
    (ii) \( \sum w_i = 1 \)

  - \( R^* \) - investor’s return goal

  - \( \sum w_i = 1 \): Weights add to 100% (Short-Selling allowed)

  - Other constraints that can be added include: (i) All \( w_i \geq 0 \) (i.e., no short selling), or (ii) All \( w_i \leq P \), where \( P \) is a fixed percentage
Mean Variance Optimization (MVO) Issues

- **Concentrated Asset Allocation**
  - Extreme portfolios
  - Easy to produce unrealistic leveraged positions

- **Stability issues**
  - Sensitivity to small changes in inputs
  - Tendency to maximize errors in input assumptions

- **Not intuitive**
  - Cannot incorporate Investor views
  - Resulting portfolio may have unappealing Risk Contribution profile
Mean Variance Regularization Techniques

- **Weight Constraints**
  - Any constraint is a cost
  - Implications for the Portfolio Manager

- **Regularization techniques**
  - Resampling methods
  - Shrinkage methods
  - Factor analysis
Re-sampled Efficient Frontier Optimization

- Mean variance optimization treats the inputs as population parameters
  - But they are only sample estimates
  - Estimation errors of the inputs will distort the optimization results

- Most important input in MV optimization is $E(r)$
  - Ziemba & Chopra (1993) show that estimation error in $E(r)$ is 10 times as important as estimation error in $\sigma$, and 20 times as important as estimation error in $\rho$
Re-sampled Efficient Frontier Optimization

- Michaud (1998)
- Re-sampling: Take several efficient portfolio simulations using different parameters for $E(r)$, $\sigma$, and $\rho$ as sensitivity analysis
  - For each level of $E(r)$ or Standard deviation, average the weights of each asset class from different efficient portfolios (that were estimated using different inputs)
  - These can then be integrated into a re-sampled efficient frontier
- Re-sampled efficient frontier tends to be more diversified and more stable over time
Shrinkage Methods


- Covariance matrix estimated from sample may not truly reflect Population Covariance
  - Especially if number of Assets high and number of return observations used low
  - Leads to ‘Error Maximization’

- Shrinkage methods
  - Estimated coefficients from Sample Covariance matrix pulled either Downwards or Upwards
  - Shrinkage of extremes towards the center
Factor Analysis

- Multi-Factor models
  - More flexible and to reduce bias
  - However, ‘Estimation Error’ increases

- Principal Component Analysis (PCA)
  - Convert set of possibly correlated variables to a set of linearly uncorrelated variables
  - Reveals the internal structure of data that best explains Variance

- MSCI Barra, APT
Sample Multi Asset Portfolio

- **Asset Classes**
  - Developed Equities – MSCI World
  - Emerging Equities – MSCI Emerging Markets
  - Global Investment Grade Bonds – BofAML Global Broad Mkt.
  - Global High Yield Bonds – BofAML Global High Yield
  - Cash – Citi Treasury Bill 3 Month

- **Horizon**
  - Jan 1999 – Sep 2012

- **Simple Constraints**
  - No Short selling or leverage
Asset Class Distributions

- Developed Equities
- Global Broad Market Bonds
- Emerging Equities
- Global High Yield Bonds
## Asset Class Correlations

<table>
<thead>
<tr>
<th></th>
<th>Developed Equities</th>
<th>Emerging Equities</th>
<th>Global Broad Bonds</th>
<th>Global High Yield</th>
<th>Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Developed Equities</strong></td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Emerging Equities</strong></td>
<td>0.87</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Global Broad Bonds</strong></td>
<td>0.26</td>
<td>0.23</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Global High Yield</strong></td>
<td>0.72</td>
<td>0.72</td>
<td>0.34</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td><strong>Cash</strong></td>
<td>-0.05</td>
<td>-0.02</td>
<td>-0.08</td>
<td>-0.16</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Asset Class Characteristics

**Top Diagram**
- **World Equity**: 16.5%
- **Emerging Equity**: 24.4%
- **Global Corporate Bond**: 14.2%
- **Global HY Bond**: 5.8%
- **Cash USD**: 2.4%

Legend:
- **Arith. Mean**
- **Ann Volatility**

**Bottom Diagram**
- **World Equity**: 54.0%
- **Emerging Equity**: 54.3%
- **Global Corporate Bond**: 61.6%
- **Global HY Bond**: 20.8%
- **Cash USD**: 34.2%

Legend:
- **Max. Drawdown**
- **Best 12 Months**
Efficient Frontier

Source: Morningstar Direct
Efficient Frontier Allocation

![Bar Chart]

- **World Equity**: 0.0% (Return 8% Normal MVO), 0.0% (Return 8% Resampled MVO)
- **Emerging Equity**: 25.9% (Return 8% Normal MVO), 21.4% (Return 8% Resampled MVO)
- **Corporate Bond**: 70.4% (Return 8% Normal MVO), 56.7% (Return 8% Resampled MVO)
- **Global HY Bond**: 3.7% (Return 8% Normal MVO), 20.7% (Return 8% Resampled MVO)
- **Cash USD**: 0.0% (Return 8% Normal MVO), 1.1% (Return 8% Resampled MVO)
Allocation Weights – Normal vs. Resampled

Source: Morningstar Direct
Ex-Post Portfolio Analytics

### Comparative Performance

<table>
<thead>
<tr>
<th>Year</th>
<th>Normal MVO</th>
<th>Resampled MVO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec-98</td>
<td>100%</td>
<td>99%</td>
</tr>
<tr>
<td>Apr-99</td>
<td>102%</td>
<td>101%</td>
</tr>
<tr>
<td>Aug-99</td>
<td>105%</td>
<td>104%</td>
</tr>
<tr>
<td>Dec-99</td>
<td>107%</td>
<td>106%</td>
</tr>
<tr>
<td>Apr-00</td>
<td>108%</td>
<td>107%</td>
</tr>
<tr>
<td>Aug-00</td>
<td>109%</td>
<td>108%</td>
</tr>
<tr>
<td>Dec-00</td>
<td>111%</td>
<td>110%</td>
</tr>
<tr>
<td>Apr-01</td>
<td>112%</td>
<td>111%</td>
</tr>
<tr>
<td>Aug-01</td>
<td>114%</td>
<td>113%</td>
</tr>
<tr>
<td>Dec-01</td>
<td>115%</td>
<td>114%</td>
</tr>
<tr>
<td>Apr-02</td>
<td>116%</td>
<td>115%</td>
</tr>
<tr>
<td>Aug-02</td>
<td>117%</td>
<td>116%</td>
</tr>
<tr>
<td>Dec-02</td>
<td>118%</td>
<td>117%</td>
</tr>
<tr>
<td>Apr-03</td>
<td>119%</td>
<td>118%</td>
</tr>
<tr>
<td>Aug-03</td>
<td>120%</td>
<td>119%</td>
</tr>
<tr>
<td>Dec-03</td>
<td>121%</td>
<td>120%</td>
</tr>
</tbody>
</table>

### Portfolio Analytics

|-----------|------------|----------------|--------------|---------------|---------------|----------------|----------------|----------------|
Sensitivity to Inputs

**Returns**
- Cash USD
  - Exp. Return: 0.0%
  - Hist. Return: 4.1%
- Global HY Bond
  - Exp. Return: 4.8%
  - Hist. Return: 5.8%
- Global Corporate Bond
  - Exp. Return: 12.5%
  - Hist. Return: 14.2%
- Emerging Equity
  - Exp. Return: 5.1%
  - Hist. Return: 14.2%
- World Equity
  - Exp. Return: 5.1%
  - Hist. Return: 4.1%

**Volatility**
- Cash USD
  - Exp. Vol: 0.0%
  - Hist. Vol: 1.1%
- Global HY Bond
  - Exp. Vol: 6.5%
  - Hist. Vol: 5.8%
- Global Corporate Bond
  - Exp. Vol: 15.0%
  - Hist. Vol: 15.0%
- Emerging Equity
  - Exp. Vol: 30.0%
  - Hist. Vol: 30.0%
- World Equity
  - Exp. Vol: 30.0%
  - Hist. Vol: 30.0%

**Portfolio Weights**
- Cash USD
  - New Inputs: 3.1%
  - Old Inputs: 1.1%
- Global HY Bond
  - New Inputs: 52.1%
  - Old Inputs: 20.7%
- Global Corporate Bond
  - New Inputs: 56.7%
  - Old Inputs: 56.7%
- Emerging Equity
  - New Inputs: 22.1%
  - Old Inputs: 21.4%
- World Equity
  - New Inputs: 0.0%
  - Old Inputs: 0.0%
Full Scale Optimization

- Mean Variance assumptions
  - Normally distributed returns
  - Investor preferences well approximated by Returns and Variance
  - Yields approximate In-Sample Optimal portfolio

- Full Scale Optimization
  - Adler, Kritzman (2007)
    - Accommodates any type of return distribution
    - Any description of Investor preferences
    - Yields ‘Truly’ In-Sample Optimal portfolio
Section 3
BLACK-LITTERMAN AND BEYOND
The Black-Litterman Approach

- Goldman Sachs, 1990s
- Reverse Optimization to determine Implied Returns
- If Global Capital Markets in Equilibrium, then prevailing market caps suggest investment weights of an efficient portfolio with highest Sharpe
- Uses Bayesian inference to blend a probability distribution for $\mu$ (Expected Returns) with a distribution for $\mu$ which is derived from investor views
- Used as inputs for Mean Variance Optimization process
Intuition behind Black-Litterman

- If Global Capital Markets in equilibrium
  - Efficient portfolio – weights equal current Market Cap
  - Disequilibrium creates Excess return opportunities

- Investor can incorporate Tactical Views
  - Absolute or Relative

- Investment Weights + Asset class volatility and correlations result in Asset class specific Risk Premia & Expected Returns

- Final weights depends on the Confidence level of those views
Black-Litterman Model Mathematical Representation

Risk Aversion Coefficient
- $\delta = \frac{(E(r) - r_f)}{\sigma^2}$

Covariance Matrix
- $\Sigma$

Market Cap. Weights
- $w_{mkt}$

$\tau_p$: Uncertainty of the CAPM Distribution (0.025-0.05)

Views
- (P) – Investor Views
- (Q) – Exp. Returns

Uncertainty of Views
- $(\Omega)$

Implied return vector
- $\Pi = \delta \Sigma w_{mkt}$
# Black-Litterman Model Portfolio

## No Views

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Market Cap ($US Billions)</th>
<th>Implied Ret</th>
<th>BL Return</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>World Equity</td>
<td>25,080</td>
<td>6.1%</td>
<td>6.1%</td>
<td>15.1%</td>
</tr>
<tr>
<td>Emerging Equity</td>
<td>10,000</td>
<td>9.4%</td>
<td>9.4%</td>
<td>24.4%</td>
</tr>
<tr>
<td>Global Corporate Bond</td>
<td>2,500</td>
<td>0.8%</td>
<td>0.8%</td>
<td>5.8%</td>
</tr>
<tr>
<td>Global HY Bond</td>
<td>1,000</td>
<td>3.4%</td>
<td>3.4%</td>
<td>10.8%</td>
</tr>
<tr>
<td>Cash USD</td>
<td>10,000</td>
<td>0.2%</td>
<td>0.2%</td>
<td>1.0%</td>
</tr>
</tbody>
</table>

## MVO Asset Allocation for 8% Expected Return

<table>
<thead>
<tr>
<th>BL - No Views</th>
<th>MSCI World</th>
<th>MSCI EM</th>
<th>BofAML Global Broad Market</th>
<th>BofAML Global High Yield</th>
<th>Citi Treasury Bill 3 Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>8% Resampled</td>
<td>21.6%</td>
<td>68.9%</td>
<td>1.7%</td>
<td>5.5%</td>
<td>2.4%</td>
</tr>
</tbody>
</table>
Ex-Post Portfolio Analytics

### Comparative Performance

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>BL-No Views</td>
<td>9.85%</td>
<td>20.43%</td>
<td>0.36</td>
<td>0.46</td>
<td>56.14%</td>
<td>21.63%</td>
<td>78.78%</td>
<td>-50.35%</td>
</tr>
</tbody>
</table>
Sensitivity to Views

- View 1: Global Corp. Bonds OP Emg. Eq. by 1% with **50% Confidence**
- View 2: Global Corp. Bonds OP Emg. Eq. by 1% with **90% Confidence**
- View 3: Global Corp. Bonds **UP** Emg. Eq. by 1% with **90% Confidence**
Black-Litterman Issues

- Same issue as with Mean Variance
  - Cannot deal with higher moments
  - Potentially unstable results

- Arbitrary parameter $\tau_p$, and parameters $\Omega$ which are hard to determine

- Expressing views casually is dangerous and may lead to model risk

- Bayesian mechanism does not work properly
Section 4
RISK BASED PORTFOLIO CONSTRUCTION
Typical Balanced Portfolio

Weight:
- Commodity: 5%
- Real Estate: 10%
- Global Equities: 35%
- Global Bonds: 50%

Contribution to Risk:
- Commodity: 4.10%
- Real Estate: 8.50%
- Global Equities: 74.60%
- Global Bonds: 12.80%
Generic Risk Budgeting Approach

- Post Credit Crisis, focus back on Risk Management
  - Lower risk tolerance of investors
  - Risk On – Risk Off

- Risk Contribution from each component is equal to the set Risk Budget

- Different Risk approach
  - In MVO, risk approach is marginal; Marginal Volatility is important
  - In Risk Budgeting, risk approach is global; Mixing of Marginal Volatility and portfolio Weights
Risk Based Asset Allocation Approach

- **Focus on Risk & Diversification**
  - Do not have to estimate Expected Returns

- **Risk based strategies**
  - Equal Weighted
  - Global Minimum Variance
  - Most Diversified Portfolio
  - Equal Risk Contribution

- **Historical Studies**
  - Outperforms on Risk Adjusted basis both Market Cap. Weighted BMs and ex-ante MVO Portfolios; In some cases, also outperforms on an Absolute basis
Equal Weighted

- Simplest approach – Naïve diversification
  - For n assets, weight equals 1/n; Completely ignores assets being invested in

- Most Heuristic portfolio
  - Completely ignores return and risk prospects

- Not a great asset allocation strategy
  - Highly sensitive to universe of assets under consideration
  - BM to assess performance of other portfolio allocation rules?
Global Minimum Variance

- Portfolio of ‘risky’ assets with the lowest possible Volatility
  - Can be uniquely determined just with a Covariance matrix

- Portfolio studies
  - Engineered Ex-ante to deliver lowest volatility, has higher return than the EF Market Portfolio
  - High sensitivity to frequency of rebalancing and weight constraints imposed

- Leads to concentrated portfolios
  - Not diversified from a portfolio weighting perspective
Global Minimum Variance Portfolio

No Constraints
- Cash: 98.0%
- Global High Yield: 0.9%
- Global Broad Bonds: 1.1%
- Emerging Equities: 0.0%
- Developed Equities: 0.0%

Cash < 5%
- Cash: 5.0%
- Global High Yield: 10.8%
- Global Broad Bonds: 84.2%
- Emerging Equities: 0.0%
- Developed Equities: 0.0%

Cash < 5%, Max. 50%
- Cash: 5.0%
- Global High Yield: 45.0%
- Global Broad Bonds: 50.0%
- Emerging Equities: 0.0%
- Developed Equities: 0.0%
Most Diversified Portfolio

- Choueifaty and Coignard (2008)
  - Introduced ‘Maximum Diversification’ Portfolios through a measure called ‘Diversification Ratio’

- Diversification Ratio (DR)
  - Asset’s Weighted average volatility to overall volatility
  - Measures Diversification gained from holding uncorrelated assets
  - Higher the ratio, the more diversified the portfolio
  - DR squared = Independent Risk Factors

- Most Diversified Portfolio maximizes the Diversification Ratio
Equal Risk Contribution

- Maillard, Roncalli, Teiletche (2008)

- Risk Contribution
  - Weight of asset x Marginal Contribution to Risk

- Equal Risk Contribution (ERC) or Risk Parity (RP) portfolio
  - Risk Contribution from each asset in portfolio equal

- Portfolio weights are endogenous in determining asset’s Risk Contribution
  - Possibility of multiple solutions cannot be ruled out
  - Achieving Risk Parity remains heuristic in nature
### MD & ERC Portfolio Weights

<table>
<thead>
<tr>
<th>Developed Equities</th>
<th>Emerging Equities</th>
<th>Global Corporate Bond</th>
<th>Global HY Bond</th>
<th>Cash USD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MDP</strong></td>
<td>0.7%</td>
<td>0.4%</td>
<td>6.0%</td>
<td>2.5%</td>
</tr>
<tr>
<td><strong>ERC</strong></td>
<td>2.9%</td>
<td>2.0%</td>
<td>8.3%</td>
<td>4.4%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constraints (Cash &lt; 5%) - Asset Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MDP</strong></td>
</tr>
<tr>
<td><strong>ERC</strong></td>
</tr>
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</table>
## Comparative Performance

<table>
<thead>
<tr>
<th></th>
<th>Resampled MVO</th>
<th>BL-No Views</th>
<th>GMV-Cons</th>
<th>MDP-Cons</th>
<th>ERC-Cons</th>
<th>EQ. Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ann Return</strong></td>
<td>8.56%</td>
<td>9.85%</td>
<td>5.96%</td>
<td>6.84%</td>
<td>7.08%</td>
<td>7.15%</td>
</tr>
<tr>
<td><strong>Ann Volatility</strong></td>
<td>8.44%</td>
<td>20.43%</td>
<td>5.42%</td>
<td>6.53%</td>
<td>8.18%</td>
<td>9.84%</td>
</tr>
<tr>
<td><strong>Sharpe Ratio</strong></td>
<td>0.72</td>
<td>0.36</td>
<td>0.64</td>
<td>0.66</td>
<td>0.56</td>
<td>0.47</td>
</tr>
<tr>
<td><strong>Sortino Ratio</strong></td>
<td>0.63</td>
<td>0.46</td>
<td>0.25</td>
<td>0.41</td>
<td>0.38</td>
<td>0.34</td>
</tr>
<tr>
<td><strong>Max. Drawdown</strong></td>
<td>21.27%</td>
<td>56.14%</td>
<td>10.82%</td>
<td>15.63%</td>
<td>22.98%</td>
<td>30.28%</td>
</tr>
<tr>
<td><strong>Semi Deviation</strong></td>
<td>8.76%</td>
<td>21.63%</td>
<td>5.56%</td>
<td>7.03%</td>
<td>9.07%</td>
<td>10.66%</td>
</tr>
<tr>
<td><strong>Best 12 Months</strong></td>
<td>42.38%</td>
<td>78.78%</td>
<td>22.45%</td>
<td>29.99%</td>
<td>39.01%</td>
<td>43.55%</td>
</tr>
<tr>
<td><strong>Worst 12 Months</strong></td>
<td>-20.38%</td>
<td>-50.35%</td>
<td>-6.58%</td>
<td>-14.70%</td>
<td>-21.35%</td>
<td>-26.59%</td>
</tr>
</tbody>
</table>
### Ex-Post Portfolio Analytics

- **Different strategy for different market regimes?**

<table>
<thead>
<tr>
<th>Year</th>
<th>Normal MVO</th>
<th>Resampled MVO</th>
<th>BL-No Views</th>
<th>GMV-Cons</th>
<th>MDP-Cons</th>
<th>ERC-Cons</th>
<th>Eq. Wt</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>15.0%</td>
<td>13.0%</td>
<td>51.4%</td>
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Regime Switching Models

- **Risk On – Risk Off**: Most important investment decision
- Dynamic flexibility to Asset Allocation process required
- Regime Switching models seek to forecast changing risk conditions
  - Persistence in Volatility and Volatility clustering, Changing correlations
  - Changes in aggregate equity market returns, treasury bond returns, currency strength, market volatility, equity dividend yield, treasury yield spread, and credit spread
Section 5

SUMMARY
Summary

- Asset Allocation/Portfolio Construction methodologies have been continually evolving over the last 60 years
- Mean-Variance Optimization still quite relevant
- Newer methods aim to improve reliability of inputs
- Risk based methods aim at eliminating unreliable expected return estimates
- Clear objective and constraints extremely important
Thank You!
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